다입력계통에 대한 새로운 슬라이딩 모드 제어기

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The Novel Sliding Mode Controller for Linear System with Multi-Input

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Abstract - In this paper, new sliding mode surfaces are proposed by defining novel virtual states. These sliding surfaces have nominal dynamics of an original system and makes it possible that the Sliding Mode Control(SMC) technique is used with the various types of controllers. Its design is based on the augmented system whose dynamics have m-th higher order than those of the original system where m is the number of inputs. The reaching phase is removed by setting the initial virtual states which makes the initial switching functions equal to zero.

1. Introduction

The SMC is a popular robust control method which has many good results and its applications, however basically two problems are in it[1]. They are the reaching phase and the input chattering[2][3]. Besides these two problems, the SMC is very conservative to be used with other controller design methods because the state trajectories of SMC system is determined by sliding mode dynamics which have one lower order than those of the original system. To overcome this conservatism and remove the reaching phase, novel virtual states are defined based on the controllable canonical form of the nominal system. With this virtual state, an augmented ststem is constructed and novel sliding mode surfaces are proposed. This makes it possible that the new sliding mode have the dynamics of the nominal system which is the desired dynamics controlled by various types of control strategies. In this paper, a state feedback controller is considered as a nominal one and sliding modes has the dynamics of the nominal state feedback control system. By using the initial virtual states which make the initial switching functions equal to zero, the reaching phases are removed.

2. Problem Statement

Consider the n-th order system with multi-input described by

$$\dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)u(t) + Df(t) \qquad (1)$$

where $x \in R^n$, $u \in R^m$, $f \in R^r$ and the bounded uncertainties ΔA, ΔB and the disturbance matrix D satisfies the following matching conditions

$$rank([B \vdots \Delta A \vdots \Delta B \vdots D]) = rank B \qquad (2)$$

The existing sliding mode surfaces have the following form[4].

$$S(x,t) = c_n(t)x_n + c_{(n-1)}(t)x_{(n-1)} + \cdots c_1(t)x_1 + c_0(t) = 0 \qquad (3)$$

where $c_0(t), c_1(t), \cdots, c_n(t)$ are given so that sliding mode dynamics can be stable.

The above sliding modes surface has (n-1)-th order dynamics which is not the same as the n-th order dynamics of the original system. The reaching phase exists when the initial $S(x,t)$ is not zero.

The following condition guarantees the sliding mode[1].

$$S(x,t)\,\dot{S}(x,t) < 0 \qquad (4)$$

Using a number of existing techniques[2], the above condition can be satisfied by solving for the functionals $u^{+}(\cdot)$ and $u^{-}(\cdot)$ of the following feedback control which is discontinuous on the surface defined by $S(x, x_v)$

$$u(\cdot) = \begin{cases} u^{+}(\cdot), & \text{for } S>0 \\ u^{-}(\cdot), & \text{for } S<0 \end{cases} \qquad (5)$$

The problems to be solved in this paper is as follows.

- to overcome the conservatism of the SMC by using novel sliding mode surfaces which have the same dynamics of the nominal original system controlled by a nominal controller.
- to remove the reaching phase.

3. SMC with New sliding surface

Various types of sliding surfaces have been proposed including time-varying sliding surface[5].

These existing sliding mode surfaces can not have the dynamics of the original system controlled by other type of controller. This makes the conventional SMC very conservative to be combined with the other type of control strategies. To overcome this conservatism completely, a new SMC, with novel sliding mode surfaces which have the dynamics of the nominal original system controlled by nominal controller, is proposed. The novel sliding mode surfaces are designed based on the augmented system which have the novel virtual states. The virtual states are defined from the controllable canonical form of the nominal system.

Let's consider the following nominal system for the original system of Eq.(1).

$$\dot{x}_o(t) = Ax_o(t) + Bu_o(t) \qquad (6)$$

where $u_o(x_o(t), t)$ is a nominal regulating control input and differentiable.

The novel virtual states are defined based on the following controllable canonical form for the above system.

$$\dot{z}_o(t) = A_c z_o(t) + B_c u_o(t) \qquad (7)$$

with

$$A_c = \left[\begin{array}{cccc:cccc:c:cccc}
0 & 1 & 0 \cdots & 0 & 0 & 0 & 0 \cdots & 0 & & 0 & 0 & 0 \cdots & 0 \\
0 & 0 & 1 \cdots & 0 & 0 & 0 & 0 \cdots & 0 & \cdots & 0 & 0 & 0 \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\
-\alpha_n & -\alpha_{n-1} & \cdots & & & \cdots & & & & & \cdots & -\alpha_2 & -\alpha_1 \\ \hdashline
0 & 0 & 0 \cdots & 0 & 0 & 1 & 0 \cdots & 0 & & 0 & 0 & 0 \cdots & 0 \\
0 & 0 & 0 \cdots & 0 & 0 & 0 & 1 \cdots & 0 & \cdots & 0 & 0 & 0 \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\
-\beta_n & -\beta_{n-1} & \cdots & & & \cdots & & & & & \cdots & -\beta_2 & -\beta_1 \\ \hdashline
\vdots & \vdots & & \vdots & & \vdots & & & \vdots & & & \vdots & \vdots \\ \hdashline
0 & 0 & 0 \cdots & 0 & 0 & 0 & 0 \cdots & 0 & & 0 & 1 & 0 \cdots & 0 \\
0 & 0 & 0 \cdots & 0 & 0 & 0 & 0 \cdots & 0 & \cdots & 0 & 0 & 1 \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\
-\omega_n & -\omega_{n-1} & \cdots & & & \cdots & & & & & \cdots & -\omega_2 & -\omega_1
\end{array}\right] \begin{array}{l} \}\,\mu_1 \\ \}\,\mu_2 \\ \}\,n \end{array}$$

$$
B_c=\begin{bmatrix}0&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&&\vdots\\0&0&\cdots&0\\1&0&\cdots&0\\\hline 0&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&&\vdots\\0&0&\cdots&0\\0&1&\cdots&0\\\hline\vdots&\vdots&&\vdots\\\hline 0&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&&\vdots\\0&0&\cdots&0\\0&0&\cdots&1\end{bmatrix}
\begin{matrix}\left.\right\}\mu_1\\ \left.\right\}\mu_2\\ \left.\right\}n\end{matrix}
$$

The novel virtual states $z_{ov1}, z_{ov2}, \cdots, z_{ovm}$, which are proposed in this paper, are defined as a derivative of $z_{o\mu1}, z_{o\mu2}, \cdots, z_{on}$ and its dynamic are

$$
\begin{aligned}
\dot z_{ov1}(t)=&-\alpha_n z_{o2}(t)-\alpha_{n-1}z_{o3}(t)-\cdots-\alpha_{n-\mu1+2}z_{o\mu1}(t)-\\
&\alpha_{n-\mu1+1}z_{ov1}(t)-\alpha_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\alpha_{n-\mu2+1}z_{ov2}(t)\\
&-\cdots-\alpha_2 z_{on}(t)-\alpha_1 z_{ovm}(t)+\dot u_{o1}(x_o,t)\\
\dot z_{ov2}(t)=&-\beta_n z_{o2}(t)-\beta_{n-1}z_{o3}(t)-\cdots-\beta_{n-\mu1+2}z_{o\mu1}(t)-\\
&\beta_{n-\mu1+1}z_{ov1}(t)-\beta_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\beta_{n-\mu2+1}z_{ov2}(t)\\
&-\cdots-\beta_2 z_{on}(t)-\beta_1 z_{ovm}(t)+\dot u_{o2}(x_o,t)\\
&\vdots\\
\dot z_{ovm}(t)=&-\omega_n z_{o2}(t)-\omega_{n-1}z_{o3}(t)-\cdots-\omega_{n-\mu1+2}z_{o\mu1}(t)-\\
&\omega_{n-\mu1+1}z_{ov1}(t)-\omega_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\omega_{n-\mu2+1}z_{ov2}(t)\\
&-\cdots-\omega_2 z_{on}(t)-\omega_1 z_{ovm}(t)+\dot u_{om}(x_o,t)
\end{aligned}\tag{8}
$$

From the above equations, the following virtual states $z_{v1}, z_{v2}, \cdots, z_{vm}$ will be defined by replacing nominal state z_o with original state z.

$$
\begin{aligned}
\dot z_{v1}(t)=&-\alpha_n z_2(t)-\alpha_{n-1}z_3(t)-\cdots-\alpha_{n-\mu1+2}z_{\mu1}(t)-\\
&\alpha_{n-\mu1+1}z_{v1}(t)-\alpha_{n-\mu1}z_{(\mu1+2)}(t)-\cdots-\alpha_{n-\mu2+1}z_{v2}(t)\\
&-\cdots-\alpha_2 z_n(t)-\alpha_1 z_{vm}(t)+\dot u_{o1}(x,t)\\
\dot z_{v2}(t)=&-\beta_n z_2(t)-\beta_{n-1}z_3(t)-\cdots-\beta_{n-\mu1+2}z_{\mu1}(t)-\\
&\beta_{n-\mu1+1}z_{v1}(t)-\beta_{n-\mu1}z_{(\mu1+2)}(t)-\cdots-\beta_{n-\mu2+1}z_{v2}(t)\\
&-\cdots-\beta_2 z_n(t)-\beta_1 z_{vm}(t)+\dot u_{o2}(x,t)\\
&\vdots\\
\dot z_{vm}(t)=&-\omega_n z_2(t)-\omega_{n-1}z_3(t)-\cdots-\omega_{n-\mu1+2}z_{\mu1}(t)-\\
&\omega_{n-\mu1+1}z_{v1}(t)-\omega_{n-\mu1}z_{(\mu1+2)}(t)-\cdots-\omega_{n-\mu2+1}z_{v2}(t)\\
&-\cdots-\omega_2 z_n(t)-\omega_1 z_{vm}(t)+\dot u_{om}(x,t)
\end{aligned}\tag{9}
$$

where $u_o(x,t)$ and $\dot u_o(x,t)$ are obtained from $u_o(x_o,t)$ and $\dot u_o(x_o,t)$ respectively by replacing nominal state x_o with original state x. Any uncertainty must not be considered in this procedure.

With the novel virtual states, the augmented system is established as follows.

$$
\begin{aligned}
\dot x(t)=&(A+\Delta A)x(t)+(B+\Delta B)u(t)+Df(t)\\
\dot z_{v1}(t)=&-\alpha_n z_2(t)-\alpha_{n-1}z_3(t)-\cdots-\alpha_{n-\mu1+2}z_{\mu1}(t)-\\
&\alpha_{n-\mu1+1}z_{v1}(t)-\alpha_{n-\mu1}z_{(\mu1+2)}(t)-\cdots-\alpha_{n-\mu2+1}z_{v2}(t)\\
&-\cdots-\alpha_2 z_n(t)-\alpha_1 z_{vm}(t)+\dot u_{o1}(x,t)\\
\dot z_{v2}(t)=&-\beta_n z_2(t)-\beta_{n-1}z_3(t)-\cdots-\beta_{n-\mu1+2}z_{\mu1}(t)-\\
&\beta_{n-\mu1+1}z_{v1}(t)-\beta_{n-\mu1}z_{(\mu1+2)}(t)-\cdots-\beta_{n-\mu2+1}z_{v2}(t)\\
&-\cdots-\beta_2 z_n(t)-\beta_1 z_{vm}(t)+\dot u_{o2}(x,t)\\
&\vdots\\
\dot z_{vm}(t)=&-\omega_n z_2(t)-\omega_{n-1}z_3(t)-\cdots-\omega_{n-\mu1+2}z_{\mu1}(t)-\\
&\omega_{n-\mu1+1}z_{v1}(t)-\omega_{n-\mu1}z_{(\mu1+2)}(t)-\cdots-\omega_{n-\mu2+1}z_{v2}(t)\\
&-\cdots-\omega_2 z_n(t)-\omega_1 z_{vm}(t)+\dot u_{om}(x,t)
\end{aligned}\tag{10}
$$

where $u(t)$ guarantees the sliding mode on $S_i(z, z_{vi})$ ($i=1,\cdots,m$).

For the above augmented system, the novel sliding surfaces are defined as

$$
\begin{aligned}
S_1(z,z_{v1})=&z_{v1}(t)+\alpha_n z_1(t)+\alpha_{n-1}z_2(t)+\cdots+\alpha_2 z_{n-1}(t)\\
&+\alpha_1 z_n(t)-u_{o1}(x,t)=0\\
S_2(z,z_{v2})=&z_{v2}(t)+\beta_n z_1(t)+\beta_{n-1}z_2(t)+\cdots+\beta_2 z_{n-1}(t)\\
&+\beta_1 z_n(t)-u_{o2}(x,t)=0\\
&\vdots\\
S_m(z,z_{vm})=&z_{vm}(t)+\omega_1 z_n(t)+\omega_2 z_{n-1}(t)\cdots+\omega_{n-1}z_2(t)\\
&+\omega_n z_1(t)-u_{om}(x,t)=0
\end{aligned}\tag{11}
$$

The reaching phase is removed easily by setting the initial values of

$$
\begin{aligned}
z_{v1}(t_0)=&-\alpha_n z_1(t_0)-\alpha_{n-1}z_2(t_0)-\cdots-\alpha_2 z_{n-1}(t_0)\\
&-\alpha_1 z_n(t_0)+u_{o1}(t_0)\\
z_{v2}(t_0)=&-\beta_n z_1(t_0)-\beta_{n-1}z_2(t_0)-\cdots-\beta_2 z_{n-1}(t_0)\\
&-\beta_1 z_n(t_0)+u_{o2}(t_0)\\
&\vdots\\
z_{vm}(t_0)=&-\omega_n z_1(t_0)-\omega_{n-1}z_2(t_0)-\cdots-\omega_2 z_{n-1}(t_0)\\
&-\omega_1 z_n(t_0)+u_{om}(t_0)
\end{aligned}\tag{12}
$$

Now the following theorem is obtained.

Theorem 1. The novel sliding mode surfaces $S_i(x, z_{vi})$ ($i=1,\cdots,m$) have the same dynamics as the nominal system of Eq.(6)(7) controlled by nominal control inputs.

Proof) Suppose $z_{o1}, z_{o2}, \ldots, z_{on}, z_{ov1}, \cdots, z_{ovm}$ are on the sliding surface where $\begin{bmatrix}z_{o1}\\z_{o2}\\\vdots\\z_{on}\end{bmatrix}=P\begin{bmatrix}x_{o1}\\x_{o2}\\\vdots\\x_{on}\end{bmatrix}$.

Then the following equations are satisfied.

$$
\begin{aligned}
&z_{ov1}(t)+\alpha_n z_{o1}(t)+\alpha_{n-1}z_{o2}(t)+\cdots+\alpha_2 z_{o(n-1)}(t)\\
&\qquad+\alpha_1 z_{on}(t)-u_{o1}(x_o,t)=0\\
&z_{ov2}(t)+\beta_n z_{o1}(t)+\beta_{n-1}z_{o2}(t)+\cdots+\beta_2 z_{o(n-1)}(t)\\
&\qquad+\beta_1 z_{on}(t)-u_{o2}(x_o,t)=0\\
&\vdots\\
&z_{ovm}(t)+\omega_n z_{o1}(t)+\omega_{n-1}z_{o2}(t)+\cdots+\omega_2 z_{o(n-1)}(t)\\
&\qquad+\omega_1 z_{on}(t)-u_{om}(x_o,t)=0
\end{aligned}\tag{13}
$$

Set $z_{o2}=\dot z_{o1}, \cdots, z_{o\mu1}=\dot z_{o(\mu1-1)}, z_{o(\mu1+2)}=\dot z_{o(\mu1+1)}, \cdots,$ $z_{on}=\dot z_{o(n-1)}$. (14)

By differentiating Eq.(13), the following equations are obtained.

$$
\begin{aligned}
\dot z_{ov1}(t)=&-\alpha_n z_{o2}(t)-\alpha_{n-1}z_{o3}(t)-\cdots-\alpha_{n-\mu1+2}z_{o\mu1}(t)-\\
&\alpha_{n-\mu1+1}\dot z_{o\mu1}(t)-\alpha_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\alpha_{n-\mu2+1}\dot z_{o\mu2}(t)\\
&-\cdots-\alpha_2 z_{on}(t)-\alpha_1\dot z_{on}(t)+\dot u_{o1}(x_o,t)\\
\dot z_{ov2}(t)=&-\beta_n z_{o2}(t)-\beta_{n-1}z_{o3}(t)-\cdots-\beta_{n-\mu1+2}z_{o\mu1}(t)-\\
&\beta_{n-\mu1+1}\dot z_{o\mu1}(t)-\beta_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\beta_{n-\mu2+1}\dot z_{o\mu2}(t)\\
&-\cdots-\beta_2 z_{on}(t)-\beta_1\dot z_{on}(t)+\dot u_{o2}(x_o,t)\\
&\vdots\\
\dot z_{ovm}(t)=&-\omega_n z_{o2}(t)-\omega_{n-1}z_{o3}(t)-\cdots-\omega_{n-\mu1+2}z_{o\mu1}(t)-\\
&\omega_{n-\mu1+1}\dot z_{o\mu1}(t)-\omega_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\omega_{n-\mu2+1}\dot z_{o\mu2}(t)\\
&-\cdots-\omega_2 z_{on}(t)-\omega_1\dot z_{on}(t)+\dot u_{om}(x_o,t)
\end{aligned}\tag{15}
$$

According to Eq.(10), $\dot z_{ov1}, \dot z_{ov2}, \cdots, \dot z_{ovm}$ have the following dynamics.

$$
\begin{aligned}
\dot z_{ov1}(t)=&-\alpha_n z_{o2}(t)-\alpha_{n-1}z_{o3}(t)-\cdots-\alpha_{n-\mu1+2}z_{o\mu1}(t)-\\
&\alpha_{n-\mu1+1}z_{ov1}(t)-\alpha_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\alpha_{n-\mu2+1}z_{ov2}(t)\\
&-\cdots-\alpha_2 z_{on}(t)-\alpha_1 z_{ovm}(t)+\dot u_{o1}(x_o,t)\\
\dot z_{ov2}(t)=&-\beta_n z_{o2}(t)-\beta_{n-1}z_{o3}(t)-\cdots-\beta_{n-\mu1+2}z_{o\mu1}(t)-\\
&\beta_{n-\mu1+1}z_{ov1}(t)-\beta_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\beta_{n-\mu2+1}z_{ov2}(t)\\
&-\cdots-\beta_2 z_{on}(t)-\beta_1 z_{ovm}(t)+\dot u_{o2}(x_o,t)\\
&\vdots\\
\dot z_{ovm}(t)=&-\omega_n z_{o2}(t)-\omega_{n-1}z_{o3}(t)-\cdots-\omega_{n-\mu1+2}z_{o\mu1}(t)-\\
&\omega_{n-\mu1+1}z_{ov1}(t)-\omega_{n-\mu1}z_{o(\mu1+2)}(t)-\cdots-\omega_{n-\mu2+1}z_{ov2}(t)\\
&-\cdots-\omega_2 z_{on}(t)-\omega_1 z_{ovm}(t)+\dot u_{om}(x_o,t)
\end{aligned}\tag{16}
$$

From Eq.(15) and Eq.(16),

$$z_{ov1} = \dot{z}_{o\mu 1}$$
$$z_{ov2} = \dot{z}_{o\mu 2}$$
$$\vdots$$
$$z_{ovm} = \dot{z}_{on}$$

Now the followings are obtained from the Eq.(13).

$$\begin{aligned} \dot{z}_{o\mu 1}(t) &= -\alpha_n z_{o1}(t) - \alpha_{n-1} z_{o2}(t) - \cdots - \alpha_2 z_{o(n-1)}(t) \\ &\quad - \alpha_1 z_{on}(t) + u_{o1}(x_o, t) \\ \dot{z}_{o\mu 2}(t) &= -\beta_n z_{o1}(t) - \beta_{n-1} z_{o2}(t) - \cdots - \beta_2 z_{o(n-1)}(t) \\ &\quad - \beta_1 z_{on}(t) + u_{o2}(x_o, t) \\ &\vdots \\ \dot{z}_{on}(t) &= -\omega_n z_{o1}(t) - \omega_{n-1} z_{o2}(t) - \cdots - \omega_2 z_{o(n-1)}(t) \\ &\quad - \omega_1 z_{on}(t) + u_{om}(x_o, t) \end{aligned} \tag{17}$$

With Eq.(17), Eq.(14) are the canonical form of the nominal system. It is transformed to the Eq.(6) by the transformation $x_o(t) = P^{-1} z_o(t)$. Therfore the novel sliding mode surfaces $S_i(x, z_{vi})(i=1,\cdots,m)$ have the same dynamic as the nominal system.
End of Proof

From Theorem 1 mentioned above and SMC theory, the following result is obtained.

Theorem 2. If SMC inputs $u(t)$ are designed to force the states of the system onto the sliding surfaces $S_i(z, z_{vi})(i=1,\cdots,m)$, then the states $x(t)$ follow the trajectories of the nominal system controlled by $u_o(x, t)$.
Proof) It is obvious from the Theorem 1 and SMC theory.

Note that the nominal control inputs $u_o(x, t)$ can be any type of control input and this makes it possible that SMC is used with the various type of controllers. This means that the conservatism of the SMC is removed.

4. State Feedback Control using the Nov Surface

In this section, a state feedback controller is designed with the new SMC. This shows that the novel SMC have flexibility to be use with the various type of controllers to improve the robustness. Using the state feedback controller as the nominal one, a robust state feedback controller which make the states follow the nominal trjectories in spite of parameter uncertainties, can be designed. For a clear explanation, let's consider the following third order system. This can be extended to the n-th order system without loss of generality.

$$\dot{x}(t) = (A + \Delta A)x(t) + Bu(t) \tag{18}$$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $\Delta A = \begin{bmatrix} \Delta a_{11} & \Delta a_{12} & \Delta a_{13} \\ \Delta a_{21} & \Delta a_{22} & \Delta a_{23} \\ \Delta a_{31} & \Delta a_{32} & \Delta a_{33} \end{bmatrix}$

$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$ and $\Delta a_{ij} < \Delta_{ij}$ (constant).

Its nominal system is as follows.

$$\dot{x}_o(t) = Ax_o(t) + Bu_o(x_o, t) \tag{19}$$

The state feedback control inputs for the nominal system are obtained.

$$u_o^*(x_o) = -\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix} x_o(t) \tag{20}$$

The nominal system with this control inputs is

$$\dot{x}_o(t) = Ax_o(t) + Bu_o^*(x_o). \tag{21}$$

$\dot{u}_o^*(x_o)$ are calculated as follows.

$$\dot{u}_o^* = -K(Ax_o + Bu_o^*(x_o)) \tag{22}$$

$$= K_4 x_{o1} + K_5 x_{o2} + K_6 x_{o3}$$

where $K_3 = -K_1(a_{11} - b_1 K_1) - K_2(a_{21} - b_2 K_1)$
$K_4 = -K_1(a_{12} - b_1 K_2) - K_2(a_{22} - b_2 K_2)$

The following canonical system is obtained by a state transformation $\begin{bmatrix} z_{o1} \\ z_{o2} \\ z_{o3} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \begin{bmatrix} x_{o1} \\ x_{o2} \\ x_{o3} \end{bmatrix}$

$$\dot{z}_o(t) = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \\ \hline -\beta_3 & -\beta_2 & -\beta_1 \end{bmatrix} z_o(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ \hline 0 & 1 \end{bmatrix} u_n^*(x_o) \tag{23}$$

The virtual states z_{v1}, z_{v2} are defined as

$$\begin{aligned} \dot{z}_{v1} &= -\alpha_3 z_{v1} - \alpha_2 z_3 - \alpha_1 z_2 + \dot{u}_{o1}^*(x) \\ &= (-\alpha_2 p_{31} + k_{41})x_1 + (-\alpha_2 p_{32} + k_{51})x_2 + (-\alpha_2 p_{33} + k_{61})x_3 \\ &\quad - \alpha_3 z_{v1} - \alpha_1 z_{v2} \\ \dot{z}_{v2} &= -\beta_3 z_{v1} - \beta_2 z_3 - \beta_1 z_{v2} + \dot{u}_{o2}^*(x) \\ &= (-\beta_2 p_{31} + k_{42})x_1 + (-\beta_2 p_{32} + k_{52})x_2 + (-\beta_2 p_{33} + k_{62})x_3 \\ &\quad - \beta_3 z_{v1} - \beta_1 z_{v2} \end{aligned} \tag{24}$$

The augmented system is constructed as follows.

$$\dot{x}(t) = (A + \Delta A)x(t) + Bu(x)$$
$$\dot{z}_{v1}(t) = -\alpha_2 p_{31} x_1 - \alpha_2 p_{32} x_2 - \alpha_2 p_{33} x_3 - \alpha_3 z_{v1} - \alpha_1 z_{v2} + \dot{u}_{o1}^* \tag{25}$$
$$\dot{z}_{v2}(t) = -\beta_2 p_{31} x_1 - \beta_2 p_{32} x_2 - \beta_2 p_{33} x_3 - \beta_3 z_{v1} - \beta_1 z_{v2} + \dot{u}_{o2}^*$$

For the above system, the proposed novel sliding surfaces are given by

$$\begin{aligned} s_1 &= z_{v1} + \alpha_3 z_1 + \alpha_2 z_2 + \alpha_1 z_3 - u_{o1}^*(x) \\ &= z_{v1} + k_{71} x_1 + k_{81} x_2 + k_{91} x_3 = 0 \\ s_2 &= z_{v2} + \beta_3 z_1 + \beta_2 z_2 + \beta_1 z_3 - u_{o2}^*(x) \\ &= z_{v2} + k_{72} x_1 + k_{82} x_2 + k_{92} x_3 = 0 \end{aligned} \tag{26}$$

where
$k_{71} = \alpha_3 p_{11} + \alpha_2 p_{21} + \alpha_1 p_{31} + k_{11}$
$k_{81} = \alpha_3 p_{12} + \alpha_2 p_{22} + \alpha_1 p_{32} + k_{12}$
$k_{91} = \alpha_3 p_{13} + \alpha_2 p_{23} + \alpha_1 p_{33} + k_{13}$
$k_{72} = \beta_3 p_{11} + \beta_2 p_{21} + \beta_1 p_{31} + k_{21}$
$k_{82} = \beta_3 p_{12} + \beta_2 p_{22} + \beta_1 p_{32} + k_{22}$
$k_{92} = \beta_3 p_{13} + \beta_2 p_{23} + \beta_1 p_{33} + k_{23}$

Sliding mode control inputs $u(t)$ are given by

$$\begin{aligned} u_1(t) &= \phi_1 x_1 + \phi_2 x_2 + \phi_3 x_3 + \phi_4 z_{v1} \\ u_2(t) &= \phi_5 x_1 + \phi_6 x_2 + \phi_7 x_3 + \phi_8 z_{v2} \end{aligned} \tag{27}$$

where $\phi_1, \phi_2, \cdots, \phi_8$ are variable gains which guarantee the sliding mode.

5. Conclusions

A novel design method of sliding mode surfaces has been proposed. With this sliding mode surfaces, a new SMC, which makes the states of the system follow the nominal trajectory controlled by a nominal controller, can be designed. Any type of controller which is differentiable, can be a nominal controller. It has shown that the robust state feedback controller with the novel sliding mode is designed using state feedback controller as a nominal one. The reaching phase is easily removed by setting the initial virtual states appropriately. The result of this paper opens up very attractive area that various type of controller can be combined with the novel sliding mode surfaces.

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