SISO 비선형 시스템의 제어를 위한 퍼지 모델 기반 제어기

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The Fuzzy Model-Based-Controller for the Control of SISO Nonlinear System

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Abstract - This paper addresses analysis and design of a fuzzy model-based-controller for the control of uncertain SISO nonlinear systems. In the design procedure, we represent the nonlinear system by using a Takagi-Sugeno fuzzy model and construct a global fuzzy logic controller via parallel distributed compensation and sliding mode control. Unlike other parallel distributed controllers, this globally stable fuzzy controller is designed without finding a common positive definite matrix for a set of Lyapunov equations, and has good tracking performance. Furthermore, stability analysis is conducted not for the fuzzy model but for the real underlying nonlinear system. A simulation is included for the control of the Duffing forced-oscillation system, to show the effectiveness and feasibility of the proposed fuzzy control method.

1. Introduction

A systematic analysis and design procedures of the fuzzy control system has been difficult since they are essentially nonlinear. In this paper, stability analysis and design of the Takagai-Sugeno (TS) fuzzy model-based-control system for uncertain SISO nonlinear systems are presented. On the basis of the TS fuzzy model, some fuzzy-model-based controls have been investigated in the literature [1-4]. Sometimes, they are called parallel distributed compensation (PDC). These kinds of design approaches suffer from a few limitations: 1) A common positive definite matrix must be found to satisfy a set of Lyapunov equations, which is difficult especially when the number of fuzzy rules required to give a good plant model is large. 2) The performance of the closed-loop system is difficult to predict. 3) The stability is guaranteed only for the simplified TS fuzzy models although they have been successfully applied to the original, underlying nonlinear systems. 4) The tracking problem of nonlinear systems is not easy to discuss.

In [5] authors presented a new kind of TS fuzzy model-based-controller for known SISO nonlinear system. In this paper, we extend the result of the above approach to the control of uncertain SISO nonlinear systems. Interesting readers can refer to [5] for more detailed explanation. A simulation is included for the control of the Duffing forced-oscillation system, to show the effectiveness and feasibility of the proposed fuzzy control method.

2. TS Fuzzy Model

Consider a class of uncertain SISO nonlinear dynamic systems :

$$x^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u \tag{1}$$

where the scalar x is the output state variable of interest, the scalar u is the system control input, and $\mathbf{x} = \begin{bmatrix} x & \dot{x} & \dots & x^{(n-1)} \end{bmatrix}^T$ is the state vector. In equation (1), $f(\mathbf{x})$ is a unknown nonlinear continuous function of $\mathbf{x}$, and similarly, the control gain $g(\mathbf{x})$ is a unknown nonlinear continuous and invertible function of $\mathbf{x}$. This SISO nonlinear system can be approximated by the TS fuzzy model, proposed in [1], which combines the fuzzy inference rule and the local linear state model [2-4]. The ith rule of the TS fuzzy model, representing a complex single input single output (SISO) system (1), is the following:

Plant Rule i: IF $x(t)$ is F_1^i and ... and $x^{(n-1)}(t)$ is F_n^i
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{b}_i u(t)$ (2)
$(i = 1, 2, \dots, r)$

where Rule i denotes the ith fuzzy inference rule, F_j^i $(j = 1, 2, \dots, n)$ are fuzzy sets, $\mathbf{x}(t) \in R^n$ is the state vector, $u(t) \in R^1$ is the input control, $\mathbf{A}_i \in R^{n \cdot n}$, $\mathbf{B}_i \in R^{n \cdot 1}$, r is the number of fuzzy IF-THEN rules

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, the dynamic fuzzy model (2) can be expressed as the following global model:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \frac{\sum_{i=1}^{r} w_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))} \\ &= \sum_{i=1}^{r} \mu_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)) \\ &= \mathbf{A}(\mu(\mathbf{x}(t)))\mathbf{x}(t) + \mathbf{B}(\mu(\mathbf{x}(t)))u(t)\end{aligned} \tag{3}$$

where

$$w_i(\mathbf{x}(t)) = \prod_{j=1}^{n} F_j^i(x^{(j-1)}(t))$$
$$\mu_i(\mathbf{x}(t)) = \frac{w_i(\mathbf{x}(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))}$$
$$\mu(\mathbf{x}(t)) = (\mu_1(\mathbf{x}(t)), \mu_2(\mathbf{x}(t)), \cdots, \mu_n(\mathbf{x}(t)))$$

and $F_j^i(x^{(j-1)}(t))$ is the grade of membership of $x^{(j-1)}(t)$ in F_j^i. It is assumed, as usual, that

$$w_i(\mathbf{x}(t)) \geq 0,\ (i = 1, 2, \ldots, r),\ \sum_{i=1}^{r} w_i(\mathbf{x}(t)) > 0$$

for all t. Therefore,

$$\mu_i(\mathbf{x}(t)) \geq 0,\ (i = 1, 2, \ldots, r),\ \sum_{i=1}^{r} \mu_i(\mathbf{x}(t)) = 1$$

for all t. For simplicity of notation, let $w_i = w_i(\mathbf{x}(t))$, $\mu_i = \mu_i(\mathbf{x}(t))$, and $\mu = \mu(\mathbf{x}(t))$.

Definition 1: Model (3) is called the global state-space model of the fuzzy system (2). If the pairs ($\mathbf{A}_i$, $\mathbf{B}_i$), i = 1, 2, ..., r are controllable, the fuzzy system (2) is called locally controllable.

3. Robust TS fuzzy model-based-controller

First, Let us define controller rule as (4).

Controller Rule i: If x_1 is F_{i1} and ... and x_n is F_{in}
THEN $u = -\mathbf{K}_i\mathbf{x} + L_i r$ (4)
(i = 1, 2, ..., r)

where L_i and r are scalar values. The scalar input r will be determined later. Equation (4) can be rewritten as

$$u = \frac{\sum_{i=1}^{r} w_i(-\mathbf{K}_i\mathbf{x} + r)}{\sum_{i=1}^{r} w_i} = -\sum_{i=1}^{r} v_i \mathbf{K}_i \mathbf{x} + r \tag{5}$$

where $v_i = w_i / \sum_{i=1}^{r} w_i$. The closed-loop system is obtained from the feedback interconnection of the nonlinear system (1) and the controller (5) and can be described by the following equation:

$$x^{(n-1)} = F(\mathbf{x}) + g(\mathbf{x})r \tag{6}$$

where $F(\mathbf{x}) = f(\mathbf{x}) - g(\mathbf{x})\sum_{i=1}^{r} v_i \mathbf{K}_i \mathbf{x}$

In order to proceed, we have to make the following assumption.

Assumption 1. We can determine function $f^U(\mathbf{x})$, $g^U(\mathbf{x})$, and $g_L(\mathbf{x})$ such that $|f(\mathbf{x})| \leq f^U(\mathbf{x})$ and $0 \leq g_L(\mathbf{x}) \leq g(\mathbf{x}) \leq g^U(\mathbf{x})$.

Based on $f^U(\mathbf{x})$, $g^U(\mathbf{x})$, and $g_L(\mathbf{x})$, and observing (6), the upper bound function of $F(\mathbf{x})$ can be easily obtained.

$$\begin{aligned} |F(\mathbf{x})| &= |f(\mathbf{x}) - g(\mathbf{x})\sum_{i=1}^{r} v_i \mathbf{K}_i \mathbf{x}| \\ &\leq f^U + g^U |\sum_{i=1}^{r} v_i \mathbf{K}_i \mathbf{x}| = F^U(\mathbf{x}) \end{aligned} \tag{7}$$

Let $\tilde{x} = x - x_d$ be the tracking error in the variable x, and let

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x} \cdots \tilde{x}^{(n-1)}]^T \tag{8}$$

In order to incorporate sliding mode control theory into the fuzzy model based control architecture, we first define a time-varying surface $S(t)$ in the state-space $\mathbf{R}^n$ by the scalar equation $s(x;t) = 0$, where

$$s(x;t) = (\frac{d}{dt} + \lambda)^{n-1}\tilde{x} = \tilde{x}^{(n-1)} + a_1\tilde{x}^{(n-2)} + \cdots + a_{n-1}\tilde{x} \tag{9}$$

where λ is a strictly positive constant.

Given an initial condition, the problem of tracking n dimensional vector $\mathbf{x}_d$ can be reduced to that of keeping the scalar quantity s at zero. More precisely, nth order tracking problem in x can be replaced by a 1st order stabilization problem in s [6].

The simplified, 1st order problem of keeping the scalar s at zero can now be achieved by choosing the control law such that outside of $S(t)$

$$\frac{d}{dt} s^T s \leq -\eta |s| \tag{10}$$

Differentiating $s(x;t)$ with respect to time, we obtain

$$\begin{aligned} &\dot{s} = \bar{F}(\mathbf{x}) + g(\mathbf{x})r, \\ &\bar{F}(\mathbf{x}) = F(\mathbf{x}) - \mathbf{x}_d^{(n)} + a_1\tilde{\mathbf{x}}^{(n-1)} + \ldots.. + a_{n-1}\dot{\tilde{\mathbf{x}}} \end{aligned} \tag{11}$$

Since $F(\mathbf{x})$ and $g(\mathbf{x})$ are unknown; only their bounds can be used to construct u. In this case, the control law r is chosen to be

$$r = -g_L^{-1}\{Q\,\mathrm{sgn}(s) - Ks\} \tag{12}$$

where
$$Q = [F^U + \left|\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1)} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}\right|]$$

Substituting (12) into (11), we have

$$\begin{aligned} &\dot{s} = \\ &\bar{F} - \{gg_L^{-1}[F^U + \left|\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}\right|]\}\mathrm{sgn}(s) - gg_L^{-1}Ks \end{aligned} \tag{13}$$

$$\begin{aligned} &s^T\dot{s} = s^T\bar{F} \\ &- s^T\{gg_L^{-1}[F^U + \left|\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}\right|]\mathrm{sgn}(s)\} \\ &- s^T gg_L^{-1}Ks \\ &\leq -s^T gg_L^{-1}Ks \\ &+ \left|s^T\right|\left|\bar{F}\right| \\ &- \left|s^T\right|\{gg_L^{-1}[F^U + \left|\mathbf{x}_d^{(n)} - a_1\tilde{\mathbf{x}}^{(n-1} - \ldots.. - a_{n-1}\dot{\tilde{\mathbf{x}}}\right|]\} \\ &\leq -s^T gg_L^{-1}Ks \leq 0 \end{aligned} \tag{14}$$

Therefore the close loop fuzzy system (6) is asymptotically stable. The results are summarized in the following theorem.

Theorem 1: If the dynamic fuzzy model described in (1) is locally controllable, then the closed-loop fuzzy system described in (6), with control law (12), is asymptotically stable.

Note that the controllability condition in Definition 1 is only required for the design of local compenstors in each rule.

4. Application to the Duffing forced-oscillation system

To illustrate the proposed method, we study the control of the Duffing forced oscillation system. The dynamic equations of the system are [7]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12\cos(t) + u(t) \end{aligned} \qquad (15)$$

The above system is chaotic without control. The trajectory of the system with $u(t) = 0$ is shown in the phase plane in Figure 6 for the initial condition $x(0) = [2 \;\; 2]^T$ and time period $t_0 = 0$ to $t_f = 20$. We now use the proposed controller to control the system state x_l to track the reference trajectory $y_m(t)=\sin(t)$. The fuzzy model for this chaotic system can also be obtained by linearizing the nonlinear equations over a number of operating points in the phase plane of (x_1, x_2). The following fuzzy model has been obtianed.

Plant Rule:
Rule 1: IF x_1 is about 0, THEN $\dot{x} = \mathbf{A}_1 x + \mathbf{B}_1 u$
Rule 2: IF x_1 is about ± 2, THEN $\dot{x} = \mathbf{A}_2 x + \mathbf{B}_2 u$
Rule 3: IF x_1 is about ± 4, THEN $\dot{x} = \mathbf{A}_3 x + \mathbf{B}_3 u$

Where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \qquad \mathbf{B}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -12 & -0.1 \end{bmatrix}, \qquad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ -48 & -0.1 \end{bmatrix}, \qquad \mathbf{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The membership functions for x_1 are chosen as in Figure 1. The rules of the new fuzzy-model-based controller are:

Controller Rule:
Rule 1: IF x_1 is about 0, THEN $u = -\mathbf{K}_1 \mathbf{x} + u_s$
Rule 2: IF x_1 is about ± 2, THEN $u = -\mathbf{K}_2 \mathbf{x} + u_s$
Rule 3: IF x_1 is about ± 4, THEN $u = -\mathbf{K}_3 \mathbf{x} + u_s$

The desired closed loop poles for each local model are chosen as –2 and –2. Thus the feedback control gains are found as:

$$\mathbf{K}_1 = [4 \;\; 3.9]$$
$$\mathbf{K}_2 = [-8 \;\; 3.9]$$
$$\mathbf{K}_3 = [-44 \;\; 3.9]$$

We choose $K = 10$, $\lambda = 5$, $g^U = g_L = 1$, $f^U = 12 + |x_1|^3$. The simulation results are shown in Figure 2 to illustrate the feasibility and effectiveness of the proposed method. Figure 2 (a) shows the state $x_l(t)$ and it desired value $y_m(t)=\sin(t)$ and Figure 2 (b) shows the state $x_2(t)$ and its desired value $\dot{y}_m(t) = \cos(t)$.

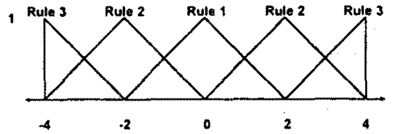
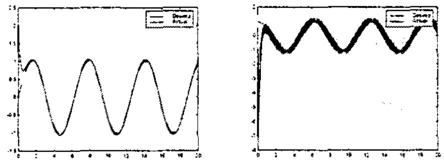


Figure 1. Membership functions of state x_1



(a) (b)

Figure. 2 Closed loop system trajectories of (a) $x_l(t)$ and (b) $x_2(t)$

5. Conclusion

In this paper, we propose a stable fuzzy logic controller architecture for an uncertain SISO nonlinear system. In the design procedure, we represent the fuzzy system as a family of local state space models, which is often called Takagi Sugeno fuzzy model and construct a global fuzzy logic controller by considering each local state feedback controller. Unlike other conventional methods, we incorporate the sliding mode control theory into the PDC approach to obtain robust tracking performance without finding common positive definite matrix **P**. Finally, Simulation is performed to control the Duffing forced-oscillation system to show the effectiveness and feasibility of the proposed fuzzy controller.

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