ESTIMATION OF DEVICE CURRENT IN PWM INVERTERS

Ho-Chul Ji
Information and Electronic Research Center
Korean Advanced Institute of Science and Technology
Kusong-dong 373-1, Yusong-ku, Taejon 305-701, Korea
Phone +82-42-869-5463, Fax +82-42-869-3410

Seung-Gi Jeong
Dept. of Electrical Engineering
Kwangwoon University
Wolgye-dong 447-1, Nowon-ku, Seoul 139-701, Korea
Phone +82-2-940-5146, Fax +82-2-943-7195

Abstract — This paper gives an analytical expression of the average and rms currents of switching devices in voltage-fed PWM inverters. It is shown that the device currents are represented by a function of the power factor of the load and the normalized output voltage of the inverter. The validity of the derived formulas is confirmed with simulation and experiment, showing that the modulation method has a minor effect on the characteristics of the device current.

1. INTRODUCTION

Design of a power converter requires determining proper current ratings of power semiconductor devices including maximum rms, average, and peak current with adequate safety margins for right selection of the components [1]. The device current is also important in evaluating conduction loss. The conduction loss affects the efficiency of converters and the effect is more significant in higher capacity converters that have lower switching frequency and higher on-state current. Recent development of high capacity and high speed power semiconductors, and as a consequence the increasing use of high capacity power converters have made the conduction loss to be more important design parameter than ever. More importantly, a reasonable estimation of overall loss of a converter including the conduction loss is indispensable for proper design of cooling system. Therefore, the accurate calculation of device currents should be an important design step for the development of efficient and reliable converter system [1, 2, 4].

To establish device current models in PWM inverters, a computer-aided analysis based on Fourier analysis was carried out in [1], and the analysis based on the simulation results was made in [2]. In particular, in [2], by examining the simulation results it has been found that if the product of the modulation index and the power factor is the same, then the magnitude of each switching device current also remains the same. Simple approximate formulas for the device currents were derived as a function of the modulation index-power factor product. But no analytical proof and/or comment were given.

This paper gives an analytical approach to the derivation of the formulas for device currents in PWM inverters. Some basic observations are made first, and the analytical closed-form expressions for the device currents are derived. Then the theoretical current values calculated with the derived formulas are compared with the results of simulation and experiments.

2. BASIC CONSIDERATIONS

Fig. 1 shows a leg of an inverter circuit composed of two active switches and two diodes. At any instant, the phase current i flows through one of four devices of associated phase leg. Which device carries the current depends on the polarity of the pole voltage, v, and the direction of the phase current, as summarized in Table 1. During the positive half period of the phase current, Q_p and D_n, and during the negative half period, Q_n and D_p conduct, alternatively. In usual steady-state inverter operation, two active switches operate in a conjugate manner so that the instantaneous currents flowing through the devices show symmetry with 180° phase difference, as shown in Fig. 2.

It is readily observed that if the current flows in the direction the pole voltage dictates, positive or negative, it flows through an active switch, Q_p or Q_n. And when the current flow is in an opposite direction to the pole voltage, a diode, D_p or D_n, will conduct. Therefore, when either one of the active switches conducts, the inverter leg delivers power to the load. And when either one of the diodes conducts, the leg draws power from the load.

The negative instantaneous power flow, when aver-

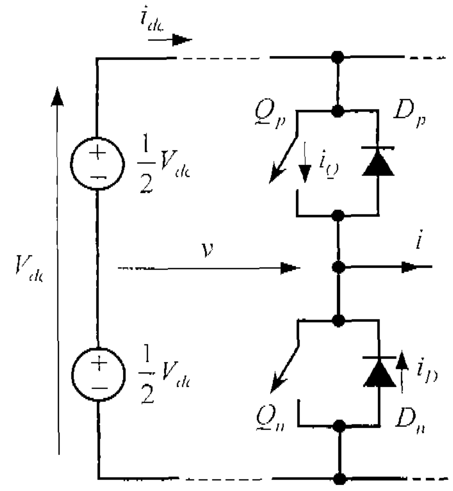
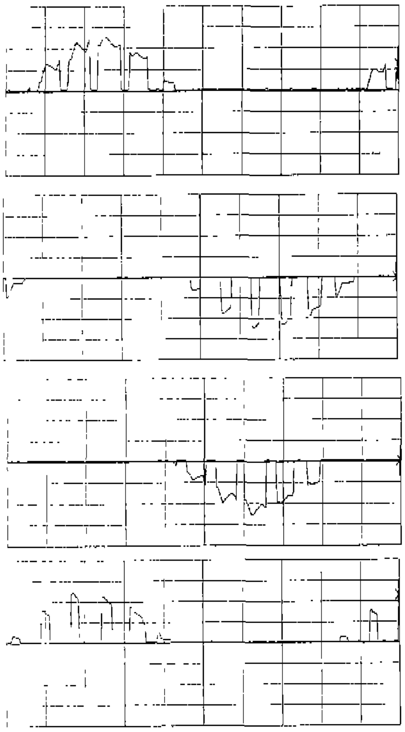


Fig. 1 Voltage-fed inverter and one leg

Table 1. Device currents and the polarity of the current and voltage

Phase Current	Pole voltage	Device currents			
		Q_p	Q_n	D_p	D_n
$i > 0$	$v > 0$	i	0	0	0
	$v < 0$	0	0	0	i
$i < 0$	$v > 0$	0	0	$-i$	0
	$v < 0$	0	$-i$	0	0


(a) Q_p

(b) D_p

(c) Q_n

(d) D_n

Fig. 2 Typical waveforms of switching device currents in a three-phase PWM inverter

aged, reflects the amount of reactive power between the inverter and load. Therefore, as the reactive power increases the diode current that contributes the power flow in the negative direction also increases while the active switch current decreases, and vice versa, for given volt-ampere. This shows a close dependency of the device currents on the power factor of the load.

In examining the characteristics of the device currents in steady state, it is sufficient to consider only a half cycle because of symmetry. Thus the positive half cycle of the output current will be considered in the following and subsequent analysis. For positive output current, the instantaneous pole voltage v will have a positive voltage level ($V_{dc}/2$) when Q_p conducts and a negative voltage level ($-V_{dc}/2$) when D_n conducts. When the power factor angle is 90°, the positive peak of the phase current occurs at the instant where the positive and the negative voltage levels hold for the same duration, so that the phase current is divided in Q_p and D_n, evenly in time. When the power factor is unity, on the other hand, at the positive peak current the duration of the positive level is much longer than that of the negative level, yielding higher active switch current than the diode current. This again confirms the dependency of the device currents on the power factor of the load.

The inverter output voltage also affects the duration of the voltage levels. For given power factor angle and load current level, the duty ratio of the active switch (diode) increases (decreases) as the output voltage increases. Therefore, it can be deduced that the effect of the power factor and the output voltage level in combination will play a key role in determining the device current level. Following section gives an analytical description of the effect.

3. DERIVATION OF DEVICE CURRENT MODEL

For analytical purpose, suppose a naturally sampled pulse-width modulation with sinusoidal reference modulating signal, commonly known as sinusoidal PWM (SPWM). The inverter pole voltage locally averaged over every switching period, $\bar{v}$, follows the reference signal to yield an approximately sinusoidal waveform as shown in Fig. 3. Assuming sufficiently high switching frequency, the phase current is considered to be very close to a sinusoid, lagging behind the associated pole voltage by the angle φ. Let θ_{sw} be the angular switching period (or the period of carrier wave). Then the pole voltage and the phase current at the k-th switching period can be expressed approximately as

$$
\begin{aligned}
v_k &= \hat{V}\sin(k\theta_{sw}) \\
i_k &= \hat{I}\sin(k\theta_{sw} - \varphi)
\end{aligned}
\tag{1}
$$

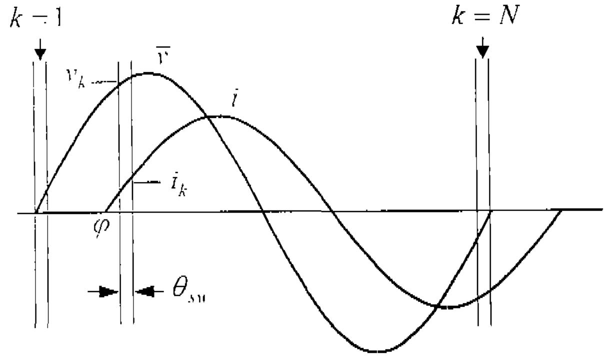


Fig. 3 Local averaged pole voltage and phase current

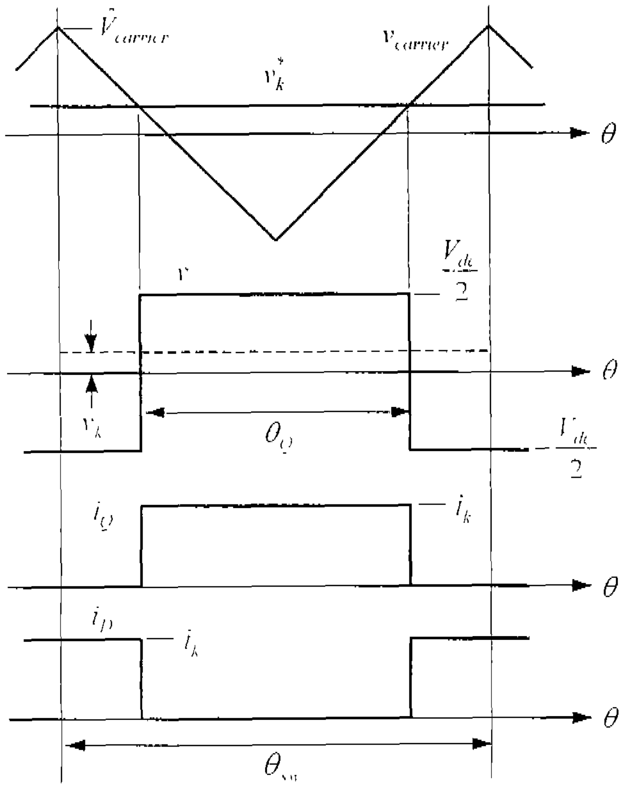


Fig. 4 Pole voltage and device current waveforms over a switching cycle for positive output current

As indicated in Fig. 3, k is measured from the zero crossing of the pole voltage and varies from 1 to N over a cycle where N is the number of switching per cycle, or the frequency modulation index.

Fig. 4 shows the carrier wave and the modulating reference signal, v_k^*, at the k-th switching period, with the waveform of the pole voltage. The active switch current and diode current, i_Q and i_D, indicated in Fig.1, are also shown assuming positive output current. For simplicity, the output current is assumed to be constant over the interval.

The proportionality between the magnitude of the reference and the local average of the pole voltage is given by

$$v_k = \frac{V_{dc}/2}{\hat{V}_{carrier}} v_k^* . \tag{2}$$

The duration of the active switch conduction is related to the reference voltage level as

$$\theta_{Qk} = \frac{\theta_{sw}}{2} + \frac{v_k^*}{\hat{V}_{carrier}} \frac{\theta_{sw}}{2} . \tag{3}$$

Accordingly, the duration of the diode conduction is

$$\theta_{Dk} = \theta_{sw} - \theta_{Qk} = \frac{\theta_{sw}}{2} - \frac{v_k^*}{\hat{V}_{carrier}} \frac{\theta_{sw}}{2} . \tag{4}$$

Over the positive half period of the current, k varies from k_φ to $N/2+k_\varphi$ where $k_\varphi \approx \varphi/\theta_{sw}$. Thus the average of the active switch current is given by

$$\begin{aligned} I_{Q,avg} &= \frac{1}{2\pi} \sum_{k=k_\varphi}^{k=N/2+k_\varphi} \theta_{Qk} i_k \\ &= \frac{1}{4\pi} \sum_{k=k_\varphi}^{k=N/2+k_\varphi} \left[1 + \frac{v_k^*}{\hat{V}_{carrier}} \right] i_k \theta_{sw} \end{aligned} \tag{5}$$

Substituting (1) and (2) into (5) results in

$$I_{Q,avg} = \frac{1}{4\pi} \sum_{k=k_\varphi}^{k=N/2+k_\varphi} \left[1 + 2\frac{\hat{V}}{V_{dc}} \sin k\theta_{sw} \right] \hat{I} \sin(k\theta_{sw} - \varphi)\theta_{sw}$$

If the number of switching over a cycle, N, is sufficiently large, then the above equation can be approximated to an integration by replacing θ_{sw} with $d\theta$ and $k\theta_{sw}$ with θ, as

$$I_{Q,avg} = \frac{\hat{I}}{4\pi} \int_{\varphi}^{\pi+\varphi} \left[1 + 2\frac{\hat{V}}{V_{dc}} \sin\theta \right] \sin(\theta - \varphi) d\theta . \tag{6}$$

The normalized output voltage is defined as

$$m = \frac{\hat{V}}{V_{dc}/2} \tag{7}$$

which is usually referred to the voltage modulation index in SPWM. Then (6) reduces to

$$I_{Q,avg} = \frac{1}{2\pi} \hat{I} + \frac{1}{8} m \cos\varphi\, \hat{I} \tag{8}$$

By normalizing with respect to the fundamental rms phase current,

$$I_{Q,avg,pu} = \frac{I_{Q,avg}}{\hat{I}/\sqrt{2}} = \frac{1}{\sqrt{2}\pi} + \frac{\sqrt{2}}{8} m \cos\varphi \tag{9}$$

or

$$I_{Q,avg,pu} = 0.225 + 0.177\, m \cos\varphi . \tag{10}$$

The above expression shows that the normalized average active switch current is a first order equation of the product of the normalized phase voltage and power factor. The product corresponds to the active component of the inverter output voltage that is in-phase with the output current.

The rms current of an active switch can be obtained in a similar way. The square of the rms current is equated as

$$I^2_{Q,rms} = \frac{1}{2\pi}\sum_{k=k_\varphi}^{N/2+k_\varphi}\theta_{Qk} i_k^2$$

$$= \frac{1}{4\pi}\sum_{k=k_\varphi}^{N/2+k_\varphi}\left[1+\frac{\hat{V}}{\hat{V}_{carrier}}\sin k\theta_{sw}\right]\hat{I}^2\sin^2(k\theta_{sw}-\varphi)\theta_{sw} \tag{11}$$

and is approximated to

$$I^2_{Q,rms} \cong \frac{1}{4\pi}\int_{\varphi}^{\pi+\varphi}\left[1+2\frac{\hat{V}}{V_{dc}}\sin\theta\right]\hat{I}^2\sin^2(\theta-\varphi)\,d\theta \tag{12}$$

that gives

$$I_{Q,rms} = \hat{I}\sqrt{\frac{1}{8}+\frac{1}{3\pi}m\cos\varphi} \tag{13}$$

or in a normalized form,

$$I_{Q,rms,pu} = \sqrt{\frac{1}{4}+\frac{2}{3\pi}m\cos\varphi} = \sqrt{0.25+0.212\,m\cos\varphi} \tag{14}$$

Because of symmetry, the half-wave current waveform is the sum of the active switch current and the diode current. Therefore, following relationships hold:

$$I_{D,avg}+I_{Q,avg} = \frac{1}{\pi}\hat{I} \tag{15}$$

$$I^2_{D,rms}+I^2_{Q,rms} = \left(\frac{1}{2}\hat{I}\right)^2 \tag{16}$$

that result in

$$I_{D,avg} = \frac{1}{2\pi}\hat{I} - \frac{1}{8}m\cos\varphi\,\hat{I} \tag{17}$$

$$I_{D,avg,pu} = \frac{1}{\sqrt{2}\pi}-\frac{\sqrt{2}}{8}m\cos\varphi = 0.225-0.177\,m\cos\varphi \tag{18}$$

and

$$I_{D,rms} = \hat{I}\sqrt{\frac{1}{8}-\frac{1}{3\pi}m\cos\varphi} \tag{19}$$

$$I_{D,rms,pu} = \sqrt{\frac{1}{4}-\frac{2}{3\pi}m\cos\varphi} = \sqrt{0.25-0.212\,m\cos\varphi} \tag{20}$$

Equations (10), (14), (18) and (20) constitute an approximate current model of an inverter leg. As no constraint on the number of legs has been made, the model is valid for both single- and three-phase inverters. Although the above model is derived for SPWM, it is expected that it applies to other pulse-width modulation methods. This is because the qualitative characteristics of the device current discussed in section 2 – the dependency on the power factor and output voltage level – is generally valid no matter what modulation method is used. Following section gives a comparison between the device current characteristics of different modulation methods with simulation.

4. ASSESMENT OF THE DEVICE CURRENT MODEL

The device current model similar to that given above has been derived in [2] by analyzing the simulation results for various power factor and modulation index conditions. The formulas for average active switch current and the diode current derived in [2] are exactly the same as those given in (10) and (18). For rms currents, however, [2] suggests the formulas

$$I_{Q,rms,pu} \approx 0.5+0.1824K \tag{21}$$

$$I_{D,rms,pu} \approx \sqrt{0.25-0.1824K-0.0333K^2} \tag{22}$$

where $K = m\cos\varphi$. Fig. 5 shows the plots of the normalized current formulas with the plots of (21) and (22) shown by thin lines. It can be observed that although the formulas for rms currents from [2] are somewhat different from those of this paper, the errors between them are not significant, bounded within only a few percent in maximum over the range.

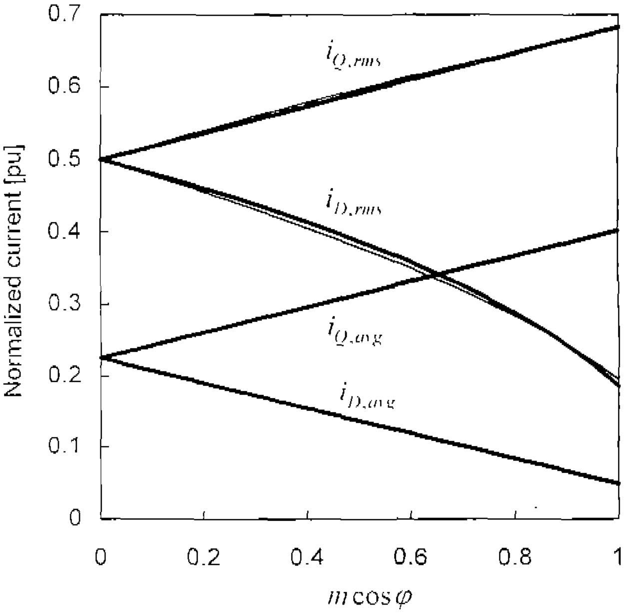


Fig. 5 Device currents versus normalized active output voltage

Another comparison has been made between the device current characteristics of the sinusoidal PWM and the space-vector PWM, one of the most widely accepted modulation techniques in recent three-phase inverter applications [3]. As the modulation index is not given explicitly in SVPWM, the comparison with SPWM is made in terms of the normalized output voltage.

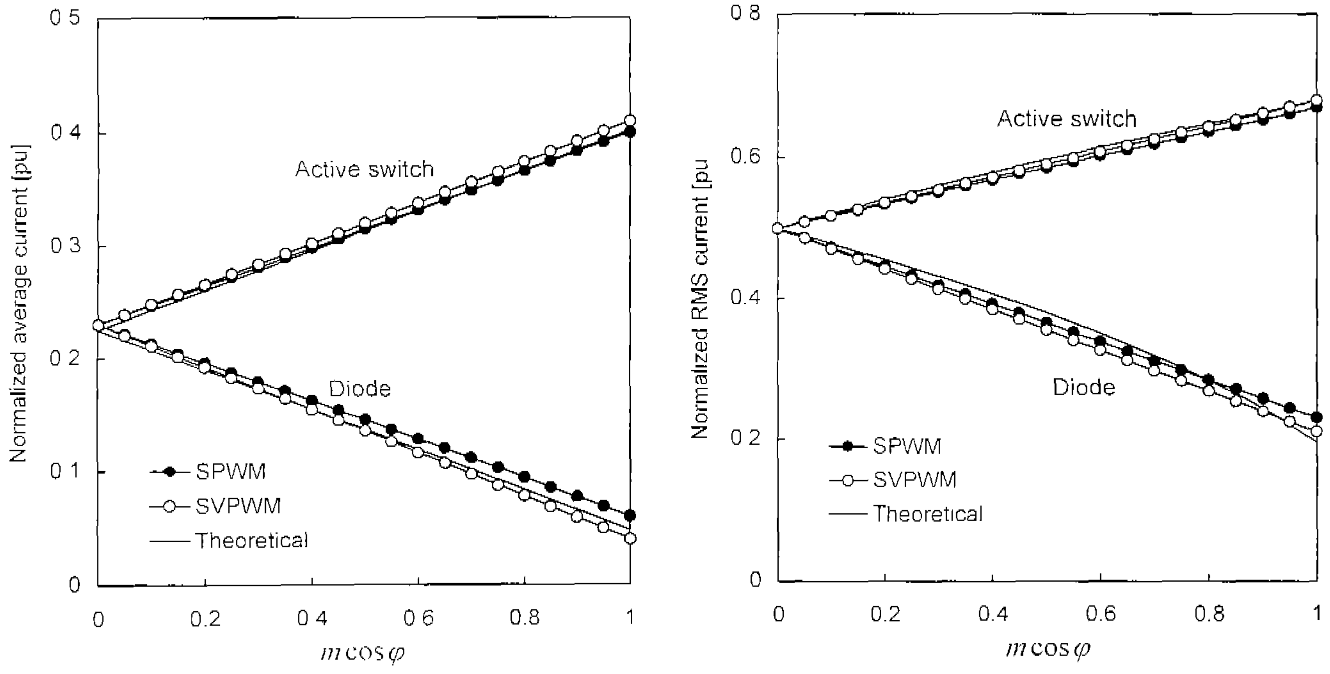
The simulation has been carried out for R-L load with the power factor varying from 0 to 1, and for the modulation index from 0 to 1. Fig. 6 shows the simulation results for SPWM and SVPWM, along with the theoretical characteristics based on the derived formulas. The experimental results for SPWM can be found in [2], which shows a good agreement with simulation results. For SVPWM, similar experiments are carried out to support the simulation, and the results are depicted in Fig. 7. Although only three cases of load power factor are shown because of practical constraints of experimental setup, they have shown to be very close to the associated simulation results and the theoretical expectation as well.

The simulation assumes a finite carrier frequency ($N = 15$), and thus the simulation results account for the effect of the switching ripple in actual current waveforms. The SVPWM features more effective use of zero-vectors in synthesizing the output voltage [4] and as a result, less ripple in its output current waveform. It shows somewhat higher average active switch current and lower average diode current than SPWM for given active voltage level as can be observed in Fig. 6(a). As for rms current, however, Fig. 6(b) shows that two PWM methods appear to have essentially the same current level through an active switch over the entire range. This is a consequence of the smaller ripple current in SVPWM in combination with the larger average current. The smaller ripple current also decreases the rms diode current and causes the difference between the rms diode currents of two modulation methods in Fig. 6(b) larger than the difference between the average diode currents in Fig. 6(a).

All the curves in Fig. 6 obtained from the simulation appear nearly straight lines and can be approximated to linear equations as shown in Table 2. The differences between the formulas for SPWM and SVPWM do not seem to be significant in a practical viewpoint, and the device current characteristics of SVPWM are considered to be generally the same as those of SPWM, as expected in section 2.

Table 2. Linear approximations of device current characteristics for SPWM and SVPWM

	SPWM	SVPWM
$I_{Q,avg}$	$0.23 + 0.17 m\cos\varphi$	$0.23 + 0.18 m\cos\varphi$
$I_{D,avg}$	$0.23 - 0.17 m\cos\varphi$	$0.23 - 0.19 m\cos\varphi$
$I_{Q,rms}$	$0.50 + 0.17 m\cos\varphi$	$0.50 + 0.18 m\cos\varphi$
$I_{D,rms}$	$0.50 - 0.27 m\cos\varphi$	$0.50 - 0.29 m\cos\varphi$



(a) (b)

Fig. 6 Comparison of device current characteristics of SPWM and SVPWM

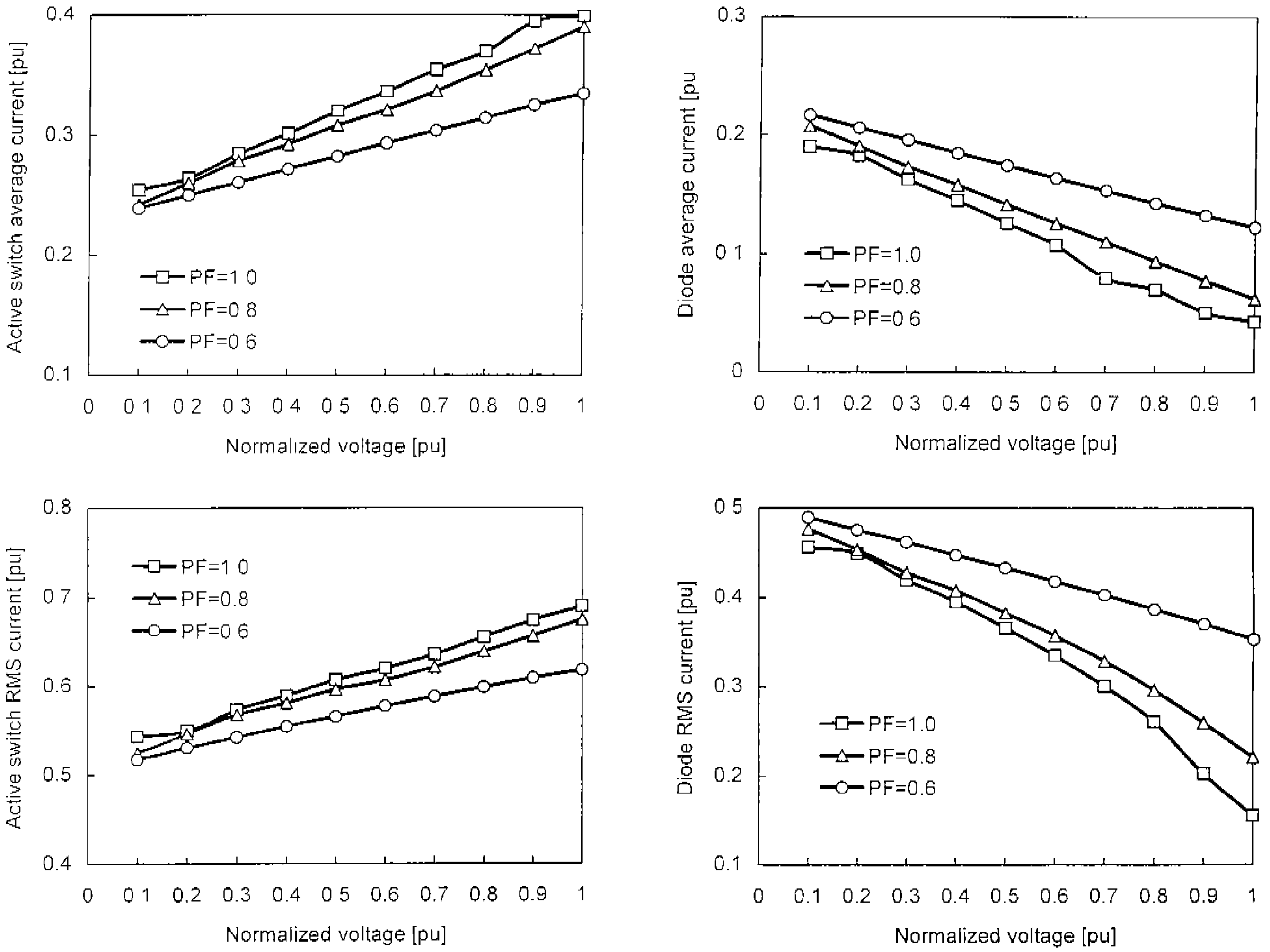


Fig. 7 Experimental results of device currents under SVPWM

5. CONCLUSION

This paper presents an analytical model of average and rms device currents of PWM inverters. The model is derived for SPWM under the assumption of sufficiently high carrier frequency, to appear as a set of simple function of the normalized output voltage and the power factor of the load. These two parameters in the derived formulas always appear as a product that corresponds to the active component of the inverter output voltage.

The simulation and experiment are carried out to confirm the validity of the model. And by examining the effect of the modulation method on the device current characteristics, it is shown that the model based on the SPWM applies to other PWM methods such as SVPWM with an accuracy good enough for practical purpose. Therefore, the device current model presented in this paper is considered to be useful in determining the device ratings and estimating the power loss of an inverter, fast and accurately, which greatly facilitates the design procedure of any kind of PWM inverter system. Further investigation is necessary to extend the model to the overmodulation range and to the case of low switching frequency.

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