NFL-FOO에 기준한 SMC의 안정도 증명 : Part 5

이 상 성° 박종근 — 서울대학교 전기공학부
이 주 장 — 한국과학기술원 전기공학과

Stability Proof of NFL-FOO-based SMC : Part 5

Sang-Seung Lee° and Jong-Keun Park
School of Electrical Engineering
Seoul National University

Ju-Jang Lee
Dept. of Electrical Engineering
KAIST

[Abstract] This paper presents a stability proof for the nonlinear feedback linearization-full order observer-based sliding mode controller (NFL-FOO-based SMC). The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

Keywords : nonlinear feedback linearization-full order observer-based sliding mode controller, Lyapunov function, stability proof

1. Introduction

In this paper, to solve the problem associated with the full state feedback [1-17], the nonlinear feedback linearization-full order observer-based sliding mode controller (NFL-FOO-based SMC) for unmeasurable state variables is developed. The proposed NFL-FOO-based SMC is obtained by the estimated state variable based on observer state in designing a sliding surface gain to ensure the stability by Lyapunov's method. The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

2. NFL-FOO-based SMC

Let us consider the general nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \tag{1}$$

$$y(t) = h(x(t)) \tag{2}$$

in which $f(x)$ and $g(x)$ are smooth vector fields, and $h(x)$ is a smooth function, defined on R^n.
The state equations based on nonlinear feedback linearization (NFL) [19] can be expressed as

$$z(t) = T(x(t)) \tag{3}$$

$$\dot{z}(t) = Az(t) + Bu(t) \tag{4}$$

$$y(t) = Cz(t) \tag{5}$$

where $x \in R^n$, $z \in R^n$, $u \in R^m$, $y \in R^p$, A is the system matrix, B is the control matrix, and C is the output matrix.
To implement the state feedback control law to plants with unmeasurable states, observer is designed to estimate the states and the estimated state is used for control input. The observer based on NFL is to construct of the form [18]

$$\begin{aligned}\dot{\hat{z}}(t) &= A\hat{z}(t) + Bu(t) + L(y(t) - C\hat{z}(t)) \\ &= (A - LC)\hat{z}(t) + Bu(t) + Ly(t)\end{aligned} \tag{6}$$

$$L = PC^T R^{-1} \tag{7}$$

$$AP + PA^T - PC^T R^{-1} CP + Q = 0 \tag{8}$$

where $\hat{z} \in R^n$ is the estimated state, L is the $n \times m$ output injection matrix, P is the symmetric positive definite solution, and Q and R are positive definite matrices.

Remark : The estimator in equation (6) is driven by the input as well as the output of the original system. The output, $y = Cz$, is compared with $\hat{y} = C\hat{z}$, and their difference is used to serve as a correcting term. The difference of y and $C\hat{x}$, $y - C\hat{z}$, is multiplied by an real constant matrix L, and fed into the input of the integrators of the estimator. This estimator is called an *asymptotic state estimator*. □

Definition 1: The system (4), or the pair (A, B), is said to be *stabilizable* if there exists a state feedback $u = -K_{LQR}z$ such that the system is stable (i.e., $A - BK$ is stable). □

Definition 2: The system (4), or the pair (A,B), is said to be *stabilizable* if there is a linear control such that the feedback system is asymptotically stable. □

Definition 3: The system (4) and (5), or the pair (C,A), is *detectable* if $A-LC$ is stable for some L. □

The estimated input vector of the NFL-O/LQR is defined as

$$u_{FL-O/LQR} = -K_{LQR}\hat{z}(t) \quad (9)$$

$$K_{LQR} = R^{-1}B^T P \quad (10)$$

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (11)$$

where K_{LQR} is an optimal feedback gain.

Remark : The relation between the weight matrices Q and R defines the tradeoff between the two contradictory desires such as to have a rapidly decaying control process, and to reduce power consumption for its realization. □

Definition 4: In equations (4) and (5), if pair (A,B) is stabilizable and pair (A,C) is detectable, $Q \geq 0$, $R \geq 0$, the Riccati equation has *unique solution* defining the optimal control, the feedback system being asymptotically stable. □

The sliding surface vector and the differential sliding surface vector to cope with the problem of the unmeasurable state variables can be expressed as

$$\sigma(\hat{z}(t)) = G_{SS}^T \hat{z}(t) \quad (12)$$

$$\dot{\sigma}(\hat{z}(t)) = G_{SS}^T \dot{\hat{z}}(t) \quad (13)$$

where G_{SS}^T is the sliding surface gain.

The Lyapunov's function candidate is chosen by

$$V(\hat{z}(t)) = \sigma^2(\hat{z}(t))/2 \quad (14)$$

The time derivative of $V(\hat{z}(t))$ can be expressed as

$$\begin{aligned}\dot{V}(\hat{z}(t)) &= \sigma(\hat{z}(t))\dot{\sigma}(\hat{z}(t)) \quad (15)\\ &= G_{SS}^T \hat{z} G_{SS}^T \dot{\hat{z}}(t)\\ &= G_{SS}^T \hat{z}(t) G_{SS}^T\left[(A-LC)\hat{z}(t) + B\hat{u}_{O-SMC}^{equal}(t) + Ly(t)\right]\\ &\leq 0 \quad (16)\end{aligned}$$

where $\hat{u}_{O-SMC}^{equal}(t)$ is the estimated input vector of the nonlinear feedback linearization-observer-based sliding mode control (NFL-O-based SMC).

The selection of the maximum and the minimum values in the following manner will guarantee sliding mode operation for any operating point. From equation (16), the estimated control input of the NFL-O-based SMC with switching function can be derived as follows:

$$\hat{u}_{O-SMC}^{+}(t) \geq -(G_{SS}^T B)^{-1}\left[G_{SS}^T(A-LC)\hat{z}(t) + G_{SS}^T Ly(t)\right]$$
$$\text{for } G_{SS}^T \hat{z}(t) > 0 \quad (17)$$

$$\hat{u}_{O-SMC}^{-}(t) \leq -(G_{SS}^T B)^{-1}\left[G_{SS}^T(A-LC)\hat{z}(t) + G_{SS}^T Ly(t)\right]$$
$$\text{for } G_{SS}^T \hat{z}(t) < 0 \quad (18)$$

From equations (17) and (18), the estimated control input of the proposed NFL-O-based SMC with sign function can be reformed as

$$\hat{u}_{O-SMC}^{sign}(t) = -(G_{SS}^T B)^{-1}\left[G_{SS}^T(A-LC)\hat{z}(t) + G_{SS}^T Ly(t)\right] sign(\sigma(\hat{z}(t))) \quad (19)$$

subject to $sign(\sigma(\hat{z}(t))) = 1$ for $\sigma(\hat{z}(t)) > 0$

$sign(\sigma(\hat{z}(t))) = 0$ for $\sigma(\hat{z}(t)) = 0$

$sign(\sigma(\hat{z}(t))) = -1$ for $\sigma(\hat{z}(t)) < 0$

Finally, the equation (19) is simplified as follows:

$$\hat{u}_{O-SMC}^{sign}(t) = -\left[OK_1^{equal}\hat{z}(t) + OK_2^{equal}y(t)\right] sign(\sigma(\hat{z}(t))) \quad (20)$$

$$OK_1^{equal} := (G_{SS}^T B)^{-1} G_{SS}^T (A-LC) \quad (21)$$

$$OK_2^{equal} := (G_{SS}^T B)^{-1} G_{SS}^T L \quad (22)$$

Theorem 1: Suppose that (A,C) is detectable and $(A-LC)$ is Hurwitz. Consider the state equations and the observer state equation based on NFL for the regulation problem

$$\dot{z} = Az + B\hat{u}_{O-SMC}^{sign}$$
$$y = Cz$$
$$\dot{\hat{z}} = A\hat{z} + B\hat{u}_{O-SMC}^{sign} + L(y - C\hat{z})$$

The estimated sliding mode control law with sign function based on NFL that keeps the system stable is guaranteed an *asymptotically stable* for the system (4)

$$\hat{u}_{O-SMC}^{sign} = -\left[OK_1^{equal}\hat{z} + OK_2^{equal}y\right] sign(\sigma(\hat{z}))$$
$$OK_1^{equal} := (G_{SS}^T B)^{-1} G_{SS}^T (A-LC)$$
$$OK_2^{equal} := (G_{SS}^T B)^{-1} G_{SS}^T L$$

subject to $sign(\sigma(\hat{z})) = 1$ *for* $\sigma(\hat{z}) > 0$

$sign(\sigma(\hat{z})) = 0$ *for* $\sigma(\hat{z}) = 0$

$sign(\sigma(\hat{z})) = -1$ *for* $\sigma(\hat{z}) < 0$

Proof. Let us define the estimation error equation

$$e = z - \hat{z}$$

The differential estimation error equation is

$$
\begin{aligned}
\dot{e} &= \dot{z} - \dot{\hat{z}} \\
&= Az + b\hat{u}^{sign}_{O-SMC} - A\hat{z} - B\hat{u}^{sign}_{O-SMC} - LCz + LC\hat{z} \\
&= Az - A\hat{z} - LCz + LC\hat{z} \\
&= (A - LC)e
\end{aligned}
$$

Lyapunov's function candidate using the addition form of the sliding surface and the estimation error is chosen by

$$V = \frac{1}{2}\sigma^T\sigma + \frac{1}{2}e^T e$$

The derivative of a Lyapunov's function candidate is obtained by

$$
\begin{aligned}
\dot{V} &= \sigma^T\dot{\sigma} + e^T\dot{e} \\
&= \sigma^T\left(G^T\dot{\hat{z}}\right) + e^T(A - LC)e \\
&= \sigma^T\left(G^T\left(A\hat{z} + B\hat{u}^{sign}_{O-SMC} + L(y - C\hat{z})\right)\right) + e^T(A - LC)e \\
&= \sigma^T G^T\Big((A - LC)\hat{z} + B\Big(-\left(OK_1^{equal}\hat{z} + OK_2^{equal}y\right) \\
&\quad sign(\sigma(\hat{z})) + LCz \quad + e^T(A - LC)e \\
&= \sigma^T\Big(G^T(A - LC)\hat{z} - G^T BOK_1^{equal}\hat{z}sign(\sigma(\hat{z})) \\
&\quad - G^T BOK_2^{equal}LCzsign(\sigma(\hat{z})) + G^T LCz\Big) + e^T(A - LC)e
\end{aligned}
$$

Let $OK_1^{equal} := \left(G_{SS}^T B\right)^{-1} G_{SS}^T (A - LC)$, and

$$OK_2^{equal} := \left(G_{SS}^T B\right)^{-1} G_{SS}^T L$$

$$
\begin{aligned}
\dot{V} &= \sigma^T\Big(G_{SS}^T(A - LC)\hat{z} - G_{SS}^T B\left(\left(G_{SS}^T B\right)^{-1} G_{SS}^T(A - LC)\right)\hat{z} \\
&\quad sign(\sigma(\hat{z})) \\
&\quad - G_{SS}^T B\left(\left(G_{SS}^T B\right)^{-1} G_{SS}^T L\right) LCzsign(\sigma(\hat{z})) + G_{SS}^T LCz\Big) \\
&\quad + e^T(A - LC)e \\
&= \sigma^T\Big(G_{SS}^T A\left(1 - sign(\sigma(\hat{z}))\right)\hat{z} - G_{SS}^T(LC)e \\
&\quad - \left(G_{SS}^T L\right)(1 - L)\big)Csign(\sigma(\hat{z}))e \quad + e^T(A - LC)e
\end{aligned}
$$

If $(A - LC)$ is stable, the estimation error is $e \to 0$ as $t \to 0$.

$$\dot{V} = \sigma^T G_{SS}^T A\left(1 - sign(\sigma(\hat{z}))\right)\hat{z} \quad \le 0$$

subject to if $\sigma > 0$, $\dot{V} = 0$

if $\sigma = 0$, $\dot{V} = 0$

if $\sigma < 0$, $\dot{V} \le -2kG_{SS}^T A\hat{z} < 0$,

k is positive constant. The above condition is satisfied on negative definite, and is an *asymptotically stable*. This completes the proof of this theorem. □

6. Conclusion

A stability proof for the nonlinear feedback linearization-full order observer-based sliding mode controller (NFL-FOO-based SMC) has been presented.

References

[1] V. I. Utkin, "Variable structure systems with sliding modes", IEEE Trans. on Automatic Control, AC-22, No.2, pp. 212-222, April, 1977.

[2] W. C. Chan and Y. Y. Hsu, "An optimal variable structure stabilizer for power system stabilization", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-102, pp. 1738-1746, Jun., 1983.

[3] J. J. Lee, "Optimal multidimensional variable structure controller for multi-interconnected power system", KIEE Trans., Vol. 38, No. 9, pp. 671-683, Sep., 1989.

[4] M. L. Kothari, J. Nanda and K. Bhattacharya, 'Design of variable structure power system stabilizers with desired eigenvalues in the sliding mode', IEE Proc. C, Vol. 140, No. 4, pp. 263-268, 1993.

[5] S. S. Lee, J. K. Park and J. J. Lee, "Sliding mode-MFAC power system stabilizer", Jour. of KIEE, Vol. 5, No. 1, pp. 1-7, Mar., 1992

[6] S. S. Lee and J. K. Park, "Sliding mode-model following power system stabilizer including closed-loop feedback", Jour. of KIEE, Vol. 9, No. 3, pp. 132-138, Sep., 1996

[7] S. S. Lee, J. K. Park et al., "Multimachine stabilizer using sliding mode-model following including closed-loop feedback", Jour. of KIEE, Vol. 9, No. 4, pp. 191-197, Dec., 1996.

[8] S. S. Lee and J. K. Park, "Sliding mode power system stabilizer based on LQR : Part I", Jour. of EEIS, Vol. 1, No. 3, pp. 32-38, 1996.

[9] S. S. Lee and J. K. Park, "Sliding mode observer power system stabilizer based on linear full-order observer : Part II", Jour. of EEIS, Vol. 1, No. 3, pp. 39-45, 1996.

[10] S. S. Lee and J. K. Park, "Full-order observer-based sliding mode power system stabilizer with desired eigenvalue-assignment for unmeasurable state variables", Jour. of EEIS, Vol. 2, No. 2, pp. 36-42, 1997.

[11] S. S. Lee and J. K. Park, "New sliding mode observer-model following power system stabilizer including CLF for unmeasurable state variables", Jour. of EEIS, Vol. 2, No. 3, pp. 88-94, 1997.

[12] S. S. Lee and J. K. Park, "Multimachine stabilizer using sliding mode observer-model following including CLF for unmeasurable state variables", Jour. of EEIS, Vol. 2, No. 4, pp. 53-58, 1997.

[13] S. S. Lee and J. K. Park, "H_∞ observer-based sliding mode power system stabilizer for unmeasurable state variables", Jour. of EEIS, Vol. 2, No. 1, pp. 70-76, 1997.

[14] S. S. Lee and J. K. Park, "Nonlinear feedback linearization-full order observer/sliding mode controller design for improving transient stability in a power system", Jour. of EEIS, Vol. 3, No. 2, pp. 184-192, 1998.

[15] S. S. Lee and J. K. Park, "Nonlinear feedback linearization-H_∞/sliding mode controller design for improving transient stability in a power system", Jour. of EEIS, Vol. 3, No. 2, pp. 193-201, 1998.

[16] S. S. Lee and J. K. Park, "Design of power system stabilizer using observer/sliding mode, observer/sliding mode-model following and H_∞/sliding mode controllers for small-signal stability study", Inter. Jour. of Electrical Power & Energy Systems, accepted, 1998.

[17] S. S. Lee and J. K. Park, "Design of reduced-order observer-based variable structure power system stabilizer for unmeasurable state variables, IEE PROC.-GEN., TRANS. AND DISTRIB., accepted, 1998.

[18] D. G. Luenberger, "Observing the state of a linear system", IEEE Trans. Mil. Electron, Vol. MIL-8, pp. 74-80, Apr, 1964.

[19] R. Marino and P. Tomei, "Nonlinear control design", Prentice-Hall Press, 1995.