A Commutation Torque Minimization Method for Brushless DC Motors with Trapezoidal Elecromotive Force

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ABSTRACT - In this paper, a commutation torque minimization method using parameter observer for a brushless DC motor fed by a voltage source inverter is described. In order to investigate the nature of the commutation torque ripple in trapezoidal brushless DC motor, a new model of the motor is proposed. The optimal drive voltage to minimize the ripple torque is represented as a function of the motor parameters. Therefore, the important parameter is estimated by least-square algorithm.

1. INTRODUCTION

In this paper, a commutation torque minimization method for a brushless DC motor fed by a voltage source inverter is described. The electromotive force of the motor has trapezoidal waveforms. Thus, the current in each phase of the brushless DC motor must be constant for ON period to achieve a constant torque. And, only one current sensor is used in this method.

The commutation torque in trapezoidal brushless DC motor takes the form of torque spikes or dips which are generated at each discrete time instant when any of the square-wave current excitation waveforms change levels. This ripple torque is caused by the combination of nonzero phase inductance and finite inverter bus voltage which prevents the phase current excitation waveforms from changing levels instantaneously [1].

Since the use of a single current sensor is one of the attractive features of the brushless DC motor with trapezoidal back EMF drives from the standpoints of simplicity and cost reduction, several investigators have attacked the problem of attenuating commutation torque ripple without adding any new current sensors [6]-[8]. However, it is required the customized turning for each set of machine parameter. And, the criterion for minimizing torque pulsation in [6] and [7] ($V = 4E$) should be modified.

Therefore, a new commutation torque minimization method is proposed to overcome the above problems. Moreover, in order to investigate the nature of the commutation torque ripple in trapezoidal brushless DC motor, a new model of the motor is introduced. The torque characteristic is analyzed by the new model. By this analysis, the optimal drive condition to minimize the commutation ripple torque can be represented as a function of the internal parameters. However, the parameters have some uncertainy and can be changed with the temperature and the other operating conditions. Therefore, the motor parameters will be estimated by parameter observer. To show the effectiveness of the proposed algorithm, the phase current and generated torque are shown in various operating conditions through the computer simulation results. It is well demonstrated from these results that the proposed algorithm provided desirable torque performance of the brushless DC motor.

2. MODEL OF BRUSHLESS DC MOTOR

A developed Cartesian axial view of the motor construction adopted for this analysis is shown in Fig.

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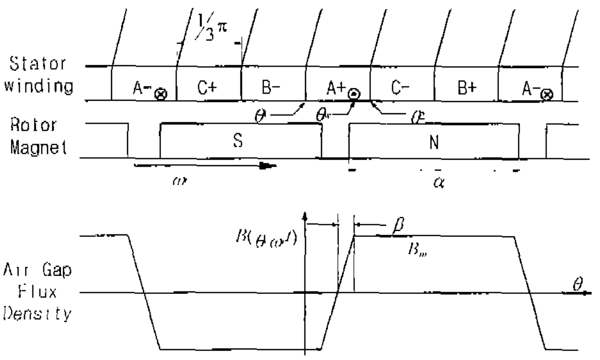

Fig. 1 Axial view of the brushless DC motor including model for magnet radial flux density distribution

1. Detailed torque calculations for such machines typically require extended finite element analyses. But, considerable insight into the torque production mechanism and a dependence of the supplied voltage can be gained by employing a simplified linear model. The simplified model assumes that the surface permanent magnets contribute a purely radial magnetic flux density distribution which can be modeled as shown in Fig. 1. There, α is the rotor magnet pole arc, β reflects the influence of rotor flux fringing, and a rotating speed of the rotor is ω_r [3].

Back EMF Calculation

In this paper, a new conceptual variable, average cross flux of each phase winding, is introduced. Using the average cross flux of each phase winding, the back EMF and the generation torque can be simply presented. As shown in Fig. 1, average cross flux of phase-A winding can be defined by

$$Fx_a'(t) = \frac{3}{\pi}\int_{\theta_1}^{\theta_2} B(\theta, t)d\theta = B_m Fx(t) \tag{1}$$

Assuming three phases balanced winding, average cross flux of each phase can be presented as follows:

$$Fx_a(t) = F(t)\,,\quad Fx_b(t) = Fx\left(t - \frac{2\pi}{3\omega_r}\right),\quad Fx_c(t) = Fx\left(t + \frac{2\pi}{3\omega_r}\right). \tag{2}$$

Consider a full-pitch stator winding with its magnetic axis displaced from that of the air-gap flux by the angle θ_w. The back EMF can be given by

$$e(\theta_w, \omega_r t) = rl_m\omega_r B(\theta_w, \omega_r t)\,. \tag{3}$$

Then, the back EMF in the phase-A is

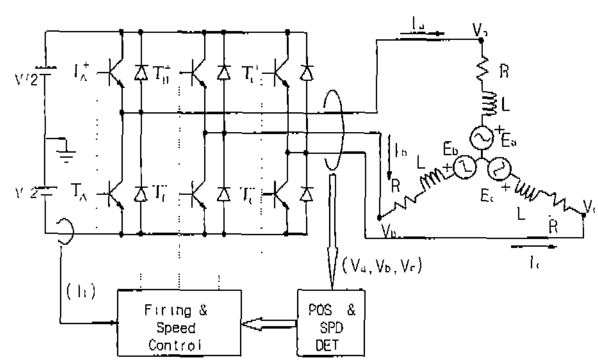

Fig. 2 Schematic diagram of brushless DC motor drive system

$$e_a(t) = 2Nrl_m\omega_r \frac{3}{\pi}\int_{\theta_1}^{\theta_2} B(\theta_w, \omega_r t)d\theta_w = EFx_a(t) \tag{4}$$

where $E = 2\omega_r Nrl_m B_m$.

Then, using average cross flux $Fx(t)$, the back EMF of each phases can be presented as follows:

$$e_a(t) = EFx_a(t)\,,\quad e_b(t) = EFx_b(t)\,,\quad e_c(t) = EFx_c(t) \tag{5}$$

BLDC Motor Drive System and Model Equation

A drive system for the brushless DC motor is shown in Fig. 2. The firing and speed controller performs two functions: (1) it developed a coordinated set of signal, based on the rotor position data, to commutate, i.e., turn on and off, the transistor in the bridge in such a way as to maintain an optimum relationship of magnet position with stator winding in the motor; and (2) it converts a user generated signal from the system control to a high frequency pulse width modulation one to control the actual voltage and current delivered to the motor. [2]

The simple circuit equations in phase variables are

$$\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} = R\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + L\frac{d}{dt}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \begin{bmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{bmatrix} + \begin{bmatrix} v_n(t) \\ v_n(t) \\ v_n(t) \end{bmatrix} \quad L = L_s - M \tag{6}$$

where R, L_s and M are resistance, self and mutual inductance.[4][5] Since the winding of machine is Y-connection and the neutral of winding is opened, the neutral voltage is given by

$$v_n(t) = \frac{1}{3}\left[\{v_a(t) - e_a(t)\} + \{v_b(t) - e_b(t)\} + \{v_c(t) - e_c(t)\}\right] \tag{7}$$

Therefore, the model equation of brushless DC motor can be summarized as follows;

$$\frac{d}{dt}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} + \frac{R}{L}\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} = \frac{1}{3L}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\left\{\begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} - E\begin{bmatrix} Fx_a(t) \\ Fx_b(t) \\ Fx_c(t) \end{bmatrix}\right\}. \quad (8)$$

Generated Torque

The generation torque can be calculated by integrating the Lorentz force density ($J \times B$) along each of the individual coil side.[3] As shown in Fig. 1, consider a full-pitch phase-A winding with its magnetic axis displaced from that of the air-gap flux by the angle θ_w, the generation torque is

$$T_c(\theta_w, t) = l_m i_a(t) B(\theta_w, t). \quad (9)$$

And, the generation torque by phase A can be presented as follows:

$$T_a(t) = 2N\frac{3}{\pi}\int_{\theta_1}^{\theta_2} T_c(\theta_w, \omega_r t) d\theta_w = 2Nl_m B_m i_a(t) Fx(t). \quad (10)$$

Considering three phases, the total generation torque is

$$T_e(t) = \frac{K_t}{2}\left[i_a(t)Fx_a(t) + i_b(t)Fx_b(t) + i_c(t)Fx_c(t)\right] \quad (11)$$

where $K_t = 4qNl_m B_m$

And the load dynamics is given by

$$T_e(t) = J\frac{d\omega_r}{dt} + B\omega_r + T_l \quad (12)$$

3. THE CURRENT COMPENSATOR for MINIMIMIZING COMMUTATION TORQUE

A new method without extra current sensor for minimizing commutation torque is introduced. In this method, a new criterion for minimizing torque pulsation is proposed and the motor parameters are observed continuously. The schematic diagram of the commutation torque minimizing algorithm is shown in Fig. 3.

A criterion for minimizing torque pulsation

In this analysis, the commutation of the current from phase C to phase A is considered. As shown in Fig. 2, this current transfer is done by switching off $+T_C$ and by switching on $+T_A$. Nevertheless, this transfer is not done directly and puts into action the free wheeling diode $-D_C$.

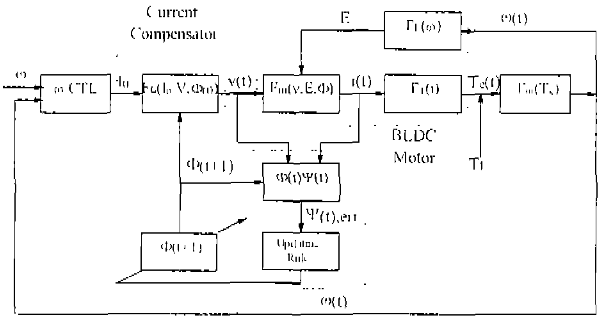

Fig. 3 Schematic diagram of current compensator for minimizing commutation torque

The free wheeling diode $-D_C$ conduct until i_c vanishes. For the immediately after switching, the input terminal voltages are given by

$$[v_a(t)\ v_b(t)\ v_c(t)]^T = \left[\tfrac{1}{2}V\ \ -\tfrac{1}{2}V\ \ -\tfrac{1}{2}V\right]^T. \quad (13)$$

Assuming that the system has ideally trapezoidal back EMF waveform, the waveform of the average cross flux can be given by

$$Fx_a(t) = 1,\quad Fx_b(t) = -1,\quad Fx_c(t) = 1 - k\omega_r t \quad \text{for } t > 0 \quad (14)$$

where k is a slope of trapezoid, typically, $k = 6/\pi$.

And the initial condition of the phase current is

$$[i_a(0)\ i_b(0)\ i_c(0)]^T = [0\ -I_0\ I_0]^T \quad (15)$$

where I_0 is target value of phase current.

Taking the beginning of the commutation as the time origin, the phase currents are presented as follows;

$$\begin{aligned} i_a(t) &= \frac{1}{3R}\left\{(2V - 4E + kE\omega_r\tau)\left(1 - e^{-t/\tau}\right) - kE\omega_r t\right\} \\ i_b(t) &= -I_0 e^{-t/\tau} - \frac{1}{3R}\left\{(V - 4E - kL\omega_r\tau)\left(1 - e^{-t/\tau}\right) - kE\omega_r t\right\} \\ i_c(t) &= I_0 e^{-t/\tau} - \frac{1}{3R}\left\{(V + 2E + 2kE\omega_r\tau)\left(1 - e^{-t/\tau}\right) - 2kE\omega_r t\right\} \end{aligned} \quad (16)$$

where $\tau = L/R$.

Using above (11) and (16), the generation torque is

$$\begin{aligned} T_e(t) &= \frac{K_t}{2}\left[i_a(t) - i_b(t) + i_c(t) - k\omega_r t \cdot i_c(t)\right] \\ &= K_t\left[-i_b(t) - \frac{k}{2} i_c(t) \cdot \omega_r t\right] \end{aligned} \quad (17)$$

The commutation time, t_{com}, can be achieved by

$i_c(t_{com}) = 0$. Assuming that $t_{com}/\tau << 1$, commutation time can be simply presented as follows;

$$i_c(t_{com}) \approx I_0 - \frac{t_{com}}{\tau}\left(I_0 + \frac{V + 2E}{3R}\right) \tag{18}$$

Then, $$t_{com} \approx \frac{3LI_0}{(3RI_0 + V + 2E)} \tag{19}$$

And, the phase current at $t = t_{com}$ can be presented as follows;

$$i_a(t_{com}) = -i_b(t_{com}) = I_{com} \tag{20}$$

By assuming that $t_{com}/\tau << 1$

$$I_{com} \approx I_0 - \frac{3RI_0 - (V - 4E)}{3RI_0 + (V + 2E)} I_0 \tag{21}$$

The generation torque at $t = t_{com}$ can be given by

$$\begin{aligned} T_e(t_{com}) &= K_t\left[-i_b(t_{com}) - \frac{k}{2} i_c(t_{com}) \cdot \omega_r t_{com}\right] = K_t I_{com} \\ &\approx K_t I_0 - K_t I_0\left(\frac{3RI_0 - (V - 4E)}{3RI_0 + (V + 2E)}\right) \end{aligned} \tag{22}$$

Consequently, the condition for minimizing commutation torque is given by

$$V_{op} = 3RI_0 + 4E \tag{23}$$

In case that $V = V_{op}$, the phase-B current can be maintained I_0 in $0 < t < t_{com}$ and, the torque pulsation can be minimized.

In case $V > V_{op}$

When the drive system has low speed or light load, the terminal voltage for minimizing commutation torque may be less then the DC link voltage. The terminal voltages can be controlled to satisfy the condition for minimizing torque pulsation. For the immediately after switching, the terminal voltages for minimizing torque pulsation are given by

$$v_a(t) = V_{op},\quad v_b(t) = -V_{op},\quad v_c(t) = -V_{op}\quad 0 < t \le t_{com} \tag{24}$$

where $$t_{com} \approx \frac{3LI_0}{(3RI_0 + V_{op} + 2E)}.$$

And, for the after commutation, the each terminal voltage of the phase-A and phase-B for minimizing torque pulsation are given by

$$v_a(t) = V_0,\quad v_b(t) = -V_0,\quad t_{com} < t \le \pi/3\omega_r \tag{25}$$

where $V_0 = 2RI + 2E$ and $v_c(t)$ is open.

In case $V < V_{op}$

When the drive system has high speed or heavy load, the terminal voltage for minimizing commutation torque is larger then the DC link voltage. In this case, the DC link voltage should be directly supplied to the input terminal until input current is reached the target value, I_0. As shown in (22), the phase current is not reached to target current at $t = t_{com}$. Then, the pulsation torque is

$$T_{pul} = K_t I_0 - K_t I_{com} \approx K_t I_0\left(\frac{3RI_0 - (V - 4E)}{3RI_0 + (V + 2E)}\right). \tag{26}$$

For the after commutation, the phase current is given by

$$\frac{di_a(t)}{dt} + \frac{i_a(t)}{\tau} = \frac{1}{2L}(V - 2E) \qquad i(t_{com}) = I_{com}. \tag{27}$$

Then, $$i_a(t) = I_{com} e^{-(t - t_{com})/\tau} + \frac{V - 2E}{2R}\left(1 - e^{-(t - t_{com})/\tau}\right). \tag{28}$$

Then, the reaching time, t_r, can be achieved by $i_a(t_r) = I_0$. Assuming that $(t_r - t_{com})/\tau << 1$, the reaching time can be obtained as follows;

$$i_a(t_r) \approx I_{com} + \left(\frac{V - 2E}{2R} - I_{com}\right)\frac{t_r - t_{com}}{\tau} \tag{29}$$

$$t_r = \tau\frac{V_0 - 2(E + RI_{com})}{V - 2(E + RI_{com})} + t_{com}. \tag{30}$$

And, in $t_r < t \le \pi/3\omega_r$, the each terminal voltages for minimizing commutation torque may be given by (25).

Motor parameter estimation

As shown above section, the compensation method for minimizing commutation torque has motor parameter sensitivities. Therefore, the motor parameters should be estimated from input current from DC link, supplied voltage, rotating speed and, back EMF. A least square algorithm is employed to estimate the motor parameters. The input current is equal to the phase-A current in this switching period. Then the current equation is given by

$$\frac{di(t)}{dt} + \frac{i(t)}{\tau} = \frac{1}{L} v_{eq}(t) \tag{31}$$

where $v_{eq}(t)$ is the equivalent voltage for current excitation. The current equation may be refined to avoid the measurement of $di(t)/dt$. Consider the current equation and the equation can be written by

$$i(t) = \frac{1}{L}\psi_1(t) + (\lambda - \frac{1}{\tau})\psi_2(t) \quad (32)$$

The signals $\psi_1(t)$ and $\psi_2(t)$ are obtained by stable filtering of the input and of the output of the plant.[9]

$$\frac{d\psi_1(t)}{dt} = -\lambda\psi_1(t) + v_{eq}(t), \quad \frac{d\psi_2(t)}{dt} = -\lambda\psi_2(t) + i(t) \quad (33)$$

Define the vector of nominal identifier parameters

$$\Phi^* := \left[\frac{1}{L} \;\; \lambda - \frac{1}{\tau}\right]^T \quad (34)$$

Knowledge of Φ^* is clearly equivalent to the knowledge of the unknown parameters $1/L$ and $1/\tau$. Similarly, define $\Phi(t)$ to be a vector of identical dimension, called the adaptive identifier parameter. $\Phi(t)$ is the estimate of Φ^* based on input-output data up to time t. Letting

$$\Psi(t) := [\psi_1(t) \;\; \psi_2(t)]^T \quad (35)$$

the current equation may be written

$$i(t) = \Phi^{*T}\Psi = \Psi^T\Phi^* \quad (36)$$

Based on measurement of $v_{eq}(t)$ and $i(t)$ up to time t, $\Psi(t)$ may be calculated, and an estimate $\Phi(t)$ derived. And, the identification error is

$$Er(t) = \Phi^T(t)\Psi(t) - i(t) = \left[\Phi^T(t) - \Phi^{*T}\right]\Psi(t) \quad (37)$$

The updating rule of the least square algorithm is given by

$$\frac{d\Phi(t)}{dt} = -P(t)\Psi(t)Er(t) \quad \Phi(0) = \Phi_0 \quad (38)$$

$$\frac{dP(t)}{dt} = -P(t)\Psi(t)\Psi^T(t)P(t) \quad P(0) = P_0 > 0 \quad (39)$$

In this algorithm, the parameter error is given by

$$\Phi(t) - \Phi^* = \left(P_0 + \int_0^t \Psi(\zeta)\Psi^T(\zeta)d\zeta\right)^{-1} P_0(\Phi_0 - \Phi^*) \quad (40)$$

It follows that $\Phi(t)$ converges asymptotically to Φ^* if $\int_0^t \Psi(\zeta)\Psi^T(\zeta)d\zeta$ is unbounded as $t \to \infty$.[9]

4. SIMMULATION RESULTS

The system parameters used in this simulation are as follows;

$K_t = 0.512[Nm/A]$, $E_0 = 0.85[V\sec/rad]$, $P = 4[pole]$,

$R = 1.8[\Omega]$, $L = 2.7[mH]$, $V = 240[V]$.

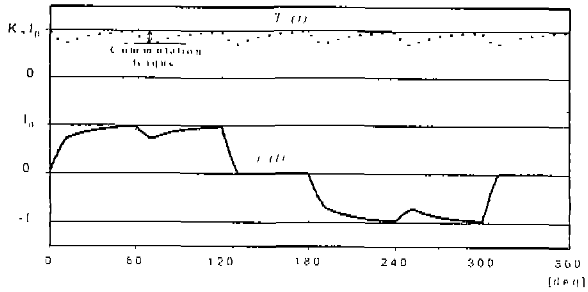

Fig 4 Generation torque and phase current

($v_a(t) = V0,\quad 0 < t \le \pi/3\omega_r$)

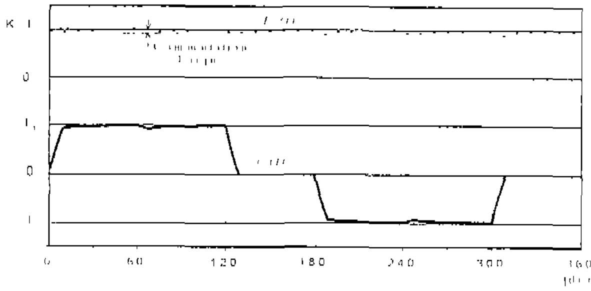

Fig 5. Current compensating result

($v_a(t) = V, 0 < t < t_1$ / $v_a(t) = V_0, t_1 < t \le \pi/3\omega_r$)

The simulation results of the phase current and generation torque in case that the constant voltage is supplied to input terminal are shown in Fig. 4, where the system has heavy load and high speed. As shown in Fig. 4, the phase current can not be arrived the target value. Then, it has quite large torque pulsation.

Fig. 5 shows a result of the current compensator for minimizing commutation torque. Since the optimum voltage for minimizing commutation torque is larger then the optimum voltage, the generation torque has a little pulsation torque. Nevertheless, the phase-A current at $t = t_1$ can not reached the target value of the phase current because t_1 is calculated from approximated equation.

A parameter sensitivity of the commutation current compensation method is shown in Fig. 6. The generation torque and the phase current in case that $R_{known} = 0.8R_{real}$ and $L_{known} = 1.2L_{real}$. The error of parameter can make large torque pulsation and current distortion.

A parameter estimation results using least square algorithm is shown in Fig. 7. Initial state of the parameter is that $R(0) = 0.8R$ and $L(0) = 1.2L$. And, $\lambda = 0.01/\tau$ and $P(0) = 10^4 I$. It shows that the estimation parameter is quickly approaches the real value.

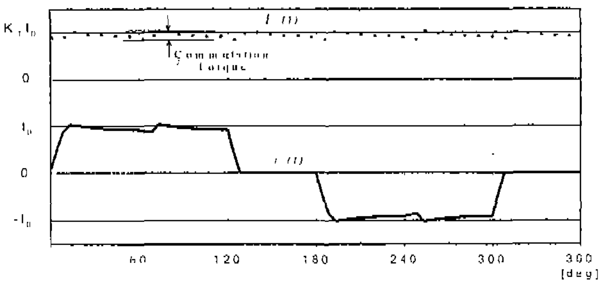

Fig. 6 Current compensating results

($R_{known} = 0.8R_{real}$, $L_{known} = 1.2L_{real}$)

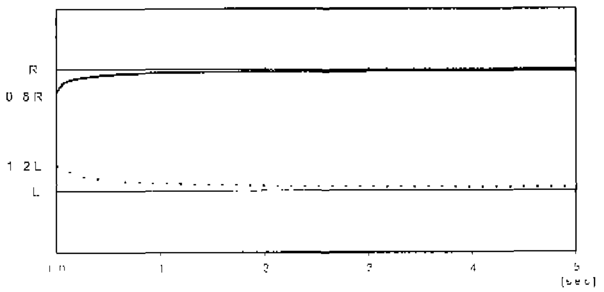

Fig. 7 Parameter estimation results

($R_{known} = 0.8R_{real}$, $L_{known} = 1.2L_{real}$)

Fig. 8 shows final results of the current compensator using parameter estimator when the motor parameters are changed. It is shown in this figure that proposed current compensator well minimizes the commutation torque, and it has robustness of the motor parameter.

5. CONCLUSIONS

In this paper, a new commutation torque minimization method has been proposed. And, in order to investigate the nature of the commutation torque ripple in brushless DC motor with trapezoidal back EMF, a new model of the motor has been introduced. An optimal drive condition to minimize the commutation torque ripple can be obtained through the torque analysis. However, even if the optimal drive condition is obtained, it has sensitivity of the motor parameters. Then, the parameter estimator using least square algorithm is added to overcome the parameter sensitivity.

The simulations are carried out for the current compensator. When motor parameter is varied, the result of the current compensator without parameter estimation has large torque pulsation. But, the result of the current

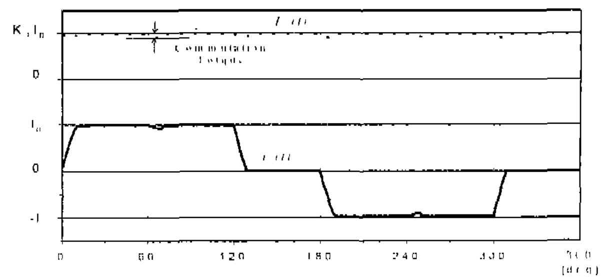

Fig. 8 Generation torque and phase current

(the current compensator using parameter estimator)

compensator using parameter estimation algorithm show a good performance. Therefore, proposed commutation torque minimizing method can really solve the commutation torque problem of the brushless DC motor with trapezoidal back EMF. Also, it may be easily applied to a real plant because of its simple structure.

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