APPLICATION OF EXTENDED LUENBERGER OBSERVER FOR INDUCTION MOTOR CONTROL

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ABSTRACT - In this paper, authors introduce an application of a nonlinear rotor flux observer, known under the name of ELO(extended Luenberger Observer), for direct rotor field oriented control(DRFOC) of induction motor. ELO requires no solution of nonlinear partial differential equation for its coordinate transformation and linearization used for the nonlinear observer design. Its simulation results concerned to different level of unknown variables of load torque and rotor resistance show high accuracy on rotor flux estimation in steady state.

I. INTRODUCTION

In the direct rotor field orientation of IM, a correctly designed flux observer can replace the flux sensor, if a mathematical model of IM is available.

In past 30 year's study, closed loop linear observer like Kalman filter(KF) or Luenberger observer(LO) used in IM drives may be successfully applied in a certain limited range of control around the predetermined operating point where IM model can be linearized. But this is not possible in wide control range since it hardly ensures that estimated variable convergent to actual one under the transient condition (such as step change of variables) and the parameter variation(such as saturation, and variation of time constants). To overcome these problems, some nonlinear type closed loop observers have been applied on states estimation of IM, for examples, extended Kalman filter (EKF)[1], modified Luenberger observer[2], nonlinear state observer [3], etc.

The observer proposed in [2] is based on operating point linearization and uses an observer gain which depends on the estimated state. There, authors argued that an ELO based on a deterministic approach is better than a EKF, a stochastic approach for IM drive because IM fed inverter is in essence a deterministic rather than stochastic and EKF have a bias problem which may cause overall system response or failure when covariance matrix was not tuned correctly, due to uncertainty in the noise characteristics.

A nonlinear state observer proposed in [3] are based on linear Luenberger observer theory and geometrical approaches. There, the error in reconstruction of rotor flux are synthesized through a feedback loop consisted by multiply of a gain matrix and an inverse matrix of observability matrix : the former is a diagonal matrix where poles can be placed arbitrary and the later is a geometrical approach.

In this paper, we propose a nonlinear state observer for rotor flux estimation of IM control. The proposed nonlinear state observer is named as an Extended Luenberger Observer because it is designed according to EKF theory, which is based on a linearization of the error dynamics along the estimated state trajectory[4][5]. ELO for state estimation could be easily designed through a few step of symbolic mathematical operation since it only requires a gain matrix derived from the inverse observability matrix. The gain matrix contains poles(eigenvalues) for the observer error dynamics in canonical coordinates.

2. SYSTEM MODELING

Induction Motor

A dynamic model of induction motor in a stationary reference frame with the state variables stator currents, rotor fluxes and rotor speed can be written in differential equations as follows:

$$\frac{di_{ds}}{dt} = -\left(\frac{R_rM^2}{\sigma L_sL_r} + \frac{R_s}{\sigma L_s}\right)i_{ds} + \frac{R_rM}{\sigma L_sL_r}\Phi_{dr} + \frac{M}{\sigma L_sL_r}\omega_r\Phi_{qr} + \frac{1}{\sigma L_s}v_{ds} \tag{1}$$

$$\frac{di_{qs}}{dt} = -\left(\frac{R_rM^2}{\sigma L_sL_r} + \frac{R_s}{\sigma L_s}\right)i_{qs} + \frac{R_rM}{\sigma L_sL_r}\Phi_{qr} - \frac{M}{\sigma L_sL_r}\omega_r\Phi_{dr} + \frac{1}{\sigma L_s}v_{qs} \tag{2}$$

$$\frac{d\Phi_{dr}}{dt} = \frac{R_rM}{L_r}i_{ds} - \frac{R_r}{L_r}\Phi_{dr} - \omega_r\Phi_{qr} \tag{3}$$

$$\frac{d\Phi_{qr}}{dt} = \frac{R_rM}{L_r}i_{qs} + \omega_r\Phi_{dr} - \frac{R_r}{L_r}\Phi_{qr} \tag{4}$$

$$\frac{d\omega_r}{dt} = \frac{N}{J}(Te - T_L) \tag{5}$$

where,

v_{ds} : d-axis stator voltage in stationary frame

v_{qs} : q-axis stator voltage in stationary frame

i_{ds} : d-axis stator current in stationary frame

i_{qs} : q-axis stator current in stationary frame

Φ_{dr} : d-axis rotor flux in stationary frame

Φ_{qr} : q-axis rotor flux in stationary frame

ω_r : mechanical speed in rad/sec

R_s : stator resistance

R_r : rotor resistance

L_s : stator inductance

L_r : rotor inductance

M : mutual inductance

$\sigma = 1 - M^2/(L_s L_r)$: leakage factor

N : number of pole pair

J : rotor inertia

$Te = \frac{3NM}{2L_r}(i_{qs}\Phi_{dr} - i_{ds}\Phi_{qr})$: electrical torque

T_L : load torque

Models of the induction motor and of the proposed observer on simulation were built with the above nonlinear state equations.

Direct Field Orientation Control

The equivalent circuits of induction motor in rotationary reference frame is shown in fig. 1.

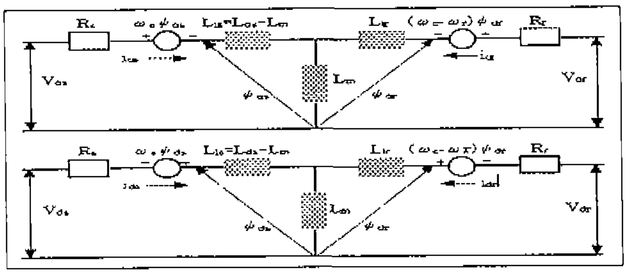


Fig 1. Equivalent circuits of induction motor in rotationary reference frame

In direct field oriented control of induction motor [6], 3ph stator currents are divided into flux and torque components : these are oriented to d and q axis of rotor flux, respectively, in rotationary reference frame by axis(coordinate) transformation. In RFOC, the angle, θ_r calculated from (6).

$$\theta_r = \tan^{-1}\left(\frac{\Phi_{qr}}{\Phi_{dr}}\right) \tag{6}$$

A dynamic IM model is derived by terms of coupled effects on two windings(rotor and stator) and on d-q axis as follows in the rotationary reference frame

$$v_{dse} = (R_s + Rr\frac{M^2}{L_r^2})i_{dse} + \sigma L_s\frac{di_{dse}}{dt} - R_r\frac{M}{L_r}\Phi_{dre} - \omega_e\sigma L_s i_{qse} - \omega_r\frac{M}{L_r}\Phi_{qre} \tag{7}$$

$$v_{qse} = (R_s + Rr\frac{M^2}{L_r^2})i_{qse} + \sigma L_s\frac{di_{qse}}{dt} - R_r\frac{M}{L_r}\Phi_{qre} + \omega_e\sigma L_s i_{dse} + \omega_r\frac{M}{L_r}\Phi_{dre} \tag{8}$$

Fluxes, Φ_{qre} and Φ_{dre} are maintained at zero and constant during field orientation control. For decoupling operation, terms involving ω_e are compensated on control circuit in terms of E_{dse} and E_{qse}. Finally, torque and flux of induction motor can be regulated by the stator voltages through PI type current controllers as follows:

$$v^*_{dse} = K_p(i^*_{dse} - i_{dse}) + K_i\int(i^*_{dse} - i_{dse}) + E_{dse} \tag{9}$$

$$v^*_{qse} = K_p(i^*_{qse} - i_{qse}) + K_i\int(i^*_{qse} - i_{qse}) + E_{qse} \tag{10}$$

Space Vector Pulse Width Modulation

In recent development in power electronic, a space vector pulse width modulation(SVPWM) have been widely applied on three phase power converter. A VSI is modeled by using the built-in blocks of SIMULINK according to SVPWM technique on reference [7].

3. EXTENDED LUENBERGER OBSERVER

In the following, the ELO design is described for multi-output nonlinear systems as far as necessary for its application on the IM model. The mathematical development of the design formulas can be found in [4][5].

Observability Assumption

Consider the nonlinear MIMO system

$$\dot{x} = f(x,u),\ x(0) = x_0, \quad y = h(x,u) \tag{11}$$

where, the state x is an n-vector, the input u is a m-vector and the output y is a p-vector. The nonlinear functions f(x,u) and h(x) are assumed to be sufficiently smooth.

It is assumed that the system (11) is locally observable, i.e. the $(n \times n)$ matrix

$$Q(x,u) = \begin{bmatrix} dh_1 \\ L_f dh_1 \\ \vdots \\ L_f^{n_1-1} dh_1 \\ \vdots \\ dh_p \\ L_f dh_p \\ \vdots \\ L_f^{n_p-1} dh_p \end{bmatrix}, \quad \sum_{j=1}^{p} n_j = n \tag{12}$$

has full rank n in the considered domain of x and u [8][9]. In (12), $L_f^i \mathrm{dh}_j$ is the Lie derivatives of the gradient dh_j along the vector field *f*. Each of these subsystems of order n_j is locally observable by the related output y_j which is used for the subsystemwise design of the ELO dynamics.

ELO Design Formula

The ELO has a structure which is similar to the well known linear Luenberger observer and comprises a simulation part and a correction part.

$$\dot{\hat{x}} = f(\hat{x},u) + G(\hat{x},u) \cdot [y - h(\hat{x},u)],\ \hat{x}(0) = \hat{x}_0 \tag{13}$$

In, general, the $n \times p$ gain matrix $G(\hat{x},u)$ depends on the reconstructed state $\hat{x}$ and input u. In order to design the gains $G(\hat{x},u)$ by ELO approach, the differential equation of the observer error $\tilde{\hat{x}}^* = \hat{x}^* - x^*$ is considered in the observer normal form coordinates x^* of (11). Moreover, the observer error dynamics is linearized along the reconstructed states which corresponds to the EKF approach. The *characteristic polynominal of the linearized observer* error dynamics is composed of the characteristic polynominals of the linearized observer subsystems

$$\prod_{j=1}^{q} \left\{ (\lambda^{n_j} + p_{jn_j}\lambda^{n_j - 1} + .. + p_{j2}\lambda + p_{j1}) \doteq \prod_{k=1}^{n_j} (\lambda - \lambda_{kj}) \right\} \tag{14}$$

here, p_{jk} is the characteristic coefficients. The gain matrix of the ELO (13) determined strait- forwardly as

$$\begin{aligned} G\ (\ \hat{x}^*, u^*) &= \{ [\ [p_{11} + p_{12} ad_f^0 +, \cdots, + \\ &\quad p_{1n_1} ad_f^{n_1 - 1} + ad_f^{n_1}] s_1(\hat{x},u), \cdots, \\ &\quad [p_{j_1} + p_{j_2} ad_f^0 +, \cdots, + p_{jn_j} ad_f^{n_q - 1} + ad_f^{n_q}] \\ &\quad s_q(\hat{x},u)\} \cdot \left[\frac{\partial \overline{h^*}}{\partial x_m^*}(\hat{x},u) \right]^{-1} \end{aligned} \tag{15}$$

This design formula is determined by n characteristic coefficients p_{jk}, p subsystem orders n_j defined in (12), the operator ad_f, p vectors $s_j(\hat{x},u)$, and the $(p \times p)$ diagonal matrix, Diag $\left[\frac{\partial \overline{h^*}}{\partial x_m^*}(\hat{x},u) \right]^{-1}$.

Here, the p vectors $s_j(x,u)$ is determined by belows

$$s_j(x,u) = \frac{\partial h_j^*}{\partial x_{n_j}^*}(\hat{x},u) Q^{-1}(\hat{x},u).e_{nj} \tag{16}$$

where e_{nj} is the $(n \times 1)$ unit vector. With that, the vectors $s_j(x,u)$ are the n_j-th columns of the inverse observability matrix (15) multiplied by the $\partial h_j^* / \partial x_{n_j}^*$ $(\hat{x},u)$. By an appropriate choice of the p *functions* $\partial h_j^* / \partial x_{n_j}^*(\hat{x},u) = 0,\ j = 1,..,p$, the vectors $s_j(\hat{x},u)$ can be simplified such that in (14) by applying the Lie bracket, ad_f. From this follows, these functions have the meaning of additional degrees of freedom in course of the ELO design.

It should be noted that the calculation of the ELO gain matrix (14) is possible for all sufficiently smooth and locally observable systems (11). Finally, the ELO design formula (14) is consistent with the well known Ackermann Formula for linear time-variant observers.

Design of ELO for Induction Motor

In order to use the ELO for a flux estimation of induction motor, the ELO design formulas are applied on the motor model derived in the previous section. As a results, states, inputs and outputs of IM are chosen as follows:

$$\begin{aligned} x &= [i_{ds},\ i_{qs},\ \Phi_{dr},\ \Phi_{qr},\ \omega_r,\ T_L]^T \\ u &= [V_{ds},\ V_{qs}]^T \\ y &= [i_{ds},\ i_{qs},\ \omega_r]^T \end{aligned} \tag{17}$$

Observability matrix,(12) is calculated from (1)~(5) and (17). All of symbols are listed in Appendix I.

$$Q(x,\ u) = [\ \mathrm{dh}_1, L_f\,\mathrm{dh}_1, \mathrm{dh}_2, L_f\,\mathrm{dh}_2, \mathrm{dh}_3, L_f\,\mathrm{dh}_3]^T$$
$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -a_1 & 0 & a_3 & a_4x_5 & a_4x_4 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -a_1 & -a_4x_5 & a_3 & -a_4x_3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -a_9x_4 & a_9x_3 & a_9x_2 & -a_9x_1 & 0 & -a_{10} \end{bmatrix} \tag{18}$$

Each row of (18) is linearly independent from each other and the system is observable as required for ELO design. The inverse observability matrix derived from (18) is

$$Q(x,\ u)^{-1}$$
$$= \frac{1}{D} \begin{bmatrix} D & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 & 0 & 0 \\ k_1 & k_2 & -k_3x_5 & -k_4x_5 & m_{35} & 0 \\ k_3x_5 & k_4x_5 & k_1 & k_2 & m_{45} & 0 \\ 0 & 0 & 0 & 0 & D & 0 \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \tag{19}$$

where, $D = a_3^2 a_{10} + a_4^2 a_{10} x_5^2$, k_c : constants, and m_{yy}: equations consisted of constants and states variables.

The subsystem orders of ELO are found easily from eq (19) : n_1=1, n_2=1 and n_3=1. Therefore, a row vector per output vector is needed in the gain matrix since n_x=1. By choosing the common determinant of inverse observability matrix as below in degree of freedom for simplification of the vectors s_1, s_2 and s_3,

$$\frac{\partial \overline{h_1^*}}{\partial x_2^*} = \frac{\partial \overline{h_2^*}}{\partial x_4^*} = \frac{\partial \overline{h_3^*}}{\partial x_6^*} = D \tag{20}$$

gain matrix (14) is calculated as

$$\begin{aligned} s_1 &= \{p_1 * \lambda_{11} + \lambda_{12}\}/D \\ s_2 &= \{p_2 * \lambda_{21} + \lambda_{22}\}/D \\ s_3 &= \{p_3 * \lambda_{31} + \lambda_{32}\}/D \end{aligned} \tag{21}$$

where,

p_1, p_2 and p_3 : characteristic coefficients of ELO error dynamics

$\lambda_{11} = [0,\ D,\ k_2,\ k_4x_5,\ 0,\ m_{62}]^T$

$\lambda_{21} = [0,\ 0,\ -k_4x_5,\ k_2,\ 0,\ m_{64}]^T$

$\lambda_{31} = [0,\ 0,\ 0,\ 0,\ 0,\ m_{66}]^T$

$$\lambda_{12} = \begin{bmatrix} g_8 + g_5x_5^2 \\ (g_5 - g_0)x_5 \\ -g_1 - g_7x_5^2 \\ (g_2 - g_6)x_5 - g_8x_2x_3 + g_8x_1x_4 + g_9x_6 \\ g_3x_2 - g_8x_4x_5 + g13x_2 - g_{18}x_1x_5 \\ G1 \end{bmatrix}$$

$$\lambda_{22} = \begin{bmatrix} (g_0 - g_4)x_5 \\ g_0 + g_5x_5^2 \\ (g_6 - g_2)x_5 + g_8x_2x_3 - g_8x_1x_4 - g_9x_6 \\ -g_7x_5^2 - g_1 \\ -g_8x_2x_5 - g_3x_4 + g_{13}x_1 - g_{18}x_2x_5 \\ G2 \end{bmatrix}$$

$$\lambda_{32} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -g_{19} - g_{21}x_5^2 \\ 2g_{20}(x_2x_3x_5 - x_1x_4x_5) - 2g_{21}x_5x_6 \end{bmatrix}$$

As seen before, a ELO, one of nonlinear state observer can be constructed with minimal effort to find the inverse observability matrix and the gain matrix from original state matrix. Finally, the proposed ELO has a form of (22) consisted with the original nonlinear system and gain matrix.

$$\dot{\hat{x}} = f(\hat{x}, u) + \{g_1(\hat{y}_1 - y_1) + g_2(\hat{y}_2 - y_2) + g_3(\hat{y}_3 - y_3)\} \tag{22}$$

4. SIMULATION

ELO designed in previous section is verified by SIMULINK simulation. Fig 2 shows the simulation block diagram of induction motor drive with ELO.

In parameters setting of ELO, the motor parameters of ELO were set at normal values of motor listed in Appendix II and the poles of gain matrix on feedback loops were initially set at $p_1 = p_2 = -10$ and $p_3 = -1000$ corresponding to states of i_{ds}, i_{qs} and ω_r, respectively. The switching frequency, f_{sw} was set at 7.2㎑.

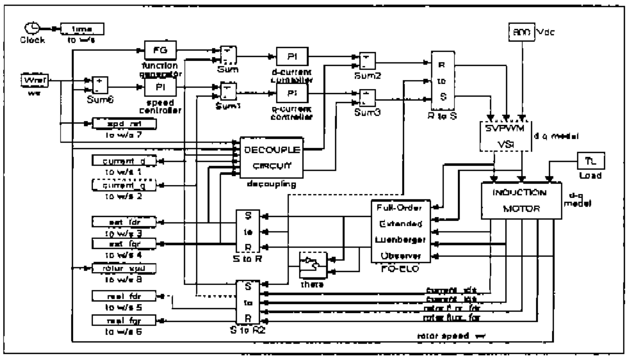
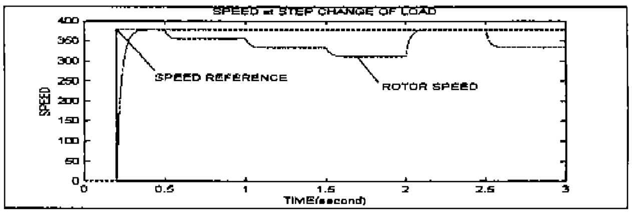


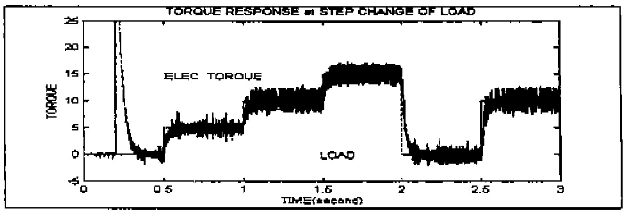
Fig 2. Simulation block diagram of induction motor

Simulations were performed with two unknown variables : load torque and rotor resistance. Saturation effect of flux is not considered in this study.

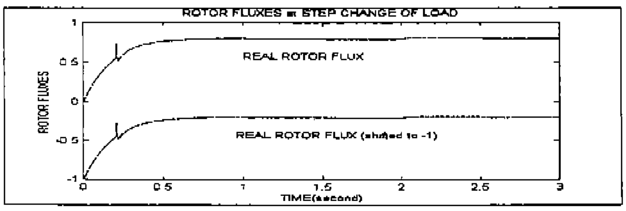
First, load torque was chosen as a unknown variable. Fig 3 show the simulation results were taken under the step change of load torque whereas that of ELO was set at zero(0 Nm). Comparing estimated rotor flux to real one, errors were tend to convergent to zero : error of estimated rotor flux with respect to measured rotor flux, $e = (\hat{f} - f)/f$ are 0.08%, 0.19% and 0.31% when load torque was step to 5Nm, 10Nm and 15Nm as shown fig. 3.



(a) speed reference and rotor speed



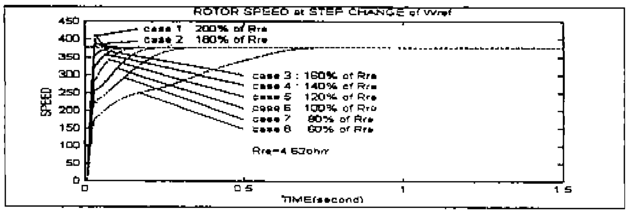
(b) load torque and generated torque



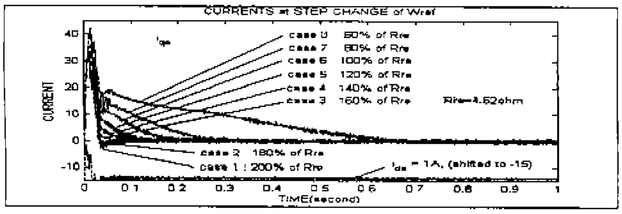
(c) real and estimated rotor flux

Fig 3. Simulation with step change of the load torque

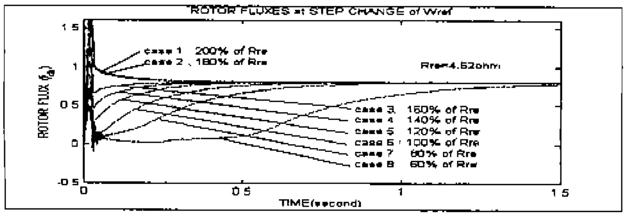
Second, rotor resistance was chosen as the unknown variables. In this case, rotor resistance was initially set at different level from that of ELO: 200%, 180%, 160%, 140%, 120%, 100%, 80% and 60% of normal value of rotor resistance(Rrn=4.62Ω). From fig. 4, the steady state error of estimated rotor flux with respect to measured rotor flux are 0.12%, 0.12%, 0.01% 0.02%, 0.04%, 0%, 0.06% and 0.12% respectively.



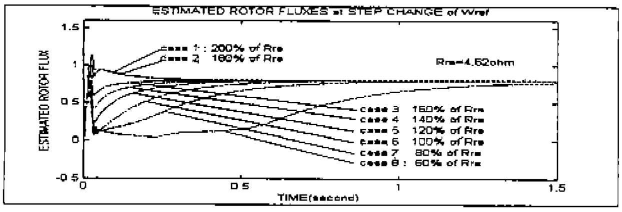
(a) speed reference and rotor speed



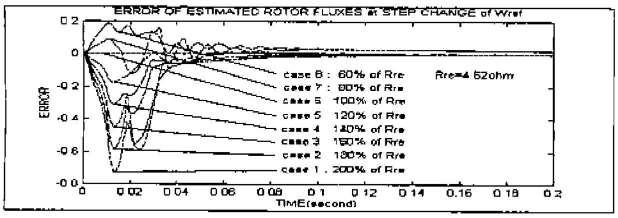
(b) stator currents



(c) real rotor fluxes



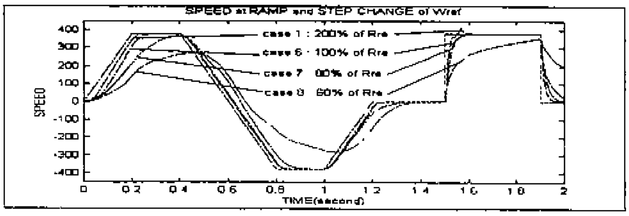
(d) estimated rotor fluxes



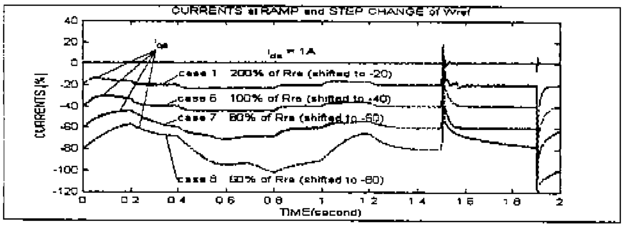
(e) errors of estimated rotor fluxes

Fig 4. Start at different rotor resistance

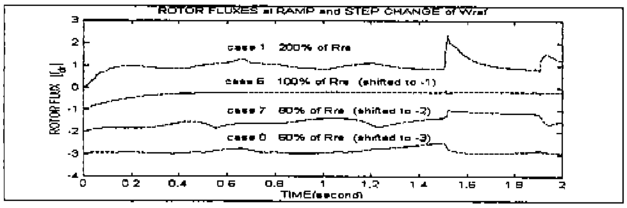
Fig. 5 show the simulation results were obtained under continuous lamp and step changes of speed. On fig. 5, initial rotor resistance of ELO were set at same values of fig.4.



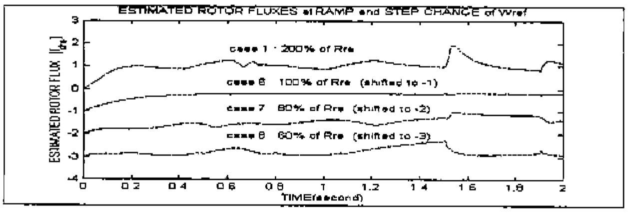
(a) speed reference and rotor speed



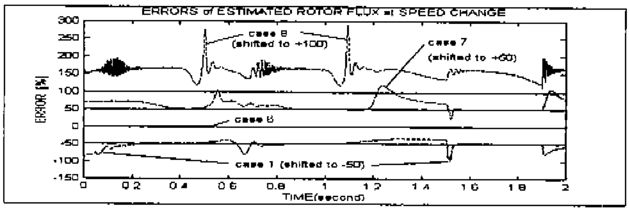
(b) stator currents



(c) real rotor fluxes



(d) estimated rotor luxes



(e) errors of estimated rotor fluxes

Fig 5. Ramp and step change of rotor speed

From the fig 4 and 5, the dynamic response of speed is not fast enough when rotor resistance of IM model is set at below of 80% where error is small enough because the gains of PI controllers are not appropriate.

5. CONCLUSION

In order to compensate the nonlinearity of induction motor and to improve the transient response in field oriented control, we proposed the application of extended Luenberger observer, as one approach of nonlinear state observer known from literatures [3-9]. We calculate the observability matrix and its inverse matrix based on 6 differential state equations of induction motor. Simulation results of induction motor drive under field oriented control with a rotor flux estimator, ELO have been presented.

The effectiveness of ELO on induction motor drive were verified by setting the unknown load torque and rotor resistance at different levels : the steady state errors of estimated rotor flux in two case are lower than 0.31% and 0.12%, respectively.

From the above results, authors believe that the proposed ELO can be applied successively in wide control range if initial setting of rotor resistance is properly chosen at its minimum level.

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Appendix I. Coefficients of ELO design

$$a_1 = \left(\frac{R_r M^2}{\sigma L_s L_r} + \frac{R_s}{\sigma L_s} \right),\ a_3 \frac{R_r M}{\sigma L_s L_r},\ a_3 = \frac{M}{\sigma L_s L_r}$$

$$a_5 = \frac{R_r M}{L_r},\quad a_7 = \frac{R_r}{L_r},\quad a_8 = 1$$

$$a_9 = \frac{N}{J} \frac{3NM}{2L_r},\quad a_{10} \frac{=N}{J}$$

$k_1 = a_1 a_3 a_{10},\ k_2 = a_3 a_{10},\ k_3 = a_1 a_4 a_{10},\ k_4 = a_4 a_{10},$
$k_5 = a_3^2 a_{10},\ k_6 = a_4^2 a_{10},\ k_7 = a_3 a_4 a_{10},\ k_8 = a_3 a_4 a_9,$
$k_9 = a_1 a_3 a_9,\ k_{10} = a_3^2 a_9,\ k_{11} = a_1 a_4 a_9,\ k_{12} = a_4^9 a_9,$
$k_{13} = a_3 a_9,\ k_{14} = a_4 a_9,\ k_{15} = a_3^2,\ k_{16} = a_4^2$

$D = k_5 - k_6 x_5^2$

$g_g = a_3 k_2 \quad g_0 = a_4 k_2 \quad g_1 = a_7 k_2 \quad g_2 = a_8 k_2$
$g_3 = a_9 k_2 \quad g_4 = a_3 k_4 \quad g_5 = a_4 k_4 \quad g_6 = a_7 k_4$
$g_7 = a_8 k_4 \quad g_8 = a_9 k_4 \quad g_9 = a_{10} k_4 \quad g_{10} = a_1 k_{13}$
$g_{11} = a_3 k_{13} \quad g_{12} = a_4 k_{13} \quad g_{13} = a_{10} k_{13} \quad g_{14} = a_1 k_{14}$
$g_{15} = a_3 k_{14} \quad g_{16} = a_4 k_{14} \quad g_{17} = a_9 k_{14} \quad g_{18} = a_{10} k_{14}$
$g_{19} = a_{10} k_{15} \quad g_{20} = a_9 k_{16} \quad g_{21} = a_{10} k_{16}$

$m_{35} = -k_7 x_4 - k_6 x_3 x_5$
$m_{45} = k_7 x_3 - k_6 x_4 x_5$
$m_{61} = k_9 x_2 - k_{10} x_4 - k_{11} x_1 x_5 + k_{12} x_4 x_5^2$
$m_{62} = k_{13} x_2 - k_{14} x_1 x_5$
$m_{63} = -k_9 x_1 + k_{10} x_5 - k_{11} x_2 x_5 + k_{12} x_3 x_5^2$
$m_{64} = k_{13} x_1 - k_{14} x_2 x_5$
$m_{65} = -k_8 x_1 x_3 + k_8 x_2 x_4 - k_{12} x_2 x_3 x_5 + k_{12} x_1 x_4 x_5$
$m_{66} = -k_{15} - k_{16} x_5^2$

$m_{35} = -k_7 x_4 - k_6 x_3 x_5$
$m_{45} = k_7 x_3 - k_6 x_4 x_5$
$m_{61} = k_9 x_2 - k_{10} x_4 - k_{11} x_1 x_5 + k_{12} x_4 x_5^2$
$m_{62} = k_{13} x_2 - k_{14} x_1 x_5$
$m_{63} = -k_9 x_1 + k_{10} x_5 - k_{11} x_2 x_5 + k_{12} x_3 x_5^2$
$m_{64} = k_{13} x_1 - k_{14} x_2 x_5$
$m_{65} = -k_8 x_1 x_3 + k_8 x_2 x_4 - k_{12} x_2 x_3 x_5 + k_{12} x_1 x_4 x_5$
$m_{66} = -k_{15} - k_{16} x_5^2$

$$G_1 = -g_{14} x_1 x_5 + g_{15} x_3 x_5 + g_{16} x_4 x_5^2 + g_1 x_1 x_2 x_3 - g_{17} x_1^2 x_2 - g_{18} x_1 x_6 + g_{10} x_2 + g_{12} x_3 x_5 - g_{11} x_4$$

$$G_2 = g_{10} x_1 - g_{11} x_3 - g_{12} x_4 x_5 - g_{14} x_2 x_5 - g_{16} x_3 x_5^2 + g_{15} x_4 x_5 + g_{17} x_2^2 x_3 - g_{17} x_1 x_2 x_4 - g_{15} x_2 x_6$$

Appendix Ⅱ. Motor parameters and controller gains

Rated power : 1.5kw,
Rated voltage : 480Vac,
Stator resistance: 5.6Ω,
Rotor resistance : 4.6Ω,
Stator inductance: 0.831 H,
Rotor inductance: 0.833 H,
Mutual inductance: 0.809 H,
Function generator: 1,
Speed controller: Kp=0.1, Kp=0,
Flux controller(FC): Kp=40, Ki=20000,
Torque Controller(TC): Kp=40, Ki=20000