

# Simultaneous Estimation of the Speed and the Secondary Resistance under the Transient State of Induction Motor

Kan Akatsu\* Atsuo Kawamura\*\*

Department of Electrical and Computer Engineering  
Yokohama National University  
79-5 Tokiwadai, Hodogaya-ku  
Yokohama 240-8501 JAPAN  
E-mail: \* aka@kawalab.dnj.ynu.ac.jp  
E-mail: \*\* kawamura@kawalab.dnj.ynu.ac.jp

*Abstract*— In the speed sensorless control of the induction motor, the machine parameters (especially the secondary resistance  $R_2$ ) have a strong influence to the speed estimation. It is known that the simultaneous estimation of the speed and  $R_2$  is impossible in the slip frequency type vector control, because the secondary flux is constant. But the secondary flux is not always constant in the speed transient state. In this paper the  $R_2$  estimation in the transient state without adding any additional signal to the stator current is proposed. This algorithm uses the least mean square algorithm and the adaptive algorithm, and it is possible to estimate the  $R_2$  exactly. This algorithm is verified by the digital simulations and the experiments.

## I. INTRODUCTION

The vector control has been widely used for the high performance control of the induction motor. Recently the sensorless vector control without the speed sensor is much focused and progressed. Though most of the vector control drives are based on the slip frequency type vector control (or indirect control), this control is strongly influenced by the  $R_2$  variation because the  $R_2$  is used for the speed estimation. It is known that the simultaneous estimation of the speed and the  $R_2$  is very difficult and it is impossible to estimate them simultaneously under the steady state based on the slip frequency type vector control [1].

To overcome the above problem, there are several algorithms for the  $R_2$  estimation in the speed sensorless control. In [2][3] the  $R_2$  was estimated from the higher order harmonics of the rotor slots, but it is difficult to estimate the  $R_2$  in the low speed because it becomes difficult to measure the higher order harmonics in the low speed. In [4] the  $R_2$  was estimated with adding the small alternating current to the secondary flux and it is fluctuated, but the ripple of the torque and the real speed oscillation are caused.

In this paper, the new algorithm for the  $R_2$  estimation under the speed transient state without any additional signal with only the stator current measurement is proposed. The least mean square algorithm and the adaptive algorithm are used for the  $R_2$  estimation and it is possible to estimate the  $R_2$  with the small calculation under the transient state.

Proceedings ICPE '98, Seoul

## II. SENSORLESS SPEED CONTROL ALGORITHM

### A. Speed estimation algorithm

The following differential equations fixed on the synchronously rotating reference frame (d-q) are assumed [5].

$$\begin{bmatrix} v_1 \\ 0 \end{bmatrix} = \begin{bmatrix} (R_1 + pL_1)I + \omega L_1 J & * \\ pL_m I + \omega_s L_m J & \\ pL_m I + \omega L_m J & \\ (R_2 + pL_2)I + \omega_s L_2 J & \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (1)$$

where

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (2)$$

All symbols are listed in the appendix.

Making the inner-product of the second row of (1) and  $J\Phi_2$ , and solving for  $\omega_s$  produces,

$$\omega_s = -R_2 \frac{i_2^T J \Phi_2}{\|\Phi_2\|^2} - \frac{p\Phi_2^T J \Phi_2}{\|\Phi_2\|^2} \quad (3)$$

$$\omega_r = \omega - \omega_s \quad (4)$$

In (3), the secondary flux vector  $\Phi_2$  and the secondary current vector  $i_2$  are replaced with the estimated values  $\hat{\Phi}_2$  and  $\hat{i}_2$ , as follows.

$$\hat{\Phi}_{1s} = \int (v_{1s}^* - R_1 i_{1s}) dt \quad (5)$$

$$\hat{i}_{2s} = \frac{1}{L_m} (\hat{\Phi}_{1s} - L_1 i_{1s}) \quad (6)$$

$$\hat{\Phi}_{2s} = L_m i_{1s} + L_2 \hat{i}_{2s} \quad (7)$$

$$\hat{i}_2 = \text{Rot}[\theta] \cdot \hat{i}_{2s} \quad (8)$$

$$\hat{\Phi}_2 = \text{Rot}[\theta] \cdot \hat{\Phi}_{2s} \quad (9)$$

where

$$\text{Rot}[\theta] = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (10)$$

$$\theta = \int \omega dt \quad (11)$$

Rated Power	0.75(kW)	$R_1$	0.435( $\Omega$ )
Rated Torque	4.8(N·m)	$R_2$	0.285 ( $\Omega$ )
Rated Speed	1500(rpm)	$L_2$	24.07(mH)
Pole No.	4	$L_1$	23.97(mH)
		$L_m$	22.94(mH)

TABLE I  
PARAMETERS OF THE ADTR MOTOR

where  $\omega$  is the electrical angular velocity.

In the all equations, the variables with the subscription "s" mean on the stationary reference frame, and the variables with "\*" mean a command reference. In this algorithm, the voltage is replaced with voltage command reference  $v_{1s}^*$ .

The slip frequency estimation  $\hat{\omega}_s$  and the rotor angular velocity estimation  $\hat{\omega}_r$  are made as follows.

$$\hat{\omega}_s = -\hat{R}_2 \frac{\hat{i}_2^T J \hat{\Phi}_2}{\|\hat{\Phi}_2\|^2} - \frac{p \hat{\Phi}_2^T J \hat{\Phi}_2}{\|\hat{\Phi}_2\|^2} \quad (12)$$

$$\hat{\omega}_r = \omega - \hat{\omega}_s \quad (13)$$

The second term of the right sides in (12) is a transient term. In the low speed range, the steady state operation is not always maintained due to the mechanical instability such as nonlinear friction-load characteristics and spatial harmonics caused by the stator slots. Thus, this transient term is very important and has strong effects to the stability.

The integration in (5) produces a problem of a dc offset and drift component in the low speed region. This problem was avoided in [6].

Fig.1 shows the block diagram of the sensorless speed estimation algorithm. In Fig.1  $e^{-T_s p}$  is the delay of the one sampling period.

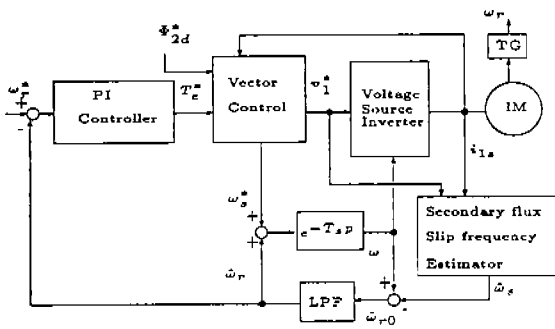


Fig. 1. Block diagram of the sensorless speed control

The motor parameters of the experiments and simulations are shown in Table I[7].

### B. Influence of $R_2$ variation

From (12) it is obvious that the  $R_2$  variation makes much influence to the speed estimation. Though the  $R_2$  variation does not make influence to the torque control,

it makes much influence to the speed control because of the speed control loop in Fig.1. Thus even if the secondary flux is estimated absolutely, the estimation error of the  $R_2$  causes the error of the speed control in proportional to the estimation error of the  $R_2$ .

Fig.2 shows the relationship between the variation of  $R_2$  and the estimation error of the slip frequency under the 50% load. From Fig.2 it is obvious that the change of  $R_2$  has much influence to the speed estimation in the low speed under the 100(rpm).

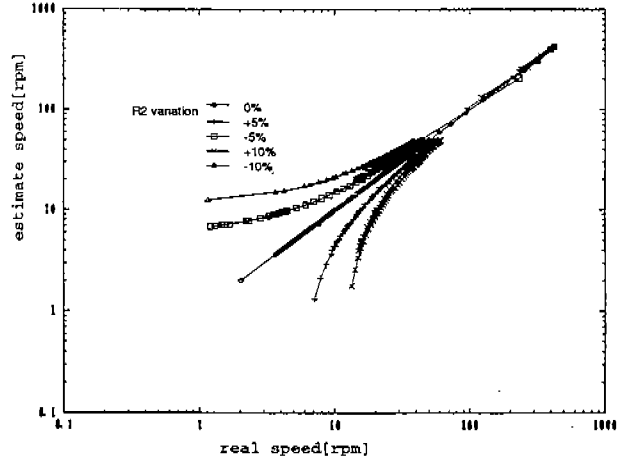


Fig. 2. the variation of  $R_2$  and estimation error of the slip frequency (log scale)

## III. $R_2$ ESTIMATION

### A. $R_2$ estimation under the steady state

Though we need to estimate  $R_2$  in the low speed, it is impossible to estimate  $R_2$  and the speed at the same time under the field oriented control, in which the secondary flux is constant [1].

Dividing the second row of (1) to d-q components, and canceling  $\omega_s$  produces  $R_2$  [8]. We can obtain the estimation value of  $\hat{R}_2$  with replacing the  $\Phi_2$  and the  $i_2$  to the estimated value.

$$\hat{R}_2 = -\frac{p \hat{\Phi}_2^T \hat{\Phi}_2}{\hat{i}_2^T \hat{\Phi}_2} \quad (14)$$

If this equation is used in the steady state, it is not possible to estimate  $R_2$  in the indirect control, because the numerator of this equation is zero if the  $\hat{\Phi}_2$  is constant, and the denominator becomes zero because the  $\hat{\Phi}_2$  and the  $\hat{i}_2$  are orthogonal. Thus in [4] a small alternating current is added to  $i_{1d}^*$  and  $\hat{\Phi}_2$  is fluctuated. But in the slip frequency control type vector control (indirect control), this alternating current makes oscillations to the real speed.

### B. $R_2$ estimation under the transient state

It is cleared that the estimation of  $R_2$  without any additional signal to the stator current is required for the low speed estimation by the algorithm in (12). In this paper  $R_2$  is estimated through the speed transient,

because the secondary flux is not exactly constant in this period. Fig.3 shows the secondary flux behavior when the speed changes from 150(rpm) to 100(rpm).

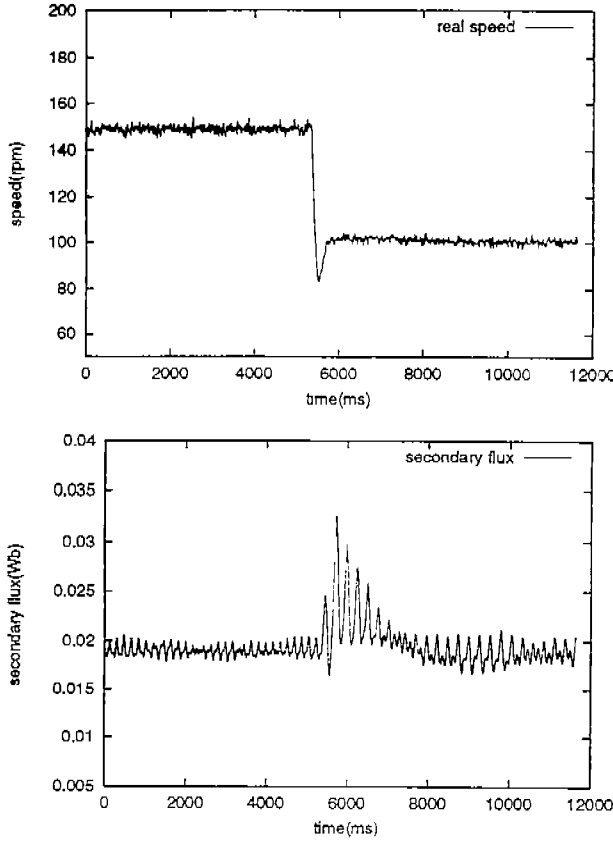


Fig. 3. The secondary flux behavior under the transient state (experiment)

Then the simultaneous estimation of the speed and the  $R_2$  is possible if the  $R_2$  is estimated exactly under the transient state and is held the previous value under the steady state. Though it is impossible to estimate  $R_2$  when the  $R_2$  is changed rapidly, it is possible to estimate  $R_2$  because generally  $R_2$  is changed slowly.

### B.1 Least mean square algorithm

To realize the above algorithm the least mean square algorithm seems to be suitable. The equation (14) is changed for the model of the least mean square algorithm as follows.

$$\frac{1}{2}p\|\hat{\Phi}_2\|^2 = -\hat{R}_2\hat{i}_2^T\hat{\Phi}_2 \quad (15)$$

To avoid the differential function,  $\frac{1}{1+\tau p}$  is multiplied to the equation (15).

$$\frac{p}{1+\tau p}\|\hat{\Phi}_2\|^2 = -2\hat{R}_2\frac{1}{1+\tau p}\hat{i}_2^T\hat{\Phi}_2 \quad (16)$$

Then the following new variables are defined.

$$y \triangleq \frac{p}{1+\tau p}\|\hat{\Phi}_2\|^2 \quad (17)$$

$$u \triangleq -2\frac{1}{1+\tau p}\hat{i}_2^T\hat{\Phi}_2 \quad (18)$$

$$\hat{\theta} \triangleq \hat{R}_2 \quad (19)$$

Then equation(16) is changed to the model for the LMS algorithm, in which  $\hat{\theta}$  is estimated from  $u$  and  $y$  as follows.

$$y = \hat{\theta}u \quad (20)$$

A low pass filter is used for  $\frac{1}{1+\tau p}$ , and a high pass filter is used for  $\frac{p}{1+\tau p}$ .

Fig.4 shows the block diagram for the estimation of  $R_2$ .

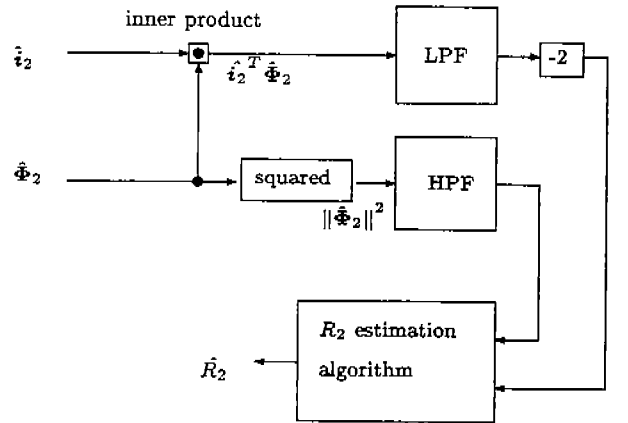


Fig. 4. Block diagram for the  $R_2$  estimation

The proposed least mean square algorithm is as follows.

$$\hat{\theta}[N] = \hat{\theta}[N-1] + k[N](y[N] - u[N]\hat{\theta}[N-1]) \quad (21)$$

$$k[N] = \begin{cases} \frac{P[N-1]u[N]}{\rho + u[N]^2 P[N-1]} & y[N-1] \geq \alpha \\ 0 & y[N-1] \leq \alpha \end{cases} \quad (22)$$

$$P[N] = (1 - k[N]u[N])P[N-1]\frac{1}{\rho} \quad (23)$$

where

$\rho$  : a forgetting factor

$\alpha$  : a threshold value

In this algorithm, a threshold value  $\alpha$  divides the state into the steady state and the transient state. In

the transient state,  $\hat{R}_2$  is obtained from (21). In the steady state, the  $\hat{R}_2$  is not renewed because  $k[N]$  becomes zero in (22). Fig.5 shows  $R_2, \hat{R}_2$  and  $\omega_r, \hat{\omega}_r$  when the rotor speed changes.

Fig.5 indicates that the true  $R_2$  is obtained only after the speed of the motor changes.

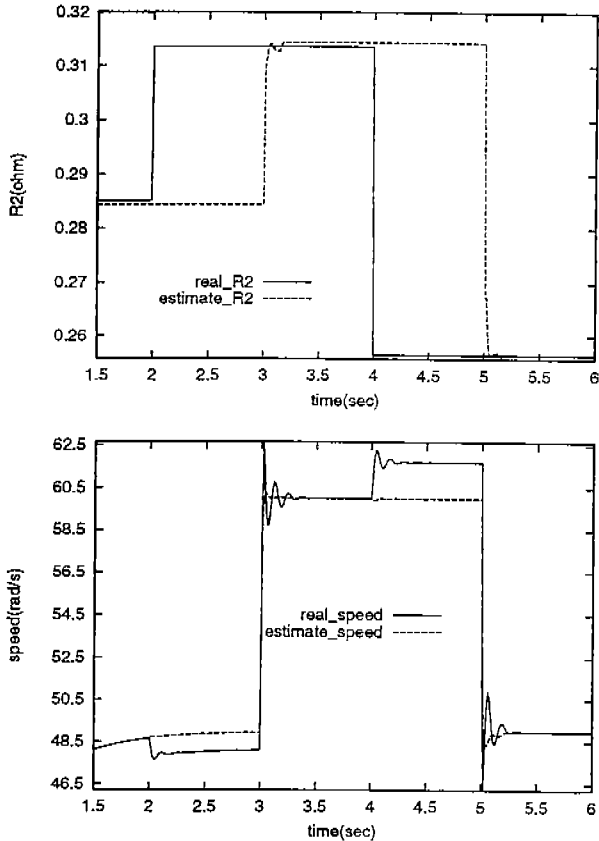


Fig. 5. Simultaneous estimation of the speed and the  $R_2$  by LMS algorithm (simulation)

## B.2 Constant gain algorithm

Although the proposed least mean square algorithm is able to estimate  $R_2$ , it is difficult to decide a forgetting factor and a threshold value. Thus, in the experiment it is difficult to estimate  $R_2$  in this algorithm. To solve this problem, we proposed a constant gain algorithm [9] to estimate  $R_2$ , in which a forgetting factor is changed automatically and we do not need a threshold value. The proposed constant gain algorithm is as follows.

$$\hat{\theta}[N] = \hat{\theta}[N-1] + P[0]u[N]e[N] \quad (24)$$

$$e[N] = \frac{1}{1 + u[N]^2 P[0]} (y[N] - \hat{\theta}[N-1]u[N]) \quad (25)$$

Where  $u[N]$  and  $y[N]$  are the same with the LMS algorithm.

In the steady state the  $u[N]$  becomes 0 and  $\hat{\theta}[N]$  ( $=\hat{R}_2$ ) is not renewed in (24). Then this algorithm does

not require for the threshold value and the forgetting factor.

In this algorithm the speed which  $\hat{\theta}[N]$  converges to the real value is decided by the gain  $P[0]$ . Although in the theory  $P[0]$  should be selected to be very large and  $\hat{\theta}[N]$  converges to the real value rapidly, in the real situation  $\hat{\theta}[N]$  has the oscillation because of a kind of disturbance. Thus  $P[0]$  must be chosen suitably.

Fig.6 shows the simulation for the estimation result of  $R_2$  and the rotor speed when  $R_2$  increases linearly. In the case when  $R_2$  increases linearly, the estimated value  $\hat{R}_2$  rapidly approaches to the true value in the transient.

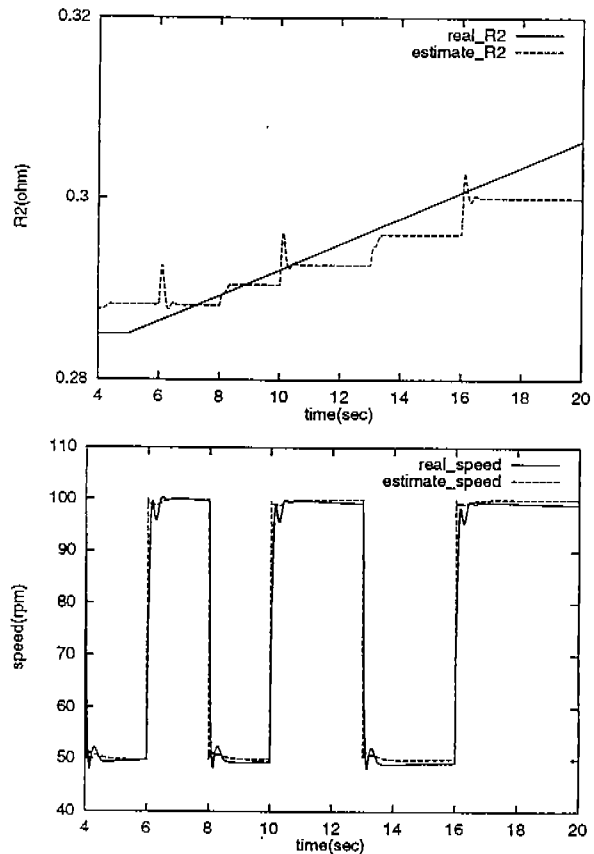


Fig. 6. Simultaneous estimation of the speed and the  $R_2$  by constant gain algorithm (simulation)

## IV. EXPERIMENTAL RESULTS

### A. Experimental equipment

Fig.8 shows the configuration of the experimental equipment. The tested induction motor is the Anti-Directional-Twin-Rotary (ADTR) motor (in Fig.7) for the EV drives (the parameter is shown in Table.I) [7]. The value of  $R_1$  in Table.I is an average value because the ADTR motor has a slip ring, by which the resistance changes as a function of the primary currents. In the experience the  $R_1$  was estimated by the algorithm in Appendix B.

The load torque is 20% rated produced by the DC

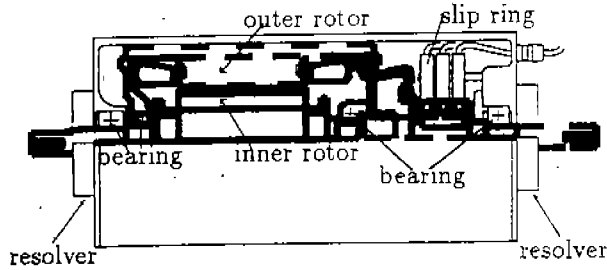


Fig. 7. Structure of the ADTR-motor

generator (0.75kw rating). The sampling period ( $T_s$ ) of this algorithm is 200(us) and switching frequency of the PWM inverter is 2.5(kHz). The inverter used the MOS-FET(2SK2586-Hitachi) and the DC voltage is 30(V) so that the influence of the voltage drop due to the dead time of the inverter is refused.

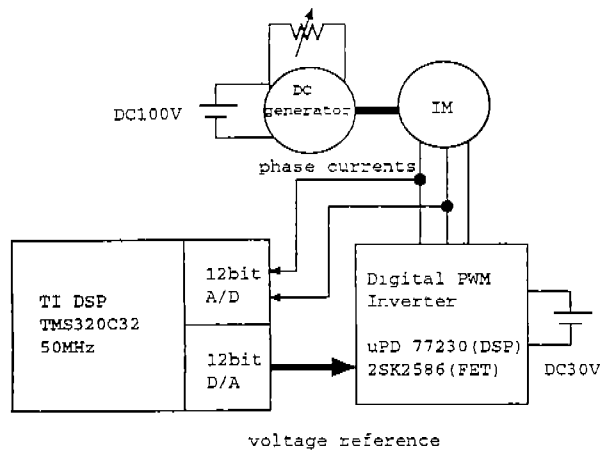


Fig. 8. Configuration of experimental equipment

### B. Experimental result of the simultaneous estimation of the speed and the $R_2$

Fig.9 shows the experimental result of the rotor speed and  $R_2$  estimation by the constant gain algorithm in which the speed reference changed from 100(rpm) to 150(rpm).

In the steady state when the rotor speed reference is 100(rpm), there is about 15% of the rotor speed estimation error because  $\hat{R}_2$  is not equal the real one.

When the rotor speed reference changes to 150(rpm), the estimate value  $\hat{R}_2$  rapidly approaches to the real value through the transient state, and the rotor speed estimation error approached zero.

### V. CONCLUSIONS

In this paper  $R_2$  is estimated by the constant gain algorithm and the LMS algorithm when the rotor speed is in the transient. The distinguished features of this algorithm are

- Simultaneous estimation of the speed and the  $R_2$  is possible

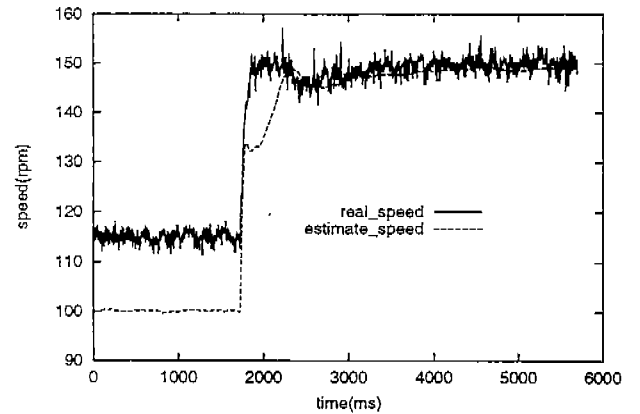
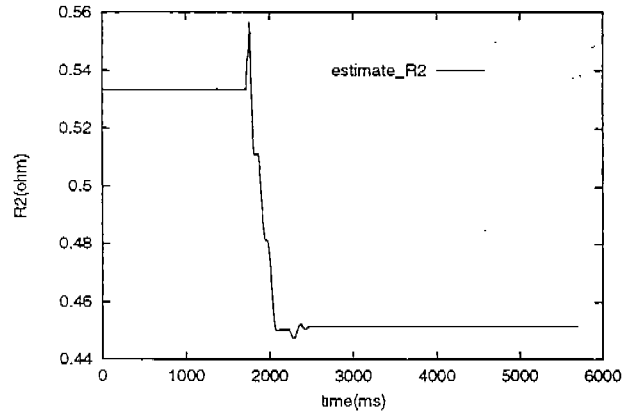


Fig. 9. Experimental result of the speed and  $R_2$  estimation

- Only the primary current is used
- It is possible to estimate  $R_2$  in the low speed

This compensation seems to be effective for the low rotor speed estimation. However there is a problem that the inverter voltage compensation algorithm without using  $R_1$  is needed, because the voltage drop of the inverter is very sensitive to the rotor speed estimation in the low speed range. And the  $R_1$  estimation in Fig.10 is not able to estimate  $R_1$  exactly when the actual inverter voltage does not have the negligible error compare with the voltage reference.

### APPENDIX

#### A. marks

- $R_1, R_2$  : stator and rotor resistance
- $L_1, L_2$  : stator and rotor self inductance
- $L_m$  : mutual inductance
- $\omega_s$  : slip frequency
- $\omega_r$  : rotor angular velocity
- $v_1$  : primary voltage vector
- $\hat{i}_1, \hat{i}_2$  : primary and secondary current vector
- $\Phi_1$  : primary flux vector ( $L_1 \hat{i}_1 + L_m \hat{i}_2$ )
- $\Phi_2$  : secondary flux vector ( $L_m \hat{i}_1 + L_2 \hat{i}_2$ )
- $p$  : derivative operator (d/dt)

#### B. $R_1$ estimation

$R_1$  is adjusted so that the secondary flux reference  $\Phi_{2d}^*$  and the estimated secondary flux  $\hat{\Phi}_{2d}$  become the

same in Fig.10 [10]. Fig.11 shows the estimation result of the  $R_1$  when the rotor speed is 160(rpm) under no load condition.

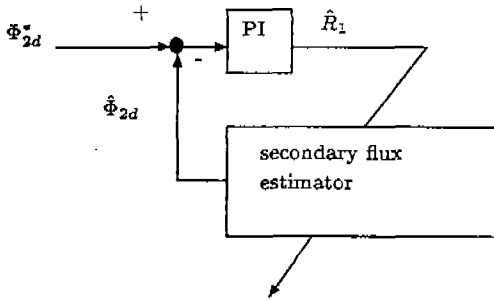


Fig. 10. the block diagram of  $R_1$  estimation

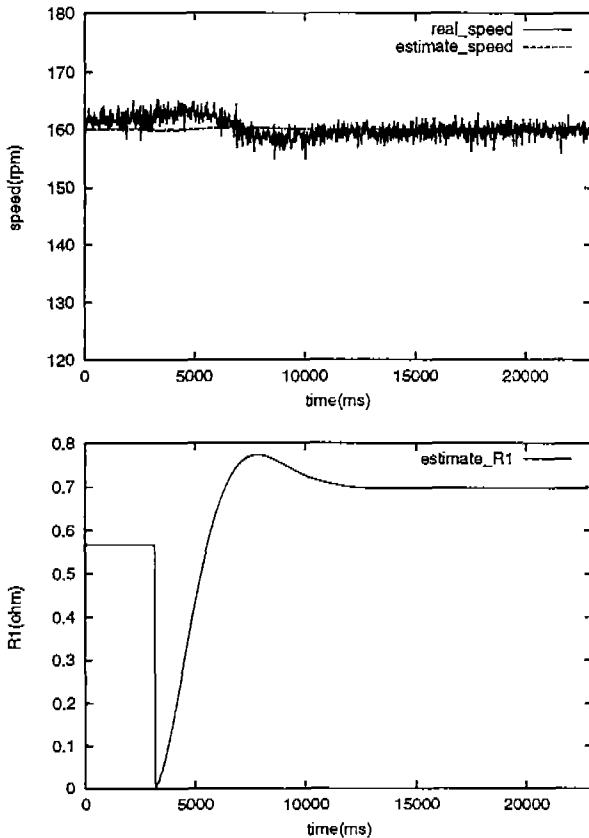


Fig. 11. Experimental result of the  $R_1$  estimation

## REFERENCES

- [1] S.Shinnaka. A unified analysis on simultaneous identification of velocity and rotor resistance of induction motor. *IEE of Japan Trans 113-D No.12 (in Japanese)*, pages 1483 – 1484, 1993.
- [2] I.Miyashita, H.Fujita, and Y.Ohmori. Speed sensorless instantaneous vector control with identification of secondary resistance. In *IEEE IAS Annual Meeting*, pages E130–E135, 1991.
- [3] J.Jiang and J.Holtz. High dynamic speed sensorless ac drive with on-line model parameter tuning for steady-state accuracy. *IEEE Industrial Electronics VOL.44*, pages 240–246, 1997.
- [4] H.Kubota and K.Matsuse. Speed sensorless field oriented control of induction motor with rotor resistance adaption. In *IEEE IAS Annual Meeting*, pages 414–418, 1993.
- [5] T.W Kim and A.Kawamura. Sensorless slip frequency estimation of induction motor in the very low speed region. *IEE Japan Trans 116-D No.6*, pages 644 – 650, 1996.
- [6] T.Ohtani. Reduction of motor parameter sensitivity in vector-controlled induction motor without shaft encoder. *IEE Japan Trans(in Japanese). Vol.110-D No.5*, pages 497–505, 1990.
- [7] A.Kawamura et al. Analysis of anti-directional-twin-rotary motor drive characteristics for electric vehicles. *IEEE Transactions on Industrial Electronics, vol.44 No.1*, pages 64–70, 1997.
- [8] T.Kanmachi and I.Takahashi. Sensorless control of an induction motor with no influence of secondary resistance variation. In *IEEE IAS Annual Meeting*, pages 408 – 413, 1993.
- [9] S.Shinnaka. *Adaptive algorithm*. industrial book, 1990.
- [10] T.Okuyama, N.Fujimoto, and H.Fujii. Simplified vector control system without speed and voltage sensors. *IEE Japan Trans 110-D No.5(in Japanese)*, pages 477 – 486, 1990.
- [11] I.Miyashita and Y.Ohmori. Speed sensorless high-speed torque and speed control of induction motor based on instantaneous spatial vector theory. In *IPEC-Tokyo*, pages 1144 – 1151, 1990.
- [12] K.Ohnishi, H.Suzuki, K.Miyachi, and M.Terashima. Decoupling control of secondary flux and secondary current in induction motor drive with controlled voltage source and its comparison with volts/hertz control. *IEEE Trans. in Industry Applications VOL.IA-21, No.1*, pages 241–247, 1985.
- [13] K.Akatsu and A.Kawamura. Sensorless speed estimation of induction motor based on the secondary and primary resistance on-line identification without any signal injection. In *IEEE Power Electronics Specialists Conference (PESC)*, pages 1575 – 1580, 1998.