발전터빈 용 다변수 제어기의 축약모델 동특성

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Dynamic performance of reduced order model of multivariable controller for generating turbine

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Abstract - This paper presents a model reduction procedure of the high order MIMO (multi input multi output) controller designed for the steam turbine in the generating plant. The application limit to reduction of the order is reviewed by variation in Hankel singular value as well as by variation in singular value Bode diagrams of transfer function matrices. Dynamic performances in the time domain are also compared for each reduced order model.

1. INTRODUCTION

The optimal controller has been designed using the robust control theory to improve the system performance in presence of external disturbances by adding the robust controller as an auxiliary compensator to the existing PID controllers. The performance measure in designing the robust controller is the shape of the singular value Bode diagram of the loop transfer function matrix. The prevailing modern methods of robust controller design are H^{∞}, frequency-weighted LQG (also known as the H^2 theory), LQG/LTR, μ synthesis theories, which make multivariable loop shaping easy.

The H^{∞} theory provides an exact loop shaping and direct one step procedure for synthesizing a controller which optimally satisfies singular value loop shaping specifications. However the order of its designed controller is usually higher than that of the design plant model. The LQG/LTR and H^2 lead to somewhat less direct, but nonetheless high effective iterative procedures for massaging singular value Bode plots to satisfy singular value loop shaping specification. The order of the designed controller is as same as the order of the design plant model.[1],[2]

The design plant model like as the prime mover of the steam turbine generating plant has a high order, and therfore its designed robust controller has also the samely high order if designed with LQG/LTR and H^2 or even the higher order with H^{∞}. Therfore effective techniques for the model reduction are needed for implementing the designed controller in the real environment.

This paper presents a model reduction procedure of the high order controller designed for the steam turbine for generation plant, and the practical limit to reduction of order, reviewing differences in time responses of the controllers with different orders.

2. PLANT MODEL

The state space representation of generation plant dynamics has the 15th order described as the equation (1) as below. Its design plant model augmented with 2 integrators has the 17th order and the designed robust auxiliary controller has the same order.

$$\dot{x}_p(t) = A_p\, x_p(t) + B_p\, u_p(t)$$
$$y_p(t) = C_p\, x_p(t) + D_p\, u_p(t) \qquad (1)$$

$$G_p(s) = C_p(sI - A_p)^{-1}B_p + D_p \qquad (2)$$

where

$y_p = [\Delta\omega \;\; \Delta p]^T$

$u_p = [\, u_1 \;\; u_2 \,]^T$

$\Delta\omega$: electric frequency variation in per unit

Δp: extraction steam pressure variation in per unit

u_1 : HP valve input

u_2 : LP valve input.

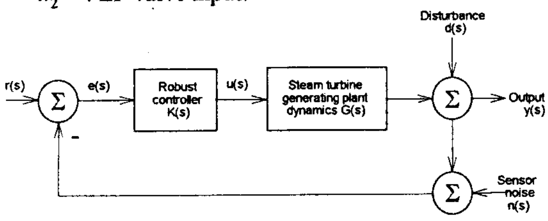

Figure 1. Steam turbine compensated by robust controller

3. REDUCTION OF CONTROLLER ORDER

Various methods for model reduction of the large scale linear systems have been studied and proposed [3], which are for example Pade approximations, modal approximation, or continued fraction expansions. In this study the method of optimal Hankel-norm approximations is utilized [4]. This technique makes it possible to calculate the achievable error between the frequency responses of the full order model and any reduced-order model. The technique has an easier computation algorithm than other methods.

The problem to reduce a model $G(s)$ is to find $\hat{G}(s)$ of McMillan degree k < n so as to minimize the norm of the error $\| G(s) - \hat{G}_r(s) \|$. The choice of norm is influenced by what norms can be minimized with reasonable computational effort and whether the chosen norm is an appropriate measure of error. It turns out that the Hankel norm is appropriate on both counts.

The Hankel norm of $G(s)$ is defined as

$$\| G(s) \|_H \overset{\Delta}{=} \bar{\sigma}(\Gamma_G) = \lambda_{\max}^{1/2}(PQ). \tag{3}$$

Here $G(s) = C(sI - A)^{-1}B$ with $\mathrm{Re}(\lambda_i(A)) < 0$ *for all i*.

Γ_G is Hankel operator where

$$(\Gamma_G v)(t) \overset{\Delta}{=} \int_0^{\infty} C \exp[A(t+\tau)] B v(\tau) d\tau. \tag{4}$$

P is the controllability gramian and defined as

$$P \overset{\Delta}{=} \int_0^{\infty} \exp(At) B B^* \exp(A^* t) dt. \tag{5}$$

Q is the observability gramian and defined as

$$Q \overset{\Delta}{=} \int_0^{\infty} \exp(A^* t) C^* C \exp(At) dt. \tag{6}$$

A^* is the complex conjugate transpose of A.

P and Q satisfy the following linear matrix equations, Lyapunov equations.

$$\begin{aligned} AP + PA^* + BB^* &= 0 \\ A^*Q + QA + C^*C &= 0 \end{aligned} \tag{7}$$

The Hankel singular values of G(s) are defined as

$$\sigma_i(G(s)) \overset{\Delta}{=} \{\lambda_i(PQ)\}^{1/2} \tag{8}$$

where by convention $\sigma_i G(s) \geq \sigma_{i+1} G(s)$.

$\hat{G}(s)$ with McMillan degree k is an optimal Hankel norm approximation to $G(s)$ and it exists in the case that $\| G(s) - \hat{G}(s) \|_H \cong \sigma_{k+1} G(s)$.

4. CASE STUDIES

4.1 Minimum order forecast by Hankel singular value

The minimum order of the desirable controller can be forecasted, even without the time domain simulation, by examining the order k where the Hankel singular values change largely. As shown in Table 1, the Hankel singular values of the designed 17th order controller change largely, to say 2 places of decimals to 1 place of decimals when the degree k moves from k=13 to k=12 and 11.

4.2 Comparison of singular values Bode diagrams

The singular value is defined as :

$$\sigma_i(A) = \sqrt{\lambda_i(A^H A)} = \sqrt{\lambda_i(AA^H)} > 0$$

where $i = 1, 2, \ldots\ldots, k$

A^H *:complex conjugate transposed matrix of matrix A*

Here σ is the singular value.

The performance and stability of the MIMO feedback control system can be evaluated by the analysis of the singular value Bode diagram of the Transfer Function Matrix (TFM) in the frequency domain. [5],[6]

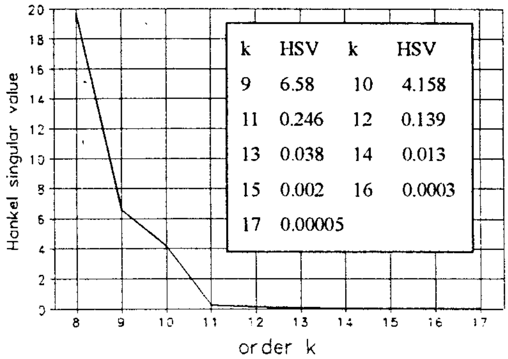

Figure 2. Hankel singular value(HSV) versus order k

$T(s) = G(s)\,K(s)$: Loop TFM (9)

$S(s) = (I + T(s))^{-1}$: Sensitivity TFM (10)

$C(s) = (I + T(s))^{-1}\,T(s)$: Closed Loop TFM (11)

The Bode diagrams of singular values $\sigma[T(j\omega)]$, $\sigma[S(j\omega)]$, $\sigma[C(j\omega)]$ are reviewed and compared for the original 17th, 13th, and 11th order models. Bode diagrams of the 17th and 13th order are closely matched each other in the frequency region of interest, but that of the 11th order deviates from those of the 17th and 13th order model. The minimum singular value of T(s) is the measure of the stability robustness, which is

$\sigma_{\min}[T(j\omega)] > 0$ dB in the frequency band of interest. The singular values of the sensitivity TFM, $\sigma[S(j\omega)]$ are compared to examine the performance of disturbance rejection. The maximum singular value is $\sigma_{\max}[S(j\omega)] < 0$ dB in the frequency band of interest.

The singular values of the closed loop TFM, $\sigma[C(j\omega)]$ are compared to check the performance of noise insensitivity. The maximum singular value $\sigma_{\max}[C(j\omega)] < 0$ dB with a high frequency bandwidth.

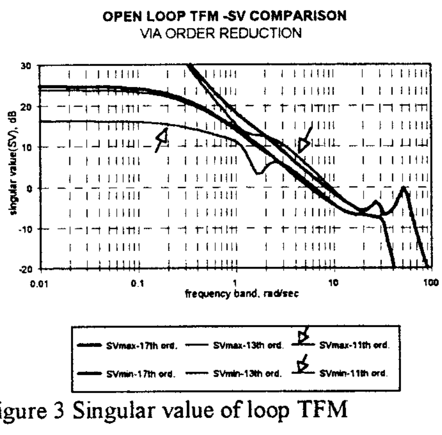

Figure 3 Singular value of loop TFM

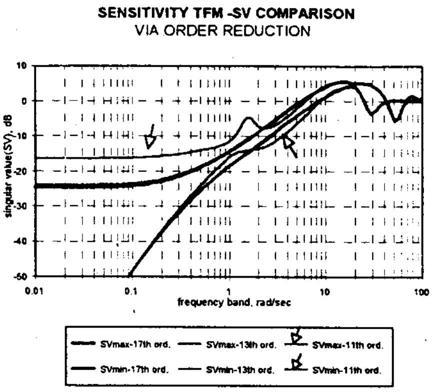

Figure 4 Singular value of sensitivity TFM

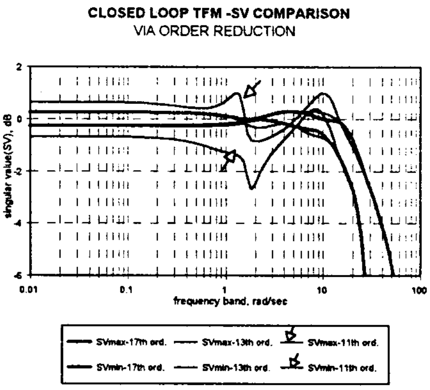

Figure 5 Singular value of closed loop TFM

4.3 Time response simulation

The time responses of steam turbine generating plant compensated by the controller are compared for each 17th, 13th, 11th order by applying stepwise increase in electrical and steam load. The 17th and 13th order controllers provide similar dynamic performances each other, however the 11th order controller provides less effective performance, which starts to deviate from the originally designed 17th order controller.

5. CONCLUSION

This paper presents a model reduction procedure of the high order MIMO controller designed for the steam turbine in the generating plant. The application limit to reduction of order is reviewed by variation in Hankel singular value as well as by variation in singular value Bode diagrams of transfer function matrices. Dynamic performances in the time domain are also compared for each reduced order model.

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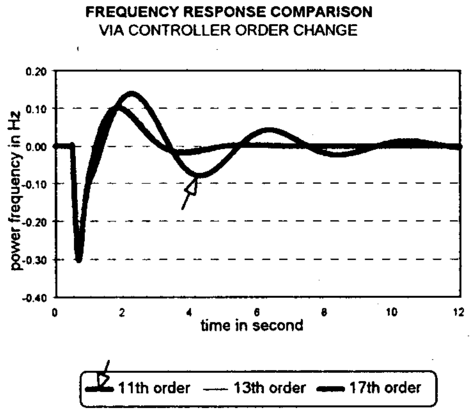

Figure 6. Frequency response comparison

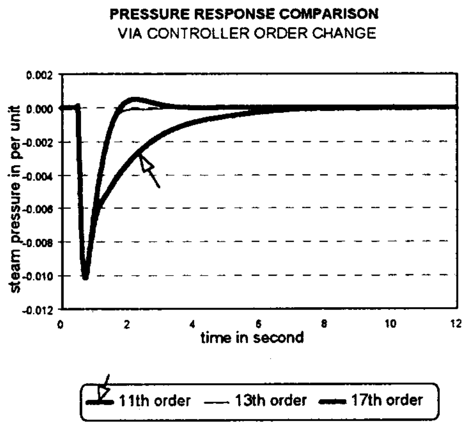

Figure 7. Pressure response comparison

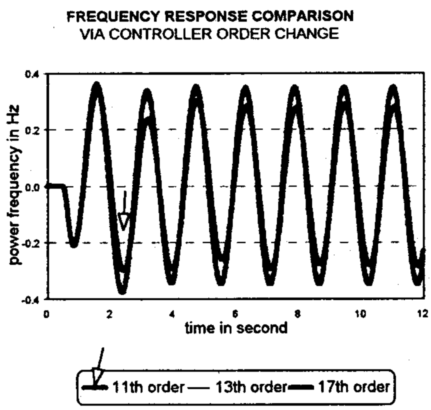

Figure 8. Frequency response comparison