Digital Simulation of Power Converters by Means of Wave Variables

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ABSTRACT - In audio techniques calculation of digital filters is accelerated considerably by use of wave variables instead of voltages and currents. The suitability of wave variables for digital simulation of power converters was investigated and the results are reported in this paper. The original method is described briefly, modelling of switches and diode rectifiers is presented, examples are given and the features of the method are discussed.

1. Introduction of wave variables

Definitions

The pair of voltage u and current i appearing at the terminals (gate) of a device or a line can be replaced by a pair of wave variables a and b which are defined by

$$a = u + Z \cdot i \qquad b = u - Z \cdot i \ . \tag{1}$$

a and b are refered to as incident wave and reflected wave. Z is called gate resistance. By a suitable choice of this arbitrary constant the relation between the wave variables becomes very simple which becomes obvious when linear devices are considered.

Linear devices

The terminal voltage u of a *voltage source* Q and its current i are related by $u = U_Q + R_Q \cdot i$. Replacing the voltage and the current by the related wave variables and solving the equation for the reflected wave b results in

$$b = \frac{2}{1 + R_Q/Z_Q} \cdot U_Q - \frac{1 - R_Q/Z_Q}{1 + R_Q/Z_Q} \cdot a \tag{2}$$

By a suitable choice of the gate resistance Z_Q this equation is simplified considerably:

$$b = U_Q \qquad \text{for} \qquad Z_Q = R_Q \tag{3}$$

In case of a *current source* this can be transformed to a voltage source and so we get

$$b = R \cdot I_Q \qquad \text{for} \qquad Z_Q = R_Q \tag{4}$$

A *resistance* R is considered a voltage source with $U_Q = 0$: Thus its wave model is given by

$$b = 0 \qquad \text{for} \qquad Z_R = R \ . \tag{5}$$

The behaviour of energy storing devices is described by differential expressions which have to be integrated. The wave models become very simple if trapezoidal integration method is used in case of digital dataprocessing.

Modelling an *inductance* starts from $L \cdot di/dt = u$. Application of trapezoidal rule to this expression delivers $i_k = i_{k-1} + T/(2L) \cdot (u_k + u_{k-1})$ where T is the sampling time or step width. Introduction of wave variables and solving for the reflected wave results in

$$b_k = \left(1 - \frac{T \cdot Z_L}{2L}\right) \cdot a_k - \left(1 + \frac{T \cdot Z_L}{2L}\right) \cdot a_{k-1} - \left(1 - \frac{T \cdot Z_L}{2L}\right) \cdot b_{k-1} \tag{6}$$

Choosing a suitable gate resistance Z_L result in

$$b_k = -a_{k-1} \qquad \text{for} \qquad Z_L = 2L/T \ . \tag{7}$$

Table 1 Wave models of linear devices .

Device	Original variables	Wave variables
Voltage source	$u = U_Q + R_Q \cdot i$	$b_k = U_Q$ for $Z_Q = R_Q$
Current source	$u = R_Q \cdot (I_Q + i)$	$b_k = R_Q \cdot U_Q$ for $Z_Q = R_Q$
Resistance	$u = R \cdot i$	$b_k = 0$ for $Z_R = R$
Capacitance	$C \cdot du/dt = i$	$b_k = a_{k-1}$ for $Z_C = T/(2C)$
Inductance	$L \cdot di/dt = u$	$b_k = -a_{k-1}$ for $Z_L = (2L)/T$

In a similar way a simple wave model for the *capacitance* is achieved.

Modelling of linear devices is summarized in Table 1. From this table the following *results* are visible (suitable choice of gate resistances provided):

- Sources and resistances are reflection-free because their reflected waves b_k do not depend on their incident waves a.
- Capacitances and inductances are reflection-free, because their reflected waves b_k do not depend on their actual incident waves a_k of the same instant k.
- Attention must be paid to the fact that the terminal voltages and currents of all devices must be assigned in the same sense as shown by example of Fig. 1(a).

Due to the facts that the reflected waves of linear devices do not depend on the actual incident waves they can be determined without any calculation. This is why the simulation algorithm is extremly simple and simulation is performed very fast.

Connection of devices

After the basic devices have been modelled they have to be connected. This procedure is explained by example of Fig. 1 showing a circuit and its wave flow graph.

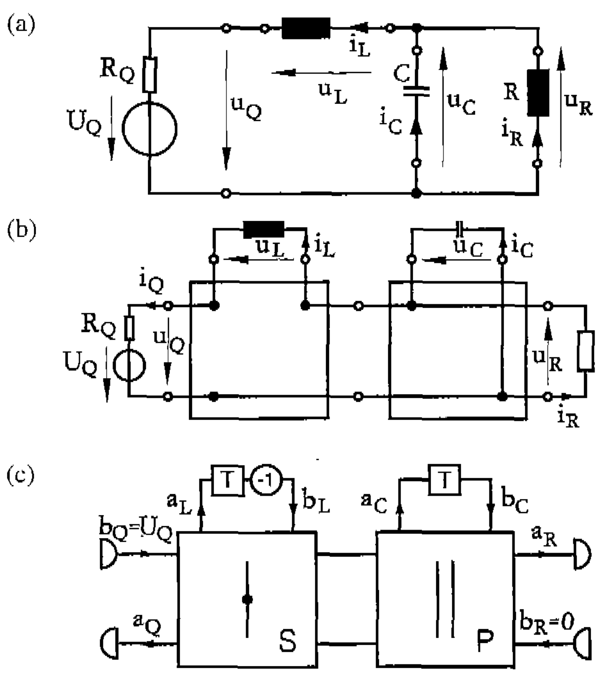

Fig. 1 Connection of devices: (a) Circuit; (b) Circuit with connecting boxes (c) Wave flow graph

At wave modelling parallel and series connections have to be performed seperately which was considered when redrawing the circuit at Fig. 1(b). In figure two connecting boxes S and P are introduced in which is performed the series and parallel connection of the devices. Note, that the parallel connection box is part of the series connection at the left and vice versa. Thus, at example three units are connected by each connection box, two devices and one subcircuit.

The related wave flow graph is shown in Fig. 1(c): The basic devices are replaced by suitable symbols and instead of the connecting boxes there are socalled adaptors, one series adaptor A and one parallel adaptor P.

The behaviour of series and parallel connections is described by Kirchhoff's laws which will be established for both types of connections. Replacing voltages and currents by wave variables will deliver the wave equations of series and parallel adaptors. But at first terminal the voltages and current of the adaptor gates have to be defined from which the wave variables of the adaptor gates are calculated.

The following *agreements* are established:

- The voltages of an adaptor's gate and of the device connected to this gate are equal, $u_{Ada} = u_{Dev}$.
  According to the general agreement how to assign a terminal current to the terminal voltage the current of the adaptor must be established according to $i_{Ada} = -i_{Dev}$.
- From the preceeding definition we get the following result for wave variables of an adaptor gate and the device being connected to this gate:

$$a_{Ada} = b_{Dev} \text{ and } b_{Ada} = a_{Dev} \text{ if}$$

$$Z_{Ada} = Z_{Dev}. \tag{8}$$

  To make use of this simplification gate resistances of connected gates must be equal.
- All units (devices and adaptors) connected to a series adaptor are arranged such that the voltage arrows of all gate are oriented either clockwise or counterclockwise.
- All units connected to a parallel adaptor are arranged such that the voltage arrows of all gates have the same orientation with regard to the internal connecting points of the adaptor.

Arbitrarily the *parallel connection is investigated* first: If N units are connected in parallel the voltages and currents are related by

$$\sum_{n=1}^{N_P} i_n = 0 \qquad \text{and for each } n \qquad u_n = u_1 \tag{9}$$

After replacing voltages and currents by wave variables the reflected wave of an arbitrary gate k can be calculated. The result can be formulated as

$$b_k = s_{kk} \cdot a_k + s_{km} \cdot \sum_{m=1; m \neq k}^{N_P} a_m \tag{10}$$

with the stray coefficients of parallel adaptor P

$$s_{kk} = \left[\frac{2 \cdot Z_k^{-1}}{\sum_{n=1}^{N_P} Z_n^{-1}} - 1 \right], \quad s_{km} = \frac{2 \cdot Z_k^{-1}}{\sum_{n=1}^{N_P} Z_n^{-1}} \tag{11}$$

Notice, that the reflected wave of an adaptor gate depends on the sum of the incident waves of the other gates by a common stray coefficient which simplifies calculation.

In case of our example the parallel adaptor has $N_P = 3$ gates to which two devices and the series adaptor are connected. At two gates the gate resistances are established by the related devices. The gate resistance Z_{P3} of the third gate $k = 3$ is not established yet. But by use of eq. (7) it can be chosen such that the related stray coefficient becomes $s_{P33} = 0$. This allows to calculate the reflected waves of adaptor gate 3 without knowledge of the incident wave a_{P3} originating from the other adaptor. Now we can proceed by considering of the series adaptor at the left part of Fig. 1(c).

For a *series connection* of N basic devices and/or subcircuits - proper choice of arrows provided - the equations

$$\sum_{n=1}^{N_s} u_n = 0 \qquad \text{and for each } n \qquad i_n = i_1 \qquad (12)$$

hold. By introduction of wave variables an equation like eq. (10) is derived, but now the stray coefficients are

$$s_{kk} = \left[1 - \frac{2 \cdot Z_k}{\sum_{n=1}^{N_s} Z_n}\right], \quad \left(s_{km} = \frac{2 \cdot Z_k}{\sum_{n=1}^{N_s} Z_n}\right). \qquad (13)$$

If this result is applied to the series adaptor of our example all gate resistances of this adaptor are already established by the linked devices and the parallel adaptor. But all incident waves, included that coming from the parallel adaptor, are known and the calculation of the reflected waves does not cause any problem. One of these waves is the missing incident wave of the parallel adaptor. Knowing this variable all wave variables reflected from the parallel adaptor to the adjacent devices can be determined.

2. Simulation procedure:

According to the preceeding section the following steps are required for digital simulation by means of wave variables:

1. From the circuit a wave flow graph has to be constructed considering the rules for voltage and current arrows.
2. The gate resistances have to be determined according to the stepwidth T. The sequence of calculation steps is established in such a manner that the reflected waves can be calculated straight forward.
3. Before starting the simulation at $k = 0$ the reflected waves of capacitances and inductances have to be initialized. For this purpose the voltages and curents of the capacitances $u_{C,0}$, $i_{C,0}$ and inductances $u_{L,0}$, $i_{L,0}$ are calculated. From this data the initial values of the reflected waves $b_{C,0}$ and $b_{L,0}$ are calculated by means of eq. (1).
4. At each simulation step k the reflected waves of the devices are considered incident waves of adaptors $a_{Ada,k} = b_{Dev,k}$. The reflected waves $b_{Ada,k}$ of adaptors are calculated stepwise as explained above for the example. These wave variables are the incident waves of the devices $a_{Dev,k} = b_{Ada,k}$. Notice, only the incident waves of energy storing devices $a_{L,k}$, $a_{C,k}$ are of interest and need to be calculated.
5. The reflected waves $b_{L,k+1}$, $b_{C,k+1}$ of the reactive components are determined for the next instant according to Table 1.
6. Steps 4 and 5 are repeated for each integration step.

A good introduction to the application of wave variables to linear circuits can be found at [1].

3. Modelling of rectifiers by characteristics

For simulation of power converters by means of wave variables wave models of transistors and diodes had to be developed [2], [3].

Diodes and diode rectifiers can be modelled either by their nonlinear characteristic or as switches which cause a change of network. Transistors and transistor stages are externally controlled and can also be modelled by switches. Often modelling by a voltage or current source is also possible. In the following modelling of diodes and diode rectifiers by characteristics is discussed more in detail.

Wave-model of ideal diode

Modelling of an ideal diode is easy. Its characteristic is derived as $b = -|a|$. This is the first device whose reflected wave depends directly on the incident wave. The device is not reflexion-free. The problem caused by this fact is not severe and solving an algebraic equation system can be avoided as long as only one diode is present.

Wave-model of diode rectifier

The same is true, if a converter comprises only one diode with finite blocking and conducting resistances or one diode rectifier, which is a subcircuit consisting of diodes and resistances and which does not comprise energy storing devices. Modelling of diode rectifiers by characteristics is now discussed refering to example of Fig. 2.

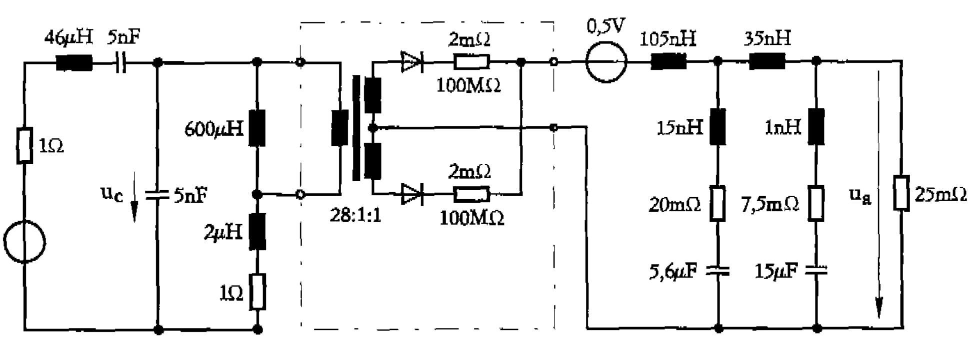

Fig. 2 Circuit of a LCC resonant converter

In the centre of Fig. 2 the diode retifier is allocated being surrounded by dashed line. It consists of an ideal, non energy storing three winding transformer and two diodes. The resistances of the diodes are shown explicitely; they can be low or high depending on the switching state of diode. By the two pais of terminals the rectifier is connected to linear networks at the right and at the left which will be discussed later in detail.

A converter like the resonant converter of Fig. 2 can be divided into two linear subsystems I and II which are connected to the gates of the rectifier G. This results in a structure shown at Fig. 3.

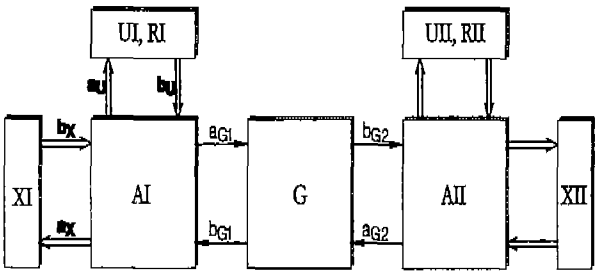

Fig. 3 Wave flow graph of a converter with rectifier

Each subsystem consists of energy storing devices X, source U and resistive devices R which - in the wave flow graph - are connected by several adaptors A. Each subsystem exchanges one pair of wave variables with the rectifier model G. The incident waves $a_{G1,2}$ of the rectifiers which are reflected waves of subcircuits' adaptors can be calculated as usual. The calculation of the reflected waves of the rectifier $b_{G1,2}$ is performed now.

Modelling of a rectifier starts at the circuit where each diode D_i is replaced by a representing resistance R_i. The value of the resistance depends on the switching state of the diode and is unknown yet. For the resistive network representing the rectifier a wave model is derived now and the reflected waves of the two rectifier gates are calculated:

$$\begin{aligned} b_1 &= s_{11} \cdot a_1 + s_{12} \cdot a_2 \\ b_2 &= s_{21} \cdot a_1 + s_{22} \cdot a_2 \end{aligned} \qquad (14)$$

In a second step the switching state of the rectifier is determined. For each diode an equation is derived which gate voltages and currents fullfill at the boundary between the conducting and the blocking state of this diode. From the requirement $u_{Di} = 0$ or $i_{Di} = 0$ a linear equation

$$k_1 \cdot u_1 + k_2 \cdot u_2 + k_3 \cdot i_1 + k_4 \cdot i_2 = 0 \qquad (15)$$

results. The coefficients k depend on the unknown switching states of the diodes, but due to $i_{Di} = 0$ they do not depend on the state of the diode under investigation D_i.

When wave variables are introduced to eq. (15) we get

$$c_1 \cdot a_1 + c_2 \cdot a_2 + c_3 \cdot b_1 + c_4 \cdot b_2 = 0 . \qquad (16)$$

In this equation the reflected waves can be eliminated by use of eq. (14) and a linear relation between the incident waves is obtained

$$a_2 = m_{Di} \cdot a_1 , \qquad (17)$$

which represents the requirement $u_{Di} = 0$ and marks the boundary between the areas where the diode is conducting and blocking, respectively. According to eq. (17) this boundary is a straight line in the a_1, a_2-plane and will be called switching line. The slope of the switching line m_{Di} is not determined yet; it depends on the switching states of the other diodes like the coefficients c and k.

When switching lines for all diodes are established they divide the a_1, a_2-plane into sectors in which different switching states of the rectifier exist.

At many circuits, like bridge topologies, pairs of diodes are switching synchronously. In these cases, of course, only one switching line per pair is existing.

<u>Simulation of converters comprising diode rectifiers</u>

In accordance with modelling, consideration of a rectifier at simulation is performed as follows, see also Appendix:

- After calculation of the incident waves of the rectifier, delivered by the linear subsystems, the position of point (a_1, a_2) with regard to the switching lines is determined in the a_1, a_2-plane. From this information the switching states and the actual resistances of all diodes result.
- From the resistances the stray coefficients s can be calculated (or precalculated values are read from the memory) which are used to calulate the reflected waves of the rectifier by use of eq. (14).

4. Modelling of rectifiers by switches

This technique which is wellknown from normal simulators requires to check the switching states at each simulation step: A switching occurs when the current of a conducting diode becomes negative ($i_{Di} < 0$) or when the voltage of a blocking diode becomes positive ($u_{Di} > 0$).

When switching takes place the topology of the circuit changes. All topologies appearing during simulation have to be modelled by a wave flow graph. Since switches are not present the circuits behave linear.

By the switching procedure the capacitance currents and inductance voltages (not being state variables) are changed. In consequence the wave variables of these devices change also and must be initialized after each switching. The initialization of wave variables does not require calculation of capacitor currents and inductance voltages but can be performed directly using wave variables as follows.

At first the currents and voltages of all inductances and capacitances are gathered in the state variable vector

$$\mathbf{x} = [i_{L1}, i_{L2}, ..., u_{C1}, u_{C2}, ...,]^T = (\mathbf{i}_L, \mathbf{u}_C)^T \qquad (18)$$

and a complementary vector

$$\mathbf{y} = [u_{L1}, u_{L2}, ..., i_{C1}, i_{C2}, ...,]^T = (\mathbf{u}_L, \mathbf{i}_C)^T \qquad (19)$$

From the values of inductances and capacitances a matrix

$$\mathbf{E} = \mathrm{diag}[L_1, L_2, ..., C_1, C_2, ...,] \qquad (20)$$

is formed. Using these definitions two equations can be established: The first equation describes the behaviour of the devices, the second - the state space equation - depends on the connection of the devices.

$$\mathbf{E} \cdot \frac{d\mathbf{x}}{dt} = \mathbf{y} \qquad \mathbf{E} \cdot \frac{d\mathbf{x}}{dt} = \tilde{\mathbf{A}} \cdot \mathbf{x} + \tilde{\mathbf{B}} \cdot \mathbf{u} \quad (21)$$

These equations are combined now and are established for the instants immediately before and after switching

$$\begin{aligned} \mathbf{y} &= \tilde{\mathbf{A}} \cdot \mathbf{x} + \tilde{\mathbf{B}} \cdot \mathbf{u} \\ \mathbf{y} + \Delta\mathbf{y} &= (\tilde{\mathbf{A}} + \Delta\tilde{\mathbf{A}}) \cdot \mathbf{x} + (\tilde{\mathbf{B}} + \Delta\tilde{\mathbf{B}}) \cdot \mathbf{u} \end{aligned} \quad (22)$$

Changes of variables and matrices are marked by Δ and, at the second equation, the continuity of state variables was already considered ($\Delta\mathbf{x} = \mathbf{0}$) and the continuity of input signals was assumed ($\Delta\mathbf{u} = \mathbf{0}$). From equations

$$\Delta\mathbf{x} = \mathbf{0} \qquad \Delta\mathbf{y} = \Delta\tilde{\mathbf{A}} \cdot \mathbf{x} + \Delta\tilde{\mathbf{B}} \cdot \mathbf{u} \quad (23)$$

resulting from the continuity and from eq. (22) the voltages and currents forming the vectors $\Delta\mathbf{x}$, $\mathbf{x}$, $\Delta\mathbf{y}$ and $\mathbf{y}$ are replaced by wave variables. Thus eq. (23) yields

$$\frac{1}{2} \cdot \begin{bmatrix} \mathbf{Z_L}^{-1} \cdot (\Delta\mathbf{a}_L - \Delta\mathbf{b}_L) \\ (\Delta\mathbf{a}_C + \Delta\mathbf{b}_C) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (24)$$

$$\begin{aligned} \frac{1}{2} \cdot \begin{bmatrix} (\Delta\mathbf{a}_L + \Delta\mathbf{b}_L) \\ \mathbf{Z_C}^{-1} \cdot (\Delta\mathbf{a}_C - \Delta\mathbf{b}_C) \end{bmatrix} &= \\ = \Delta\tilde{\mathbf{A}} \cdot \frac{1}{2} \cdot \begin{bmatrix} \mathbf{Z_L}^{-1} \cdot (\mathbf{a}_L - \mathbf{b}_L) \\ (\mathbf{a}_C + \mathbf{b}_C) \end{bmatrix} &+ \Delta\tilde{\mathbf{B}} \cdot \mathbf{u} \end{aligned} \quad (25)$$

where $\mathbf{Z_L}$ and $\mathbf{Z_C}$ are diagonal matrices comprising the gate resistances of inductances and capacitances.

From equations given above the variations of the wave variables result by a simple calculation as

$$\begin{aligned} \begin{bmatrix} \Delta\mathbf{b}_L \\ \Delta\mathbf{b}_C \end{bmatrix} &= \begin{bmatrix} \Delta\mathbf{a}_L \\ -\Delta\mathbf{a}_C \end{bmatrix} \\ \begin{bmatrix} \Delta\mathbf{a}_L \\ \mathbf{Z_C}^{-1} \cdot \Delta\mathbf{a}_C \end{bmatrix} &= \Delta\tilde{\mathbf{A}} \cdot \frac{1}{2} \cdot \begin{bmatrix} \mathbf{Z_L}^{-1} \cdot (\mathbf{a}_L - \mathbf{b}_L) \\ (\mathbf{a}_C + \mathbf{b}_C) \end{bmatrix} + \Delta\tilde{\mathbf{B}} \cdot \mathbf{u} \end{aligned} \quad (26)$$

5. Example

A simulation program based on wave variables was written and tested by simulating different circuits comprising different rectifier circuits.

LCC resonant converter of Fig. 1 is selected as simulation example here, which shows different operation modes.

In the center of the figure the rectifier stage is surrounded by dashed line. It consists of two diodes which are fed by an ideal three winding transformer. The diode resistances which depend on the switching states the resistance of the secondary windings are shown explicitely. In these resistances the resistances of the secondary windings can be included. The other parasitics of the transformer (mutual inductance, leakage inductance and resistance of the primary winding) are modelled at the vertical branch left of the rectifier stage. At the left side of the figure a voltage source is used to model the square wave voltage generated by the inverter. It is linked to the transformer via the LCC resonant circuit. At the right side of the retifier stage a dc voltage source represents the threshhold voltage of the rectifier diodes. The model of the two stage LCLC filter comprises the parasitic resistances and inductances of the capacitors which influence the output voltage ripple considerably.

The simulation of a start-up procedure is shown at Fig. 4 showing the rise of output voltage during ### ms.

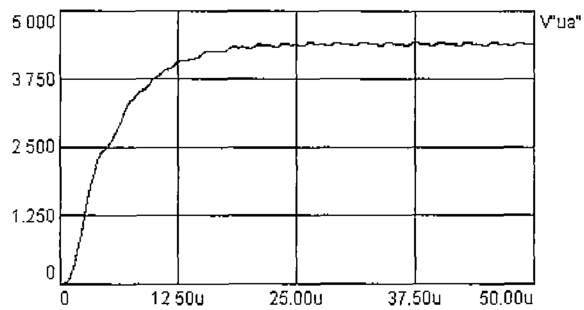

Fig. 4 Start-up procedure of LCC resonant converter

Fig. 5(a) and Fig. 5(b) show some signals during two switching periods at steady state operation at the end of the start-up. At Fig. 5(a) the rectangular voltage of the inverter is shown as well as the output voltage of the LCC resonant circuit which is applied to the transformer. The fundametal wave has a lagging phase due to switching frequency being larger than resonant frequency. The harmonics are caused by the switching rectifier stage. At Fig. 5(b) the ripple of the output voltage is shown. The rapid changes of voltage

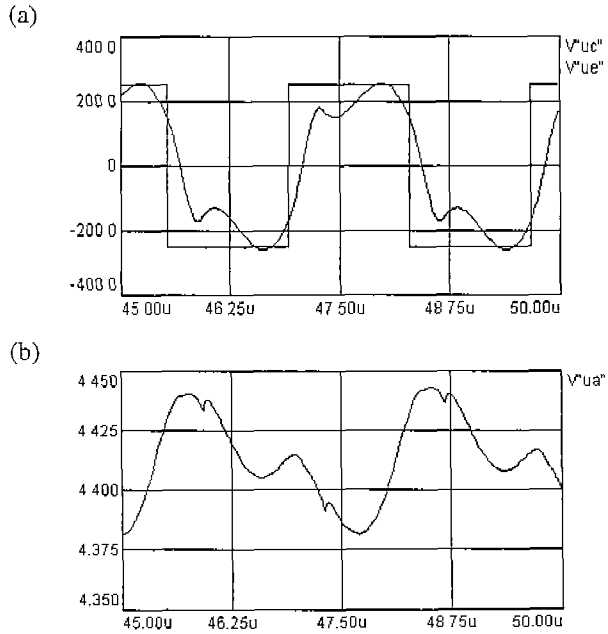

Fig. 5 Voltages at end of start-up procedure
(a) Voltages of inverter and resonant capacitor
(b) Ripple of output voltage

are caused by the parasitic inductance and resistance of the output capacitor.

Simulation of the same event with a standard simulator like Saber delivers exactly the same result as far as trapezoidal integration rule and the same step width are used. But with the same computer the simulation routine with wave variables is four times faster than with normal simulation (pre- and postprocessing not included) .

6. Conclusion

By use of wave variables instead of voltages and currents simulation speed can be increased considerably as far as the circuit does not comprise nonlinearities with exception of one diode or one rectifier stage.

Nonlinearities like the magnetization curve of a saturated inductor have already been modelled. However the simplicity of the method and its main advantage of high simulation speed are reduced when nonlinear characteristics - like the magnetization curve of a saturated inductor - have to be modelled. This is why simulation by means of wave variables cannot replace conventional simulation tools.

Nevertheless, when all devices are linear or when nonlinearities can be neglected, the new method is suitable for applications at whose simulation is very time consuming and/or has to be repeated frequently.

An example for long simulation time is the start up procedure of a switch mode power supply where many switching periods have to be calculated and where different modes of operation can appear. Timeconsuming simulation time is also necessary when power factor correcting circuits are investigated. Simulation must be extended at least for one half of theline voltage period and a large number of integration steps is necessary because stepwidth must be adapted to the short period of switching frequency.

An example, where simulation has to be repeated very often is the determination of steady state characteristics. For this purpose steady state operating points for a large variety of input voltage and output current have to be determined.

Another application, using the new simulation techniques effectively for repetetive simulation, is the optimal design of filters. In [2] it was used successfully for the design of multi-stage output filters which cannot be calculated analytically due to the high system order. The program for filter simulation in combination with an optimzation algorithm which varies the filter parameters will be part of a CAE-tool for the design of switch mode power suppplies which is developed at the department of the author.

Last not least, a very practical aspect has to be considered. If simulation by means of wave variables is not used frequently its implementation can take more time than the method can save during simulation.

Concluding the paper it should be mentioned that actually a manufacturer of a simulator is interested in implementing the wave variable method as a special integration tool in his product.

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Appendix

Consideration of a rectifier stage can also be explained by use of equations. For this purpose a matrix equation is established for adaptor AI in Fig. 3:

$$\begin{bmatrix} \mathbf{a}_{X,k} \\ \mathbf{a}_{U,k} \\ a_{G1,k} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{XX} & \mathbf{S}_{XU} & \mathbf{s}_{XG} \\ \mathbf{S}_{UX} & \mathbf{S}_{UU} & \mathbf{s}_{UG} \\ \mathbf{s}^T_{GX} & \mathbf{s}^T_{GU} & s_{GG} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{b}_{X,k} \\ \mathbf{b}_{U,k} \\ b_{G1,k} \end{bmatrix}, \qquad (27)$$

In this equation the incident and reflected waves of the devices connected to adaptor I are gathered in vectors $\mathbf{a}_{X,k}$, $\mathbf{b}_{X,k}$ and $\mathbf{a}_{U,k}$, $\mathbf{b}_{U,k}$ (Notice that the reflected waves $\mathbf{b}$ of devices are the incident waves of the adaptor). In accordance $a_{G1,k}$ and $b_{G1,k}$ are the wave variables of rectifier gate 1 being connected to adaptor I. In matrices $\mathbf{S}$ and vectors $\mathbf{s}$ the stray coefficients of the adaptor are collected.

If adaptor gate 1 is assumed to be reflexion-free $s_{GG} = 0$ holds and the reflected wave of adaptor gate 1 can be determined by use of the last line of eq. (27).

$$a_{G1,k} = \mathbf{s}^T_{GX} \cdot \mathbf{b}_{X,k-1} + \mathbf{s}^T_{GU} \cdot \mathbf{b}_{U,k} . \qquad (28)$$

After determination of reflected waves $b_{G1,k}$, $b_{G2,k}$ coming from the rectifier the incident waves of the devices can be calculated by use of lines 1 and 2 of eq. (27):

$$\begin{bmatrix} \mathbf{a}_{X,k} \\ \mathbf{a}_{U,k} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{XX} & \mathbf{S}_{XU} \\ \mathbf{S}_{UX} & \mathbf{S}_{UU} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{b}_{X,k-1} \\ \mathbf{b}_{U,k} \end{bmatrix} + \begin{bmatrix} \mathbf{s}_{XG,k} \\ \mathbf{s}_{UG,k} \end{bmatrix} \cdot b_{G1,k} . \qquad (29)$$