퍼지 추론을 이용한 디지털 재설계에 관한 연구

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Study on the Digital Redesign Using Fuzzy Inference Systems

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Abstract – In this paper, the optimal digital redesign is studied within the framework of fuzzy systems and dual-rate sampling control theory. An equivalent fast-rate discrete-time state-space model of the continuous-time system is constructed by using fuzzy inference systems. To obtain the optimal feedback gains developed in the continuous-time system, the constructed fuzzy system is converted into a continuous-time system. The developed continuous-time control law is converted into an equivalent slow-rate digital control law using the proposed digital redesign method. The digital redesign technique using a fuzzy model is employed to simulate the inverted pendulum dynamics.

1. Introduction

Most real dynamic systems are described in a continuous-time framework. In the application of digital electronics, however, the digital framework needs to be realized. Since the design of a continuous-time system using the digital counterpart is not closely coupled with the continuous aspects of the real environments, the responses of the designed system may be undesirable[1].

In the past years, hybrid self-tuning control concepts[2-5] have been improved to combine a discrete-time adaptive mechanism for updating the controller parameters with a continuous-time control theory. Karwick[3] employed a state space system to provide a pole placement scheme via state feedback. In Helliot[4], an alternative approach was proposed to the problem of applying discrete adaptation techniques for the control of continuous time processes.

For the realizations of advanced algorithms for the adaptive control of systems, it becomes necessary to consider dual-rate sampling schemes. A fast-rate sampling method is used to identify the state parameters and a slow-rate sampling method has enhanced the control performance. However, the conventional digital redesign schemes can hardly solve a nonlinear state-space system and require much more computational effort to analysis the state system. To overcome these, this paper proposes the digital redesign scheme via fuzzy inference systems.

Fuzzy inference systems employing fuzzy if-then rules can model the qualitative[7]. By embedding the fuzzy inference system into the framework of a digital redesign, hybrid state-space fuzzy self-tuning control is obtained. In this paper, new digital redesign scheme is applied to simulate the inverted pendulum.

2. Fuzzy Inference System

Fuzzy inference systems are known as fuzzy rule based systems, fuzzy models or fuzzy controllers when used as controllers. Most fuzzy inference systems can be classified into three types. In this paper, Takagi-Sugeno(TS) fuzzy inference system, which combines the fuzzy inference rule and the local linear state model, is used to model a digital system with a fast-rate sampling.

The ith rule of the TS fuzzy model, representing a complex single input single output (SISO) system, is the following:

$$\text{Rule } i: \text{ IF } x(t) \text{ is } F_1^i \text{ and ... and } x^{(n-1)}(t) \text{ is } F_n^i$$
$$\text{THEN } \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t) \quad (1)$$
$$(i = 1, 2, \ldots, r)$$

where $x(t)$ is state variables, $u(t)$ is the control input, and r is the number of rules. F is the fuzzy sets and A_i and B_i are state matrices.

Using the defuzzification method to obtain the overall output of the dynamic fuzzy model, it can be expressed as the following :

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \frac{\sum_{i=1}^{r} w_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))} \\ &= \sum_{i=1}^{r} \mu_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i u(t)) \quad (2) \\ &= \mathbf{A}(\mu(\mathbf{x}(t)))\mathbf{x}(t) + \mathbf{B}(\mu(\mathbf{x}(t)))u(t)\end{aligned}$$

where

$$w_i(\mathbf{x}(t)) = \prod_{j=1}^{n} F_j^i(x^{(j-1)}(t))$$

$$\mu_i(\mathbf{x}(t)) = \frac{w_i(\mathbf{x}(t))}{\sum_{i=1}^{r} w_i(\mathbf{x}(t))}$$

$$\mu(\mathbf{x}(t)) = (\mu_1(\mathbf{x}(t)), \mu_2(\mathbf{x}(t)), \cdots, \mu_n(\mathbf{x}(t)))$$

and $F_j^i(x^{(j-1)}(t))$ is the grade of membership of $x^{(j-1)}(t)$ in F_j^i and $w_i(\mathbf{x}(t))$ is the firing strength of ith rule.

3. Digital Redesign Technique

3.1. Optimal control with pole placement

Consider the linear controllable continuous-time system described by

$$\begin{aligned} \dot{x}_c(t) &= \mathrm{A}x_c(t) + \mathrm{B}u_c(t) \\ y(t) &= Cx_c(t) \end{aligned} \tag{3}$$

The cost function for the system in Eqn. (3) can be expressed as

$$J = \int_0^{\infty} \left[x_c^T(t)Qx_c(t) + u_c^T(t)Ru_c(t)\right]dt \tag{4}$$

where Q and R are n x n nonnegative definite and m x m positive definite symmetric matrices. The feedback control law, which minimizes the performance index, is expressed as

$$\begin{aligned} u_c(t) &= -K_c x_c(t) + E_c r(t) \\ &= -R^{-1}B^T P x_c(t) + E_c r(t) \end{aligned} \tag{5}$$

where K_c is the feedback gain, E_c is the forward gain, $r(t)$ is a reference input, and P is a non-negative symmetric matrix. It can be solved by Riccati equation

$$PBR^{-1}BP - PA - A^T P - Q = 0_n \tag{6}$$

with (Q, A) detectable. By this solution, the overall closed-loop system becomes

$$\dot{x}_c(t) = (A - BK_c)x_c(t) + BE_c u_c(t) \tag{7}$$

where K_c is $R^{-1}B^T P$ and the eigenvalues of $A\text{-}BK_c$ exist in the open left-half plane of the complex s-plane. The forward gain is computed by following

$$E_c = -\left[C(A - BK_c)^{-1}B\right]^{-1} \tag{8}$$

3.2. Digital redesign by state matching

For the implementation of the digital-time control law, the obtained continuous-time control law is converted into an equivalent digital law. The digital redesign method matches the state $x_c(t)$ at $t=kT_s$ and digitally controlled state $x_d(t)$ at $t=kT_s$.

Using the bilinear method with a sampling time, the control law is developed by the following.

The equivalent discrete-time control law and state equation are described by

$$\begin{aligned} u_d(kT_s) &= -K_d x_d(kT_s) + E_d r(kT_s) \\ &\quad for \quad kT_s \le t < kT_s + T_s \end{aligned} \tag{9}$$

$$\begin{aligned} x_d(kT_s + T_s) &= Fx_d(kT_s) + Gu_d(kT_s) \\ y(kT_s) &= Hx_d(kT_s) \end{aligned} \tag{10}$$

Therefore, the closed-loop discrete-time system with the sampling time T_s is

$$x_d(kT_s + T_s) = (F - GK_d)x_d(kT_s) + GE_d r(kT_s) \tag{11}$$

where F and G are the equivalent discrete-time state matrices, and K_d and E_d is the equivalent discrete-time feedback gain and forward gain, respectively. The continuous-time feedback gain, K_c, and forward gain, E_c, are converted into the equivalent discrete-time forms.

$$K_d = \frac{1}{2}(I_n + \frac{1}{2}K_c G)^{-1} K_c (F + I_n) \tag{12}$$

$$E_d = (I_m + \frac{1}{2}K_c G)^{-1} E_c \tag{13}$$

4. Simulation

We will illustrate the principle of imposing a linear controller design constraint here for the example of swinging up an inverted pendulum system.

Let us consider the following single input continuous time nonlinear system :

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{14}$$

A state space model is given by Eqn. (14) with

$$f(x) = \begin{bmatrix} x_2 \\ \dfrac{\frac{4}{3}mlx_4^2\sin(x_3) - \frac{mg}{2}\sin(2x_3)}{\frac{4}{3}m_t - m\cos^2(x_3)} \\ x_4 \\ \dfrac{m_t g\sin(x_3) - \frac{ml}{2}x_4^2\sin(2x_3)}{l\left(\frac{4}{3}m_t - m\cos^2(x_3)\right)} \end{bmatrix} \tag{15}$$

$$g(x) = \begin{bmatrix} 0 \\ \dfrac{1}{m_t - \frac{3}{4}m\cos^2(x_3)} \\ 0 \\ \dfrac{-\cos(x_3)}{l\left(\frac{4}{3}m_t - m\cos^2(x_3)\right)} \end{bmatrix}, \quad h(x) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{16}$$

where state variable x_1, x_2, x_3, x_4 are position and velocity of the cart, angle of the pole with vertical and rate of change of the angle, respectively. The input signal u is the force applied to the cart's center of mass. The m, m_t, l, g mean respectively mass of the pole, total mass of cart and pole, half pole length and the acceleration due to gravity.

A TS fuzzy model used to approximate the above system has four rules :

<u>Plant Rules</u>

Rule 1 : IF x_1 is small and x_3 is about 0,

$$\text{THEN } x(k+1) = F_1 x(k) + G_1 u(k)$$

Rule 2 : IF x_1 is large and x_3 is about 0,

$$\text{THEN } x(k+1) = F_1 x(k) + G_1 u(k)$$

Rule 3 : IF x_1 is small and x_3 is about $\pi/2$,

$$\text{THEN } x(k+1) = F_2 x(k) + G_2 u(k)$$

Rule 4 : IF x_1 is large and x_3 is about $\pi/2$,

$$\text{THEN } x(k+1) = F_2 x(k) + G_2 u(k)$$

where F_1, G_1 are approximated discrete-time state matrices in a small angle and F_2, G_2 in a large angle.

Figure 1 shows the fuzzy membership functions for the position and angle. The type of the fuzzy membership functions is triangle.

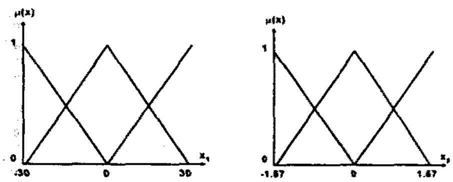

Figure 1 Fuzzy membership functions

The initial state variables are [0, 1, $-\pi/8$, 0] for the simulation. Figure 2 shows the behaviour of the closed loop system in discrete-time state-space. One shows the changes of the position, the second figure shows the angle of the pole with vertical.

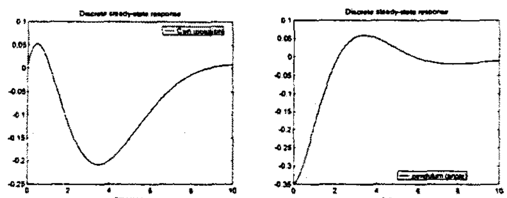

Figure 2 Simulation results of the swinging up problem

Figure 3 contains the control signal or the applied force with respect to time.

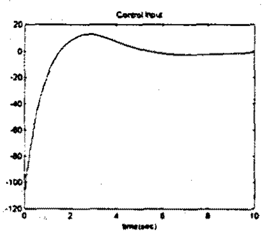

Figure 3 Result of the control input (applied force)

At this simulation, the performance of the hybrid state-space self-tuning control is shown. The position and angle of pole converges at zero.

5. Discussion

The digital redesign scheme using fuzzy inference systems is proposed in this paper. The fuzzy inference systems are used to get the discrete-time state-space model and control the applied force. This scheme has advantages of the adaptive and robust results. By inverted pendulum simulation, the performance of the proposed scheme is better than the conventional digital redesign schemes in the viewpoint of the computational efforts and performance. This scheme will be applied to the control based digital systems.

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