

# A ROBUST VECTOR CONTROL FOR PARAMETER VARIATIONS OF INDUCTION MOTOR

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**ABSTRACT** – In this paper, the robust vector control method of induction motor for the purpose of improving the system performance deterioration caused by parameter variations is proposed. The estimations of the stator current and the rotor flux are obtained by the full order state observer with corrective prediction error feedback, and the adaptive scheme is constructed to estimate the rotor speed with the error signal between real and estimation value of the stator current. Adaptive sliding observer based on the variable structure control is applied to parameter identification. Consequently, predictive current control and speed sensorless vector control can be obtained simultaneously, regardless of the parameter variations.

## 1. INTRODUCTION

Due to the advantages such as ruggedness, high reliability, low cost and minimum maintenance, induction motors are now widely used in industrial applications. Over the past decades many control strategies have been proposed to achieve better performance. As far as quick response is concerned, vector control is perhaps one of the best choice for implementing a high performance AC drives.

In these strategies, the current controlled vector control generally requires that the actual stator current should be adjusted instantaneously and precisely to its reference. In order to satisfy above requirement, deadbeat control is used in current control system of induction motor[1].

However, deadbeat control has tendency to become an unstable and deteriorate the performance since it is sensitive to parameter variations.

Also, a rotational transducer such as a shaft encoder is used in vector control system. However, a rotational transducer cannot be mounted in some cases such as motor drives in a hostile environment and high speed motor drives, etc. In recent years, several vector control of induction motor drive methods without rotational transducer have been proposed[2][3]. These methods require an accurate information of system parameters, especially the variation of rotor resistance.

In this paper, the robust vector control method for the purpose of improving the system performance deterioration caused by parameter variations is proposed. The full order state observer estimates the stator current and the rotor flux with corrective prediction error feedback of stator current. And, deadbeat current control based on this state observer is used so that the actual stator current should be adjusted instantaneously and precisely to its reference. Adaptive scheme is constructed to estimate the rotor speed with the error signal between real and estimation value of the stator current without speed sensor. Estimating error of stator current is compensated by variable pole placement technique. In order to compensate estimating error of rotor flux, adaptive sliding observer based on the variable structure control is applied to current model where stator current is input and rotor flux is output of this model. Consequently, an accurate state prediction of deadbeat current control is performed, and robustness of speed identifier is presented. The results of simulation

show that robust deadbeat current control and speed sensorless vector control can be obtained simultaneously, unaffected by the influence of parameter variations.

## 2. DEADBEAT CURRENT CONTROL

An induction motor can be described by following state equations in the stationary reference frame.

$$pX = AX + Bv_s$$

$$\begin{bmatrix} pi_s \\ p\lambda_r \end{bmatrix} = \begin{bmatrix} -A_{11} & -A_{12} \\ -A_{21} & -A_{22} \end{bmatrix} \begin{bmatrix} i_s \\ \lambda_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s \quad (1)$$

The output equation is

$$Y = CX$$

$$i_s = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} i_s \\ \lambda_r \end{bmatrix} \quad (2)$$

where.  $i_s = [i_{ds} \quad i_{qs}]^T$  : stator current

$\lambda_r = [\lambda_{dr} \quad \lambda_{qr}]^T$  : rotor flux

$v_s = [v_{ds} \quad v_{qs}]^T$  : stator voltage

$$A_{11} = -\left\{ \frac{R_s}{\sigma L_s} + \frac{1-\sigma}{\sigma \tau_r} \right\} I$$

$$A_{12} = \frac{M}{\sigma L_s L_r} \left\{ \frac{1}{\tau_r} I - \omega_r J \right\}$$

$$A_{21} = \frac{M}{\tau_r} I$$

$$A_{22} = -\frac{1}{\tau_r} I + \omega_r J$$

$$B_1 = \frac{1}{\sigma L_s} I$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$R_s, R_r$  : stator and rotor resistance

$L_s, L_r$  : stator and rotor self inductance

$M$  : mutual inductance

$\omega_r$  : electrical rotor angular velocity

$$\sigma = 1 - \frac{M^2}{L_s L_r} \quad : \text{leakage coefficient}$$

$$\tau_r = \frac{L_r}{R_r} \quad : \text{rotor time constant}$$

$p$  : differential operator

In order to derive a discrete time model of "(1)" and "(2)", inverter output voltages  $v_{ab}$ ,  $v_{bc}$  and  $v_{ca}$  are chosen as one or two voltage pulses of magnitude  $+E$  or  $-E$ , satisfying the relation of  $v_{ab} + v_{bc} + v_{ca} = 0$  at any moment.  $\Delta T_{ab}(k)$ ,  $\Delta T_{bc}(k)$  and  $\Delta T_{ca}(k)$  are the pulse widths of  $v_{ab}$ ,  $v_{bc}$  and  $v_{ca}$ , respectively in the  $k$ th sampling. The two phase pulse widths  $\Delta T_d(k)$  and  $\Delta T_q(k)$  are given by "(3)".

$$\begin{bmatrix} \Delta T_d(k) \\ \Delta T_q(k) \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & 0 \end{bmatrix} \begin{bmatrix} \Delta T_{ab}(k) \\ \Delta T_{bc}(k) \\ \Delta T_{ca}(k) \end{bmatrix} \quad (3)$$

As long as these pulses are symmetrical at the center of the sampling interval, so are  $v_{ds}$  and  $v_{qs}$ , and assuming that the rotor speed is constant during a sampling interval  $T$ . The discrete time model of "(1)" and "(2)" become

$$X(k+1) = \Phi X(k) + \Gamma \Delta T(k)$$

$$\begin{bmatrix} i_s(k+1) \\ \lambda_r(k+1) \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} i_s(k) \\ \lambda_r(k) \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \Delta T(k) \quad (4)$$

where,  $\Phi_y$  is the corresponding element of  $e^{AT}$ ,  $\Gamma_i$  is the corresponding element of  $e^{AT/2} B E$  and  $\Delta T(k) = [\Delta T_d(k) \quad \Delta T_q(k)]^T$ .  $i_s(k)$ ,  $\lambda_r(k)$  and  $\Delta T(k)$  are the values at the sampling instant of  $kT$ . Thus, using "(4)" and replacing  $i_s(k+1)$  with the reference  $i_s^*(k+1)$ , the deadbeat control law is given as follows.

$$\Delta T(k) = (\Gamma_1)^{-1} [i_s^*(k+1) - \Phi_{11} i_s(k) - \Phi_{12} \lambda_r(k)] \quad (5)$$

This control law forces the stator current to be exactly equal to the its reference value at the  $(k+1)$ th sampling

instant. This reference current is one sampling ahead preview value of stator current, which is provided by the estimated rotor flux as follows.

$$\begin{aligned} i_{ds}^*(k) &= i_m^*(k) \cos \hat{\theta}(k) - i_r^*(k) \sin \hat{\theta}(k) \\ i_{qs}^*(k) &= i_m^*(k) \sin \hat{\theta}(k) + i_r^*(k) \cos \hat{\theta}(k) \end{aligned} \quad (6)$$

where,  $\cos \hat{\theta}(k) = \hat{\lambda}_{dr}(k) / \hat{\lambda}_r(k)$

$$\sin \hat{\theta}(k) = \hat{\lambda}_{qr}(k) / \hat{\lambda}_r(k)$$

$$\hat{\lambda}_r(k) = \sqrt{\hat{\lambda}_{dr}^2(k) + \hat{\lambda}_{qr}^2(k)}$$

$i_m^*(k)$  : field current command

$i_r^*(k)$  : torque current command

### 3. FULL ORDER STATE OBSERVER

To realize the predictive current control and the vector control, the full order state observer which estimates the stator current and the rotor flux together with corrective prediction error feedback is written by the following equation[4].

$$\begin{aligned} \hat{X}(k+1) &= \Phi \hat{X}(k) + \Gamma \Delta T(k) + GC(\hat{X}(k) - X(k)) \\ \begin{bmatrix} \hat{i}_s(k+1) \\ \hat{\lambda}_r(k+1) \end{bmatrix} &= \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \begin{bmatrix} \hat{i}_s(k) \\ \hat{\lambda}_r(k) \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \Delta T(k) \\ &\quad + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} (\hat{i}_s(k) - i_s(k)) \end{aligned} \quad (7)$$

where, “ $\hat{\phantom{x}}$ ” means the estimated values and  $G$  is the observer gain matrix.

The error dynamics of the observer is

$$e(k+1) = (\Phi + GC)e(k) \quad (8)$$

where,  $e(k) = [e_{\lambda d}(k) \ e_{i q}(k) \ e_{\lambda d}(k) \ e_{\lambda q}(k)]^T$

$$e_{\lambda d}(k) = \hat{i}_{ds}(k) - i_{ds}(k), \quad e_{i q}(k) = \hat{i}_{qs}(k) - i_{qs}(k)$$

$$e_{\lambda d}(k) = \hat{\lambda}_{dr}(k) - \lambda_{dr}(k), \quad e_{\lambda q}(k) = \hat{\lambda}_{qr}(k) - \lambda_{qr}(k)$$

The gain matrix  $G$  is designed such that the poles of

“(8)” take the desired values. Thus, the pole placement technique is used in which “(4)” is first transformed into a controllable canonical form, then four poles of the observer are assigned[5]. With choice of the observer gain matrix  $G$  as shown in “(9)”, it is possible to place the poles at any specified conjugate complex pairs in the  $s$  domain.

$$G_1 = g_1 I + g_2 J = \begin{bmatrix} g_1 & -g_2 \\ g_2 & g_1 \end{bmatrix} \quad (9)$$

$$G_2 = g_3 I + g_4 J = \begin{bmatrix} g_3 & -g_4 \\ g_4 & g_3 \end{bmatrix}$$

In this paper,  $G$  is calculated by the following equation so that the observer poles are proportional to those of the induction motor.

$$\alpha_1 \pm j\beta_1 = k(p_1 \pm jp_1) \quad (10)$$

$$\alpha_2 \pm j\beta_2 = k(p_2 \pm jp_2)$$

where,  $p_1$  and  $p_2$  are the real parts of the induction motor poles in “(1)”.  $k$  is a proportional constant ( $k > 0$ ) and the corresponding  $z$  domain observer poles are given by

$$\xi_1 \pm j\eta_1 = e^{\alpha_1 T} \cos \beta_1 T \quad (11)$$

$$\xi_2 \pm j\eta_2 = e^{\alpha_2 T} \cos \beta_2 T$$

The predicted state variables are then used for deadbeat control law in “(5)”, which is transformed into

$$\Delta T(k) = (I_1)^{-1} [i_s^*(k+1) - \Phi_{11} \hat{i}_s(k) - \Phi_{12} \hat{\lambda}_r(k)] \quad (12)$$

### 4. SPEED SENSORLESS VECTOR CONTROL

In case of elimination rotational transducer, there are unknown parameters that involves  $\omega_r$  in matrix  $\Phi$ . Adaptive full order state observer is constructed by adding to adaptive scheme, and it estimates the rotor speed using the error signal between real and estimation value of the

stator current as shown in Fig. 1.

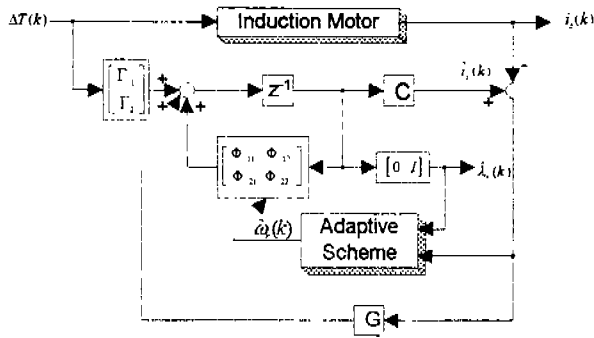


Fig. 1 Block diagram of adaptive full order state observer

$$\begin{aligned} \hat{\omega}_r(k) = & K_p (e_{id}(k) \hat{\lambda}_{qr}(k) - e_{iq}(k) \hat{\lambda}_{dr}(k)) \\ & + K_I \int (e_{id}(k) \hat{\lambda}_{qr}(k) - e_{iq}(k) \hat{\lambda}_{dr}(k)) dt \end{aligned} \quad (13)$$

where.  $K_p$  and  $K_I$  are an arbitrary positive constant.

The stability of the adaptive scheme is proved by the theory of Lyapunov's stability[6]. If the estimating error exist. "(7)" and "(8)" can be described as follows.

$$\hat{X}(k+1) = \hat{\Phi} \hat{X}(k) + \Gamma \Delta T(k) + GC(\hat{X}(k) - X(k)) \quad (14)$$

$$= (\Phi + \Delta\Phi) \hat{X}(k) + \Gamma \Delta T(k) + GC(\hat{X}(k) - X(k))$$

$$e(k+1) = (\Phi + GC)e(k) + \Delta\Phi \hat{X}(k) \quad (15)$$

where.  $\Delta\Phi$  is error matrix of  $\Phi$  caused by estimating error of the speed.

Now, the Lyapunov function is defined as follows.

$$V(k) = e(k)^T e(k) + f(k) \quad (16)$$

where.  $f(k)$  is an arbitrary function. To guarantee asymptotic stability.  $\Delta V(k)$  must be negative define.

$$\Delta V(k) = V(k) - V(k-1) < 0 \quad (17)$$

## 5. SLIDING OBSERVER

Estimating error of stator current in adaptive scheme is compensated by the full order state observer with variable

pole placement which is used so that the observer poles vary proportional to those variation of the induction motor. To accurate estimation of rotor flux, current model where stator current is input and rotor flux is output of this model is presented as follows

$$\lambda_r(k+1) = F\lambda_r(k) + Hi_s(k) \quad (18)$$

where,  $F = \Phi_{22}$ ,  $H = \Phi_{21}$

The flux observer based on sliding observer which has robustness against parameter variations and measurement noises is constructed with sign function feedback of flux error as shown in Fig. 3.

$$\hat{\lambda}_r(k+1) = F\hat{\lambda}_r(k) + Hi_s(k) + K \text{sgn}(e_{\lambda_r}(k)) \quad (19)$$

where.  $e_{\lambda_r}(k) = \hat{\lambda}_r(k) - \lambda_r(k)$

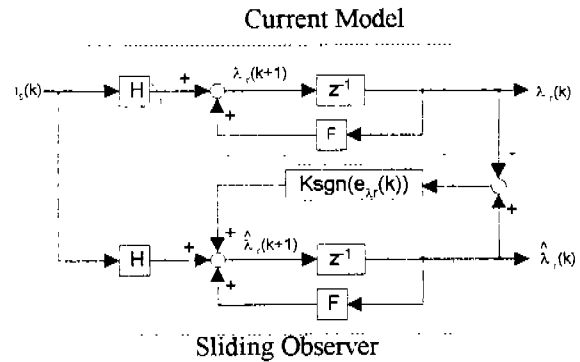


Fig. 2 Configuration of the sliding observer

The error equation of flux observer becomes

$$e_{\lambda_r}(k+1) = Fe_{\lambda_r}(k) + K \text{sgn}(e_{\lambda_r}(k)) \quad (20)$$

The sliding hyperplane is defined by

$$\begin{aligned} s(k) &= [s_1(k) \quad s_2(k)]^T = [e_{\lambda_{dr}}(k) \quad e_{\lambda_{qr}}(k)]^T \\ &= \hat{\lambda}_r(k) - \lambda_r(k) = 0 \end{aligned} \quad (21)$$

The sufficient conditions that guarantee the existence of the sliding mode are given by

$$\begin{aligned}
 s^T(k) \cdot s(k+1) &< 0 \\
 s_1(k) \cdot s_1(k+1) &= e_{\lambda d}(k) e_{\lambda d}(k+1) < 0 \\
 s_2(k) \cdot s_2(k+1) &= e_{\lambda q}(k) e_{\lambda q}(k+1) < 0
 \end{aligned} \tag{22}$$

To satisfy conditions in "(22)", it is sufficient to choose the feedback gain  $K$  large enough.

Sliding observer can estimate parameter variations using information of estimating error which exists in switching signal  $K \operatorname{sgn}(e_{\lambda}(k))$ . Therefore, an accurate state prediction of deadbeat current control is performed, and robustness of speed identifier is presented.

## 6. SIMULATION RESULTS

The construction of the proposed system is shown in Fig. 3. Simulation is carried out to verify the effectiveness of the proposed control method using the system parameters as shown in Table 1.

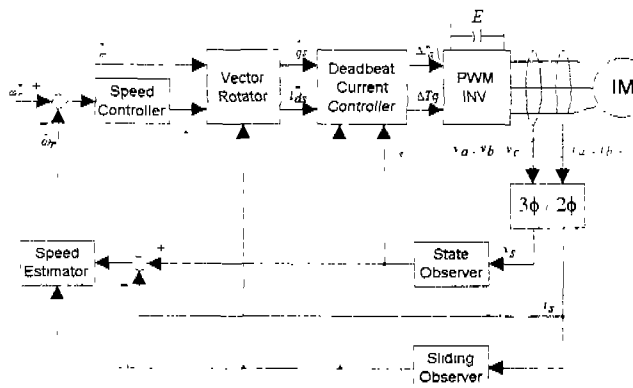


Fig. 3 The construction of the proposed system

Table 1 The system parameters for simulation

Rated voltage	220/380[V]	$R_r$	1.67[ $\Omega$ ]
Rated current	8.3/4.8[A]	$R_s$	1.5[ $\Omega$ ]
Rated speed	1740[rpm]	$L_s$	100[mH]
Rated power	2.2[Kw]	$L_r$	100[mH]
Number of poles	4	$M$	95[mH]

For the comparison and evaluation of estimation performances, simulations are accomplished for both state observer and sliding observer. Fig. 4 and Fig. 5 show the

responses of current at 0.5 times rated load when the variations of the rotor resistance are 150%. Fig. 4 depict the comparisons of the current response characteristics between conventional method and proposed method where the upper waveform is conventional method and the lower is proposed method in 50[rpm]. Fig. 5 shows the comparisons of the current response characteristics under the same condition in 500[rpm]. Fig. 6 and Fig. 7 depict the comparisons of the speed response characteristics between conventional method and proposed method in the reversible change characteristics from 50[rpm] to -50[rpm]. Fig. 6 shows the real and estimated speed of conventional method where the upper waveform is real speed and the lower is estimated speed. Fig. 7 shows the real and estimated speed of proposed method under the same condition. Fig. 8 shows the real and estimated speed response characteristics of proposed method when the variations of the rotor resistance are 150% in 50[rpm].

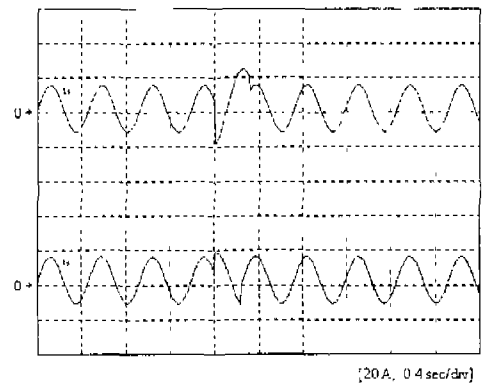


Fig. 4 Current response of a low speed region(50rpm)

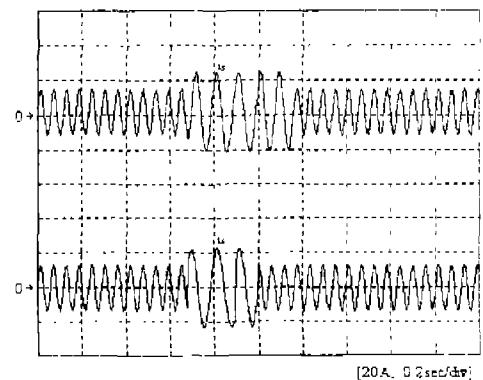


Fig. 5 Current response of a high speed region(500rpm)

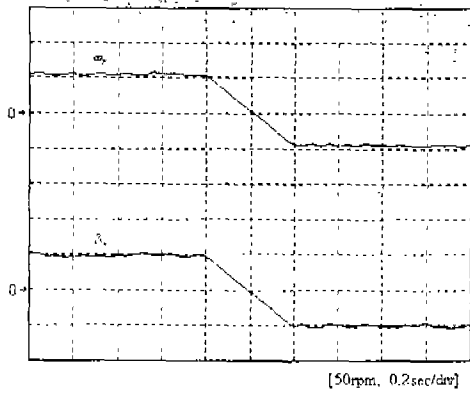


Fig. 6 Real and estimated speed response of conventional method

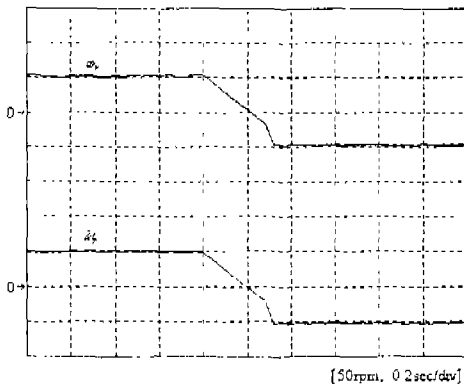


Fig. 7 Real and estimated speed response of proposed method

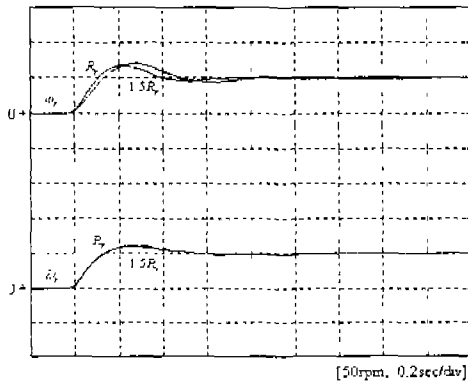


Fig. 8 Real and estimated speed response of proposed method with the variations of rotor resistance

## 7. CONCLUSIONS

In this paper, the robust vector control method for the purpose of improving the system performance deterioration caused by parameter variations is proposed. The full order state observer estimates the stator current and the rotor flux with corrective prediction error feedback of stator current. Adaptive scheme is constructed to estimate the rotor speed with the error signal between real and estimation value of the stator current without speed sensor. Estimating error of stator current is compensated by variable pole placement technique. In order to compensate estimating error of rotor flux, adaptive sliding observer is applied to current model. Consequently, an accurate state prediction of deadbeat current control is performed, and robustness of speed identifier is presented. The results of simulation show that robust deadbeat current control and speed sensorless vector control can be obtained simultaneously, regardless of parameter variations.

## 8. REFERENCES

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