DIRECT FREQUENCY REPRESENTATION OF PULSE PATTERNS FOR CONTROLLED VOLTAGE SOURCE INVERTERS

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ABSTRACT – The paper describes developed method of feedforward digital modulation of line-to-line voltage of 3-phase inverter for drive application. It is based on representation of parameters of output voltage of inverter in function of operating frequency of drive system. Pure algebraic control laws and big computational simplicity characterize this scheme of modulation. It has been presented results of simulation of adjustable drive systems with the method of pulse-width modulation described.

1. INTRODUCTION

Output signals of the majority of power converters are sequences of pulses, so principle of pulse-width modulation (PWM) is one of the basic for control in Power Electronics. Last years are characterized by intensive development of digital methods of control and modulation for converters [1,2].

Adjustable speed AC drives fed from voltage source inverters are nowadays ones of the most popular power conversion systems. Development of techniques of PWM for these systems began from carrier-based methods of modulation where comparison of reference signal with carrier signal is executed during control process [3].

Next stage of progress of PWM techniques was connected with optimized methods of modulation, based on preliminary computation of pulse patterns in accordance with different criteria of optimization [4,5]. Last decade was marked by the fast application of methods of space vector modulation, which are based on space vector representation of the set of output signals of 3-phase converters and are ones of the most suitable for induction motor drives [6,7]. Modern period of development of methods of PWM is characterized by investigation of combined techniques of modulation based on symbiosis of different concepts, including combinations of scalar and vector principles [8-11].

Majority of the methods of PWM mentioned above is based on voltage representation of control (modulation) parameters in function of relative output voltage of inverter. At the same time well known property of near-optimal control for adjustable speed induction motor drive under condition of constancy of voltage/frequency ratio during adjusting process allows to move to the frequency scheme of PWM based on representation of voltage parameters in function of output frequency of inverter [10,11]. The paper presents development of this method of PWM, based on algorithmic modification of laws of modulation and on improvement of spectra of output voltage and current of inverter.

2. BASES OF THE FREQUENCY SCHEME OF MODULATION

It is known, that for waveforms consisting from rectangular pulses with equal amplitude, a magnitude of the first voltage harmonic is proportional to reference duration of pulses in output half-wave of inverter. Maintenance of constancy of total absolute duration of pulses inside voltage half-wave during control provides direct proportion of amplitude of the first voltage harmonic from operating frequency of system. This convenient property is one of the basic in organization of the scheme of modulation described.

Fig.1 shows two basic stages of the scheme of PWM. Upper curves of the both parts are here switching state sequences of 3-phase inverter in accordance with conventional designation for the switches of the phases *abc*: 1 – 100, 2 – 110, 3 – 010, 4 – 011, 5 – 001, 6 – 101, 7 – 111, 0 – 000; lower curves show corresponding negative quarter-wave of line-to-line output voltage V_{ab} of inverter. Signals β_j represent total switch-on duration during sub-cycle τ. Signals γ_k are generated in the middle of corresponding β, spectra of output voltage and current of inverter depend from these signals a lot. Widths of notches λ_j represent duration of zero state sequences. Mutual variation of the β - and λ -parameters is one of the main differences of the method described.

To provide continuos synchronous and symmetrical variation of voltage waveform, special signals β' and λ' (with the neighboring λ'' and β'') are formed step-by-step in the clock-points of output curve. They are changing each other at the boundary frequencies F_i' (where

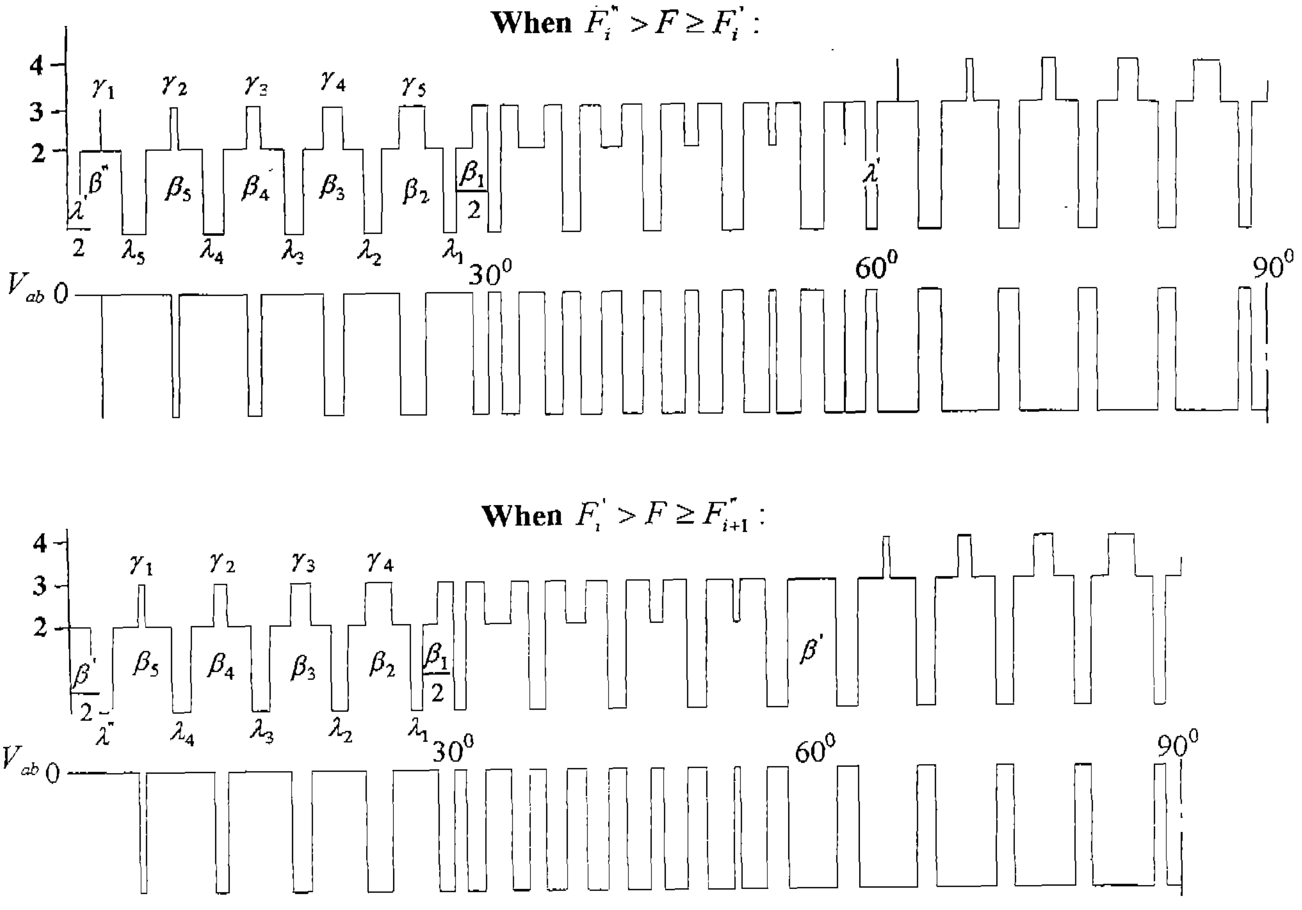

Fig.1 Switching state sequences and line-to-line output voltage of inverter for two basic stages of modulation

$\beta' \Rightarrow 0$) and $F_i^{''}$ (where $\lambda' \Rightarrow 0$), which are calculated in general form in function of width of sub-cycles τ in accordance with (1)-(2). Index i is equal here to the number of notches inside a half of 60^0 clock intervals and is determined from (3), where fraction being rounded off to nearest higher integer:

$$F_i^{'} = \frac{1}{6(2i-1.5)\tau}; \tag{1}$$

$$F_i^{''} = \frac{1}{6(2i-2.5)\tau}; \tag{2}$$

$$i = \frac{1/6F + 0.5\tau}{2\tau}. \tag{3}$$

Eqs.(4)-(15) present full set of dependences describing parameters of modulated line voltage of inverter in absolute value (seconds) for every point of control range. Control correlations presented are based on principle of approximation of voltage space vector trajectory by the polygon with 12 sites in this case (Fig.2). Every parameter is described here by algebraic dependence in function of the operating F and the maximum F_m frequencies of drive system. In particular, if $F_i^{''} > F \geq F_i^{'}$:

$$\beta_1 = \frac{0.167/F_m}{2i-3.5-0.804(i-2)(2i-3)\tau F + [1-1.608(i-1.25)\tau F][2-(F-F_i^{'})/(F_i^{''}-F_i^{'})]}; \tag{4}$$

$$\beta_i = \beta'' = \beta_1[1-1.608(i-1.25)\tau F]\left(1-\frac{0.5(F-F_i^{'})}{F_i^{''}-F_i^{'}}\right); \tag{5}$$

for $j=2,\dots i\text{-}1$: $$\beta_j = \beta_1[1-1.608(j-1.25)\tau F]; \tag{6}$$

$$\lambda_i = \lambda' = \{\tau-\beta_1[1-1.608(i-1.25)\tau F]\}\left(1-\frac{F-F_i^{'}}{F_i^{''}-F_i^{'}}\right); \tag{7}$$

for $j=1,\dots i\text{-}1$: $$\lambda_j = \tau-(\beta_j+\beta_{j+1})/2; \tag{8}$$

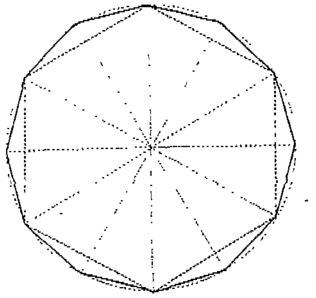

Fig.2 Approximation of the circle by polygon

$$\gamma_1 = 3\beta^{"}(\lambda^{'} + \beta^{"})F\left(1 - \frac{F - F_i^{'}}{F_i^{"} - F_i^{'}}\right); \qquad (9)$$

and for *k=2,... (i-1):*

$$\gamma_k = 6\beta_{i-k+1}\left[\frac{1}{12F} - (i-1.75)\tau + \frac{\lambda_{i-1}}{2} + \frac{\beta_{i-1}}{2} + (k-2)\tau\right]F. \qquad (10)$$

For another control sub-zones, when $F_i^{'} > F \geq F_{i+1}^{"}$:

$$\beta^{'} = \beta_{i+1} = \frac{0.144 / F_m}{2i - 0.634 - 0.804(i-1)(2i-1)\tau F}\left(1 - \frac{F - F_{i+1}^{"}}{F_i^{'} - F_{i+1}^{"}}\right); \qquad (11)$$

$$\beta_1 = \frac{0.167 / F_m - \beta^{'}}{2i - 1.5 - 0.804(i-1)(2i-1)\tau F}; \qquad (12)$$

eqs. (5), (6) and (8) are available in this sub-zone too, and

$$\lambda_i = \tau - \beta_i; \qquad (13)$$

$$\lambda^{"} = \lambda_{i+1} = \frac{1}{12F} - \frac{\beta^{'}}{2} - \frac{\beta_i}{2} - (i-1.25)\tau; \qquad (14)$$

for *k=1,... i-1:* $\gamma_k = 6\beta_{i-k+1}\left[\frac{\beta^{'}}{2} + \frac{\beta_i}{2} + \frac{\lambda_i}{2} + \lambda^{"} + (k-1)\tau\right]F.$ (15)

Fig.3 illustrates variation of parameters of line-to-line voltage of inverter during control process from initial frequency *5Hz* of the rated control mode till the maximum frequency $F_m = 60Hz$. Speed ratio $D = 12$ in this case. Here is non-linear dependence of τ from F:

$$\tau = \frac{F_m}{6F(DF_m + F_m - DF)}. \qquad (16)$$

Fig.3,a shows variation of width of $\tau, \beta_1, \beta^{'}$ and $\lambda^{'}$. Here is step-by-step changing of $\lambda^{'}$ by $\beta^{'}$, then changing of $\beta^{'}$ by $\lambda^{'}$, and so on, this smooth transience is executed at the boundary frequencies. Fig.3,b presents variation of the signals $\beta^{"}$ and $\lambda^{"}$, next to the signals $\lambda^{'}$ or $\beta^{'}$.

Fig.4 presents results of simulation of drive system with 5-hp 4-pole induction motor fed by voltage source inverter with PWM described. Here are spectrum of line voltage V_{ab}, phase current i_a and its spectrum (without 1st harmonic) for some switching frequencies F_s. Fig.5 illustrates dependence of normalized current distortion factor d from modulation index m (curves 1), compared with conventional voltage space vector PWM [6] (curves 2).

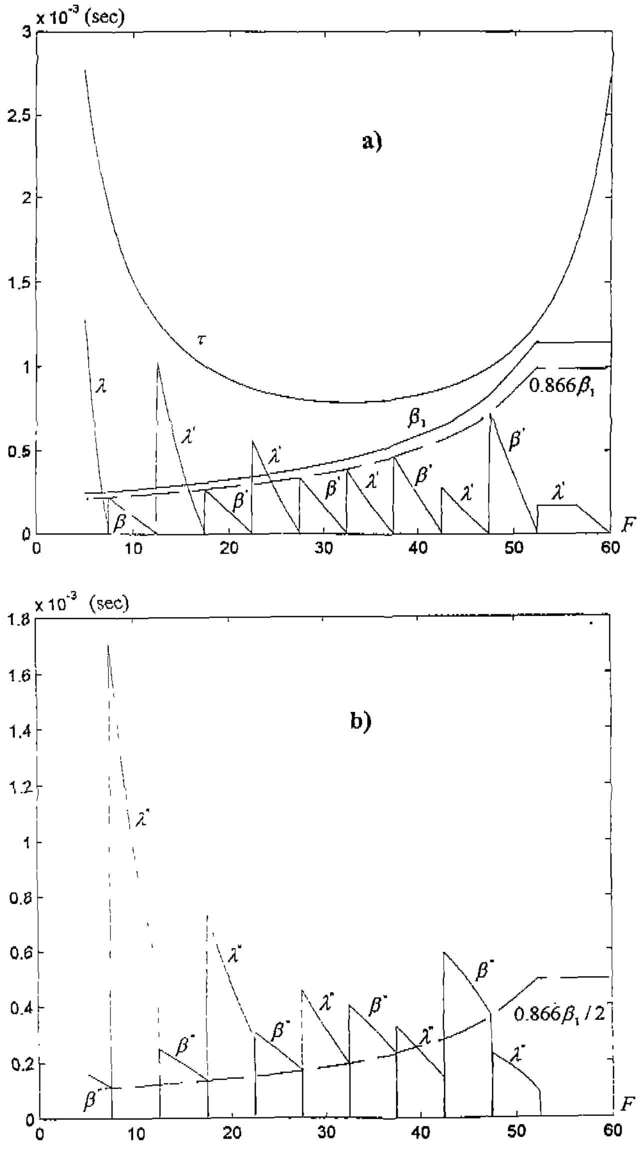

Fig.3 Variation of parameters of the scheme of PWM

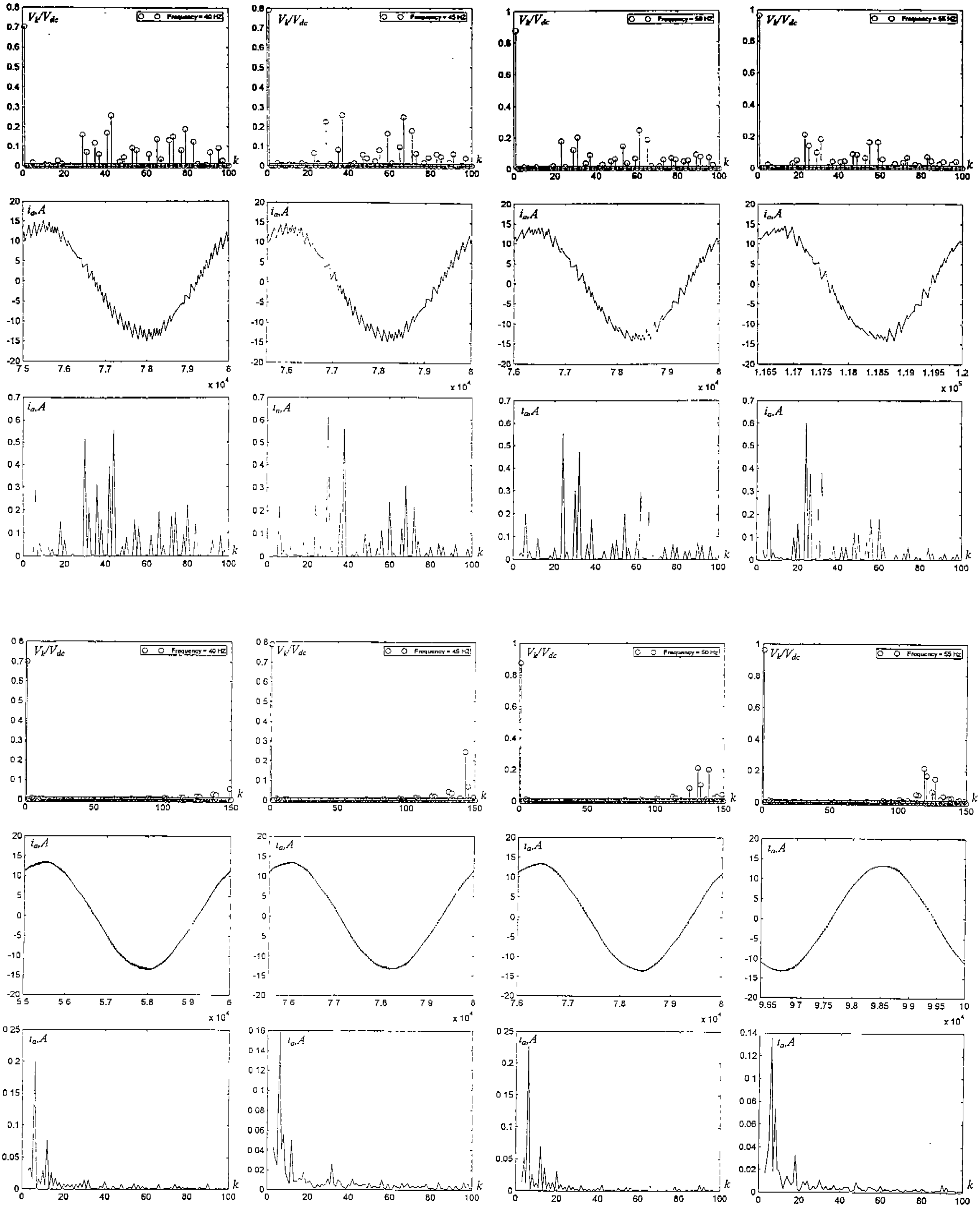

Fig.4 Simulation of systems at $F=40,45,50,55$ *Hz*, $F_s=1$ *KHz* (upper curves) and $F_s=4.5$ *KHz* (lower curves)

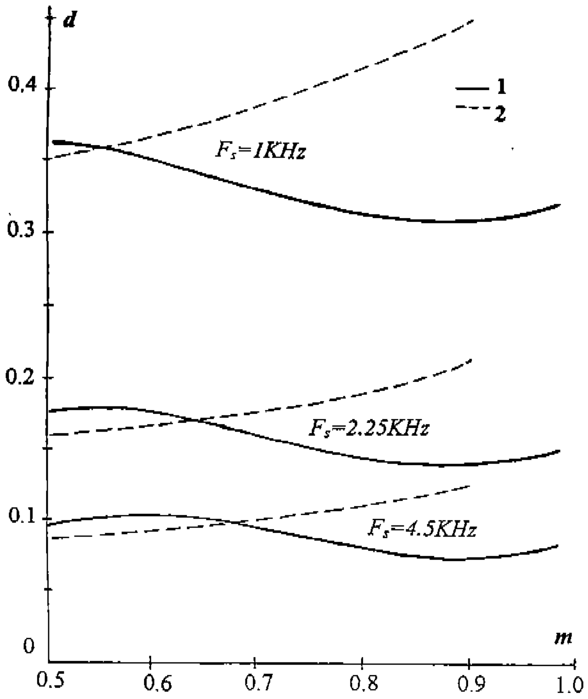

Fig.5 Current distortion factor at different F_s

Results of simulation of system, presented in Fig.5, show better performance of the method of PWM described at higher values of modulation index m. Additional advantage is here possibility of voltage control in accordance with basic algorithm till the highest values of m (basic algorithm of conventional voltage space vector modulation is limited by $m = 0.907$ [6,7]).

Control in the zone of overmodulation

To provide smooth transition from PWM to square-wave line-to-line voltage of inverter at the maximum output frequency F_m of rated control mode, general algorithm of modulation is modified at highest output frequencies beginning from the $F_2' = 1/15\tau$ frequency (Fig.6,a). Here is simple two-stage control process. The first step is connected with movement from the waveform presented in Fig.6,a to voltage waveform shown in Fig.6,b, which is observed at the F(4) boundary frequency where quarter-wave of line voltage consists from four pulses:

$$F(4) = \frac{5F_2' F_m}{3.035F_2' + 2F_m}. \tag{17}$$

There are the next control dependences in this sub-zone:

$$\beta_1 = \frac{0.071}{F_m}; \tag{18}$$

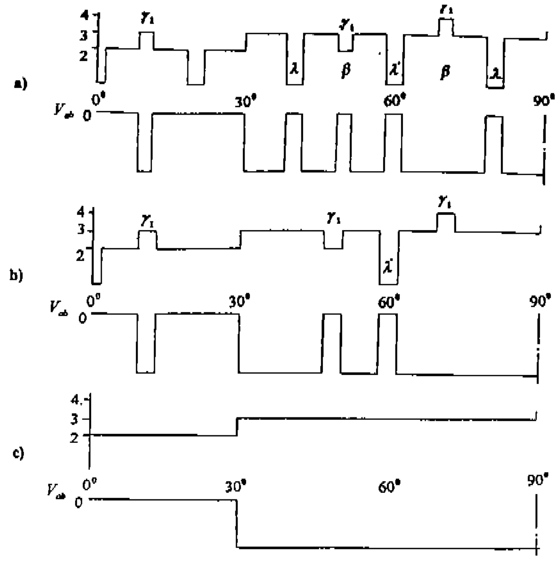

Fig.6 Control process during zone of overmodulation

$$\beta_2 = \frac{0.066}{F_m}; \tag{19}$$

$$\lambda' = \frac{0.067}{F_2'} - \frac{0.066}{F_m}; \tag{20}$$

$$\lambda = \frac{0.083}{F} - \frac{0.033}{F_2'} - \frac{0.051}{F_m}; \tag{21}$$

$$\gamma_1 = 3\beta''(\lambda' + \beta'')F. \tag{22}$$

The next stage of control is accompanied by movement from wave-form presented in Fig.6,b, to square-wave voltage (Fig.6,c). Here is smooth decreasing of widths of λ' and γ_1 till zero at the F_m frequency:

$$\lambda' = \frac{1}{6F} - \frac{1}{6F_m}; \tag{23}$$

$$\gamma_1 = 3\beta''(\lambda' + \beta'')F\left(1 - \frac{F - F(4)}{F_m - F(4)}\right). \tag{24}$$

Fig.7 shows variation of the first harmonic of line-to-line output voltage of inverter for the whole adjustment range (rated control mode). Here is continuous linear shock-less changing of the 1st voltage harmonic till the zone of overmodulation, and then in the zone of overmodulation too.

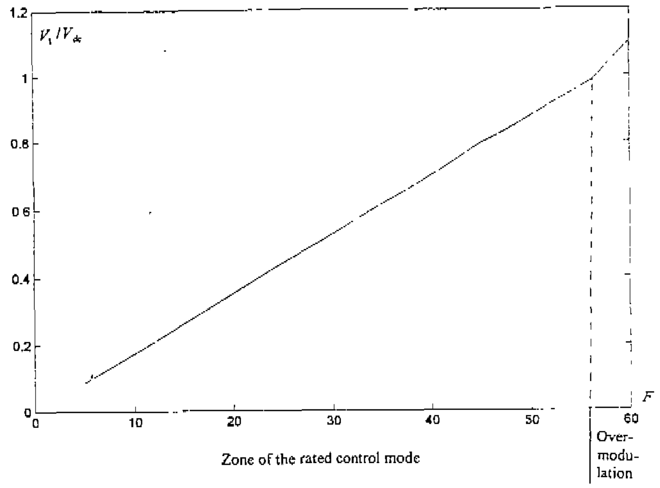

Fig.7 Variation of the first voltage harmonic

3. CONCLUSIONS

The method of direct modulation of line-to-line output voltage of 3-phase voltage source inverter has been developed in the paper. It has been analyzed variant of approximation of voltage space vector circle by the polygon with 12 sites. It is characterized by mutual variation of switch-on durations inside sub-cycles.

Results of simulation of systems with the method of PWM described approved their good performance in the zone of higher value of modulation index.

Specialized signals formed step-by step in clock-points of half-wave, provide smooth shock-less pulses' ratio changing and continuous symmetrical variation of voltage waveform during the whole adjustment range. Spectrum of output voltage of inverter does not contain even harmonics, sub-harmonics and combined harmonics.

Problem of high quality linear control in the zone of overmodlation is solved by the method proposed easily.

Modulation technique described is particularly suited for DSP-implementation due to pure algebraic description of control laws, characterized by computational simplicity.

4. ACKNOWLEDGMENT

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