NFL-O 에 기준한 SMMFC 의 안정도 증명 : Part 7

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Stability Proof of NFL-O-based SMMFC : Part 7

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[Abstract] This paper presents a stability proof for the nonlinear feedback linearization-observer-based sliding mode model following controller (NFL-O-based SMMFC). The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

Keywords : nonlinear feedback linearization-observer-based sliding mode model following controller, Lyapunov function, stability proof

1. Introduction

In this paper, the nonlinear feedback linearization-observer-based sliding mode model following controller (NFL-O-based SMMFC) for unmeasurable plant state variables to solve the problem associated with the full state feedback is developed [1-17,19]. The proposed NFL-O-based SMMFC is obtained by the estimated state variable based on observer state in designing a sliding surface gain to ensure the stability by Lyapunov's method. The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

2. NFL-O-based SMMFC

The NFL-based reference model state equation is

$$z_m(t) = T(x_m(t)) \quad (1)$$

$$\dot{z}_m(t) = A_m z_m(t) + B_m u_m(t) \quad (2)$$

where $x_m \in R^n$ is the state vector for model, $z_m \in R^n$ is the transformed state vector for model, $u_m \in R^p$ is the control input for model, A_m is the $n \times n$ system matrix for model, and B_m is the $n \times p$ control vector for model.

The control input for a reference model is

$$u_m(t) = -K_m z_m(t) \quad (3)$$

$$K_m = R_m^{-1} B_m^T P_m \quad (4)$$

$$P_m A_m + A_m^T P_m - P_m B_m R_m^{-1} B_m^T P_m + Q_m = 0 \quad (5)$$

where K_m is a $p \times n$ optimal feedback gain for model, and P_m is the algebraic matrix Riccati equation.

The closed loop feedback system is

$$\dot{z}_m(t) = (A_m - B_m K_m) z_m(t) \quad (6)$$

$$A_{km} := A_m - B_m K_m \quad (7)$$

The NFL-based state equation including CLF is reformed as

$$\dot{z}_m(t) = A_{km} z_m(t) \quad (8)$$

The control input for a controlled plant is

$$u_p(t) = -K_p z_p(t) \quad (9)$$

$$K_p = R_p^{-1} B_p^T P_p \quad (10)$$

$$P_p A_p + A_p^T P_p - P_p B_p R_p^{-1} B_p^T P_p + Q_p = 0 \quad (11)$$

The NFL-based state equation for the controlled plant and the output equation are

$$z_p(t) = T(x_p(t)) \quad (12)$$

$$\dot{z}_p(t) = A_p z_p(t) + B_p u_p(t) \quad (13)$$

$$y_p(t) = C_p z_p(t) \quad (14)$$

The NFL-based observer equation is [18]

$$\begin{aligned}\dot{\hat{z}}_p(t) &= A_p \hat{z}_p(t) + B_p u_p(t) + L_p(y_p(t) - C_p \hat{z}_p(t)) \\ &= (A_p - L_p C_p)\hat{z}_p(t) + B_p u_p(t) + L_p y_p(t) \quad (15)\end{aligned}$$

$$L_p = P_p C_p^T R_p^{-1} \quad (16)$$

$$A_p P_p + P_p A_p^T - P_p C_p^T R_p^{-1} C_p P_p + Q_p = 0 \quad (17)$$

The NFL-based state equation for the controlled plant including CLF is expressed as

$$\dot{z}_p(t) = (A_p - B_p K_p) z_p(t) \quad (18)$$

Let $A_{kp} := A_p - B_p K_p$ (19)

$$\dot{z}_p(t) = A_{kp} z_p(t) + B_p u_{cp}(t) \quad (20)$$

The error and the differential error vectors are

$$e(t) = z_m(t) - \hat{z}_p(t) \quad (21)$$

$$\dot{e}(t) = \dot{z}_m(t) - \dot{\hat{z}}_p(t) \quad (22)$$

$$\dot{e}(t) = \dot{z}_m(t) - \dot{\hat{z}}_p(t) = [A_{km} z_m(t)] - [(A_{kp} - L_p C_p)\hat{z}_p(t) + B_p \hat{u}_{O\text{-}SMMFC}(t) + L_p y_p(t)] \quad (23)$$

$$z_m(t) = e(t) + \hat{z}_p(t) \quad (24)$$

By substituting (24) into (23), we have

$$\begin{aligned}\dot{e}(t) &= A_{km} z_m(t) - (A_{kp} - L_p C_p)\hat{z}_p(t) - B_p \hat{u}_{O\text{-}SMMFC}(t) - L_p y_p(t) \\ &= A_{km}(e(t) + \hat{z}_p(t)) - (A_{kp} - L_p C_p)\hat{z}_p(t) - B_p \hat{u}_{O\text{-}SMMFC}(t) - L_p y_p(t) \\ &= A_{km} e(t) + (A_{km} + L_p C_p - A_{kp})\hat{z}_p(t) - B_p \hat{u}_{O\text{-}SMMFC}(t) - L_p y_p(t) \quad (25)\end{aligned}$$

The sliding surface vector and the differential sliding surface vector can be expressed as

$$\sigma(e(t)) = G_{SS}^T e(t) = G_{SS}^T z_m(t) - G_{SS}^T \hat{z}_p(t) \Rightarrow 0 \quad (26)$$

$$\dot{\sigma}(e(t)) = G_{SS}^T \dot{e}(t) = G_{SS}^T A_{km} e(t) + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat{z}_p(t) - G_{SS}^T B_p \hat{u}_{O\text{-}SMMFC}(t) - G_{SS}^T L_p y_p(t) \Rightarrow 0 \quad (27)$$

where G_{SS}^T is the sliding surface gain [ ].

The Lyapunov's function candidate is chosen as

$$V(e(t)) = \sigma^2(e(t))/2 \quad (28)$$

The time derivative of $V(e(t))$ is given by

$$\dot{V}(e(t)) = \sigma(e(t))\dot{\sigma}(e(t)) \quad (29)$$

$$= G_{SS}^T e(t) G_{SS}^T \dot{e}(t) \quad (30)$$

$$\begin{aligned}&= G_{SS}^T e(t) G_{SS}^T [A_{km} e(t) + (A_{km} + L_p C_p - A_{kp})\hat{z}_p(t) - B_p \hat{u}_{O\text{-}SMMFC}(t) - L_p y_p(t)] \\ &= G_{SS}^T e(t) [G_{SS}^T A_{km} e(t) + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat{z}_p(t) - G_{SS}^T B_p \hat{u}_{O\text{-}SMMFC}(t) - G_{SS}^T L_p y_p(t)] \le 0 \quad (31)\end{aligned}$$

From equation (31), the estimated control inputs with switching function are represented by

$$\hat{u}_{O\text{-}SMMFC}(t) \ge (G_{SS}^T B_p)^{-1} [G_{SS}^T A_{km} e(t) + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat{z}_p(t) - G_{SS}^T L_p y_p(t)] \quad for \ G_{SS}^T e(t) > 0 \quad (32)$$

$$\hat{u}_{O\text{-}SMMFC}(t) \le (G_{SS}^T B_p)^{-1} [G_{SS}^T A_{km} e(t) + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat{z}_p(t) - G_{SS}^T L_p y_p(t)] \quad for \ G_{SS}^T e(t) < 0 \quad (33)$$

The estimated control input vector with sign function for the controlled plant is reformed as

$$\hat{u}_{O\text{-}SMMFC}^{sign}(t) = [E_{O\text{-}SMMFC}^{equal} e(t) + P_{O\text{-}SMMFC}^{equal} \hat{z}_p(t) + O_{O\text{-}SMMFC}^{equal} y_p(t)] sign(\sigma(e(t))) \quad (34)$$

subject to $sign(\sigma(e(t))) = 1$ for $\sigma(e(t)) > 0$

$sign(\sigma(e(t))) = 0$ for $\sigma(e(t)) = 0$

$sign(\sigma(e(t))) = -1$ for $\sigma(e(t)) < 0$

where $E_{O\text{-}SMMFC}^{equal} := (G_{SS}^T B_p)^{-1} G_{SS}^T A_{Km}$ (35)

$$P_{O\text{-}SMMFC}^{equal} := (G_{SS}^T B_p)^{-1} G_{SS}^T (A_{km} + L_p C_p - A_{kp}) \quad (36)$$

$$O_{O\text{-}SMMFC}^{equal} := -(G_{SS}^T B_p)^{-1} G_{SS}^T L_p \quad (37)$$

Theorem 1: Consider the state equations of the reference model and the controlled plant based on NFL for the regulation problem

$$\dot{z}_m = A_{km} z_m \text{ and } y_m = C_m z_m$$

$$\dot{z}_p = A_{kp} z_p + B_p \hat{u}_{O\text{-}SMMFC}^{sign} \text{ and } y_p = C_p z_p$$

Consider the observer state equation based on NFL

$$\dot{\hat{z}}_p = A_p \hat{z}_p + B_p \hat{u}_{O\text{-}SMMFC}^{sign} + L_p (y_p - C_p \hat{z}_p)$$

Consider $G_{SS}^T B_p (G_{SS}^T B_p)^{-1} = I$, $y_p = C_p z_p$, $e = z_m - \hat{z}_p$, and $z_p = e_p + \hat{z}_p$. Suppose that (A_p, C_p) is detectable and $(A_p - L_p C_p)$ is Hurwitz. The estimated sliding mode model following control law with sign function based on NFL is guaranteed an *asymptotically stable* for the system (13)

$$\hat{u}_{O\text{-}SMMFC}^{sign} = [E_{O\text{-}SMMFC}^{equal} e + P_{O\text{-}SMMFC}^{equal} \hat{z}_p + O_{O\text{-}SMMFC}^{equal} y_p] sign(\sigma(e))$$

$$E_{O\text{-}SMMFC}^{equal} := (G_{SS}^T B_p)^{-1} G_{SS}^T A_{Km}$$

$$P_{O\text{-}SMMFC}^{equal} := (G_{SS}^T B_p)^{-1} G_{SS}^T (A_{km} + L_p C_p - A_{kp})$$

$$O_{O\text{-}SMMFC}^{equal} := -(G_{SS}^T B_p)^{-1} G_{SS}^T L_p$$

subject to $sign(\sigma(e)) = 1$ for $\sigma(e) > 0$

$sign(\sigma(e)) = 0$ for $\sigma(e) = 0$

$sign(\sigma(e)) = -1$ for $\sigma(e) < 0$

Proof. Let us define the error equation

$$e_p = z_p - \hat{z}_p$$

The differential error equation is represented by

$$\begin{aligned}\dot{e}_p &= \dot{z}_p - \dot{\hat{z}}_p \\ &= A_{kp} z_p + B_p \hat{u}_{O\text{-}SMMFC}^{sign} - A_{kp} \hat{z}_p - B_p \hat{u}_{O\text{-}SMMFC}^{sign} - L_p C_p z_p + L_p C_p \hat{z}_p \\ &= A_{kp} z_p - A_p \hat{z}_p - L_p C_p z_p + L_p C_p \hat{z}_p \\ &= (A_{kp} - L_p C_p) e_p\end{aligned}$$

A Lyapunov's function candidate is chosen by

$$V = \frac{1}{2}\sigma^T(e)\sigma(e) + \frac{1}{2} e_p^T e_p$$

The derivative is obtained by

$$\begin{aligned}\dot{V} &= \sigma^T(e)\dot{\sigma}(e) + e_p^T \dot{e}_p \\ &= \sigma^T(e)(G_{SS}^T A_{km} e + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat{z}_p - G_{SS}^T B_p \hat{u}_{O\text{-}SMMFC}^{sign} - G_{SS}^T L_p y_p) + e_p^T (A_{kp} - L_p C_p) e_p \\ &= \sigma^T(e)(G_{SS}^T A_{km} e + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat{z}_p - (G_{SS}^T B_p E_{O\text{-}SMMFC}^{equal} e + G_{SS}^T B_p P_{O\text{-}SMMFC}^{equal} \hat{z}_p\end{aligned}$$

$$+G_{SS}^T B_p O_{O-SMMFC}^{equal} y_p\big)sign(\sigma(e)) - G_{SS}^T L_p y_p\big)$$
$$+e_p^T (A_{kp} - L_p C_p) e_p$$

Let $E_{O-SMMFC}^{equal} := (G_{SS}^T B_p)^{-1} G_{SS}^T A_{Km}$

$$P_{O-SMMFC}^{equal} := (G_{SS}^T B_p)^{-1} G_{SS}^T (A_{km} + L_p C_p - A_{kp})$$
$$O_{O-SMMFC}^{equal} := -(G_{SS}^T B_p)^{-1} G_{SS}^T L_p$$

Therefore,

$$\dot V = \sigma^T(e)\big(G_{SS}^T A_{km} e + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat z_p$$
$$-G_{SS}^T B_p \big((G_{SS}^T B_p)^{-1} G_{SS}^T A_{Km}\big) e \ \ sign(\sigma(e))$$
$$-G_{SS}^T B_p \big((G_{SS}^T B_p)^{-1} G_{SS}^T (A_{km} + L_p C_p - A_{kp})\big)\hat z_p sign(\sigma(e))$$
$$+G_{SS}^T B_p \big((G_{SS}^T B_p)^{-1} G_{SS}^T L_p\big) y_p sign(\sigma(e)) - G_{SS}^T L_p y_p\big)$$
$$+e_p^T (A_{kp} - L_p C_p) e_p$$

Consider $G_{SS}^T B_p (G_{SS}^T B_p)^{-1} = I$, $y_p = C_p z_p$, $e = z_m - \hat z_p$, and $z_p = e_p + \hat z_p$

$$\dot V = \sigma^T(e)\big(G_{SS}^T A_{km} e + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat z_p$$
$$-G_{SS}^T A_{Km} e \ \ sign(\sigma(e)) - G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat z_p sign(\sigma(e))$$
$$+G_{SS}^T L_p y_p sign(\sigma(e)) - G_{SS}^T L_p y_p\big) + e_p^T (A_{kp} - L_p C_p) e_p$$
$$= \sigma^T(e)\big(G_{SS}^T A_{km} e + G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat z_p$$
$$-G_{SS}^T A_{Km} e \ \ sign(\sigma(e)) - G_{SS}^T (A_{km} + L_p C_p - A_{kp})\hat z_p sign(\sigma(e))$$
$$+G_{SS}^T L_p C_p z_p sign(\sigma(e)) - G_{SS}^T L_p C_p z_p\big) + e_p^T (A_{kp} - L_p C_p) e_p$$
$$= \sigma^T(e) G_{SS}^T (A_{km} + L_p C_p - A_{kp})(1 - sign(\sigma(e)))\hat z_p$$
$$+\sigma^T(e) G_{SS}^T L_p C_p (e_p + \hat z_p) sign(\sigma(e))$$
$$-\sigma^T(e) G_{SS}^T L_p C_p (e_p + \hat z_p) + \sigma^T(e) G_{SS}^T A_{km} e$$
$$-\sigma^T(e) G_{SS}^T A_{Km} e \ \ sign(\sigma(e)) + e_p^T (A_{kp} - L_p C_p) e_p$$
$$= \sigma^T(e) G_{SS}^T (A_{km} + L_p C_p - A_{kp})(1 - sign(\sigma(e)))\hat z_p$$
$$-\sigma^T(e) G_{SS}^T L_p C_p (1 - sign(\sigma(e)))\hat z_p$$
$$+\sigma^T(e) G_{SS}^T L_p C_p e_p sign(\sigma(e)) - \sigma^T(e) G_{SS}^T L_p C_p e_p$$
$$+\sigma^T(e) G_{SS}^T A_{km} e - \sigma^T(e) G_{SS}^T A_{Km} e \ \ sign(\sigma(e))$$
$$+e_p^T (A_{kp} - L_p C_p) e_p$$
$$= \sigma^T(e) G_{SS}^T (A_{km} - A_{kp})(1 - sign(\sigma(e)))\hat z_p$$
$$+\sigma^T(e) G_{SS}^T L_p C_p e_p sign(\sigma(e)) - \sigma^T(e) G_{SS}^T L_p C_p e_p$$
$$+\sigma^T(e) G_{SS}^T A_{km} e - \sigma^T(e) G_{SS}^T A_{Km} e \ \ sign(\sigma(e))$$
$$+e_p^T (A_{kp} - L_p C_p) e_p$$

If $(A_{kp} - L_p C_p)$ is stable, the error is $e_p \to 0$, and $e \to 0$ as $t \to 0$.

$$\dot V = \sigma^T(e) G_{SS}^T (A_{km} - A_{kp})(1 - sign(\sigma(e)))\hat z_p \le 0$$

subject to if $\sigma(e) > 0$, $\dot V = 0$

if $\sigma(e) = 0$, $\dot V = 0$

if $\sigma(e) < 0$, $\dot V = -2k G_{SS}^T (A_{km} - A_{kp})\hat z_p < 0$

, k is positive constant.

i.e., $\dot V < 0$ and so the system is *asymptotically stable*.

This completes the proof of this theorem. □

3. Conclusion

A stability proof of a nonlinear feedback linearization-observer-based sliding mode model following controller (NFL-O-based SMMFC) has been done.

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