NFL-ROO/SMC 의 안정도 증명 : Part 2

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Stability Proof of NFL-ROO/SMC : Part 2

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[Abstract] This paper presents the stability proof of a nonlinear feedback linearization-reduced order observer/sliding mode controller (NFL-ROO/SMC). The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

Keywords : nonlinear feedback linearization-reduced order observer/sliding mode controller, Lyapunov function, separation principle, stability proof

1. Introduction

By the separation principle, a nonlinear feedback linearization-reduced order observer/sliding mode controller (NFL-ROO/SMC) is developed [1-18]. The closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

2. NFL-ROO/SMC design

The state equation for full-state feedback and the output equation based on nonlinear feedback linearization (NFL) can be expressed as [19]

$$z(t) = T(x(t)) \tag{1}$$

$$\dot{z}(t) = Az(t) + Bu(t) \tag{2}$$

$$y(t) = Cz(t) \tag{3}$$

The transformation matrix is introduced by

$$q(t) = Tz(t) \tag{4}$$

$$\begin{bmatrix} y(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} C \\ T \end{bmatrix} z(t) \tag{5}$$

$$z(t) = \begin{bmatrix} C \\ T \end{bmatrix}^{-1} \begin{bmatrix} y(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} P & M \end{bmatrix} \begin{bmatrix} y(t) \\ q(t) \end{bmatrix} = Py(t) + Mq(t) \tag{6}$$

The estimate $\hat{z}$ of z is generated as

$$\hat{z}(t) = Py(t) + M\hat{q}(t) \tag{7}$$

A new realization for the system is expressed as

$$E\dot{z}(t) = EAz(t) + EBu(t) \tag{8}$$

Substituting for z and E, we get

$$E\dot{z}(t) = \begin{bmatrix} C \\ T \end{bmatrix} \dot{z}(t) = \begin{bmatrix} \dot{y}(t) \\ \dot{q}(t) \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} C \\ T \end{bmatrix} Az(t) + \begin{bmatrix} C \\ T \end{bmatrix} Bu(t) \tag{10}$$

$$= \begin{bmatrix} C \\ T \end{bmatrix} A \begin{bmatrix} P & M \end{bmatrix} \begin{bmatrix} y(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} C \\ T \end{bmatrix} Bu(t) \tag{11}$$

Therefore, we get

$$\begin{bmatrix} \dot{y}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} CAP & CAM \\ TAP & TAM \end{bmatrix} \begin{bmatrix} y(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} CB \\ TB \end{bmatrix} u(t) \tag{12}$$

$$:= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t) \tag{13}$$

$$\dot{y}(t) = A_{11}y(t) + A_{12}q(t) + B_1u(t) \tag{14}$$

$$\dot{q}(t) = A_{22}q(t) + A_{21}y(t) + B_2u(t) \tag{15}$$

The estimator equation for q is

$$\dot{\hat{q}}(t) = A_{22}\hat{q}(t) + A_{21}y(t) + B_2u(t) + L(y(t) - C\hat{z}(t)) \tag{16}$$

Let $CP = I$ and $CM = 0$

$$y(t) - C\hat{z} = y(t) - C(Py(t) + M\hat{q}) \tag{17}$$

$$= y(t) - CPy(t) - CM\hat{q} \tag{18}$$

$$= y(t) - y(t) - 0 = 0 \tag{19}$$

The reduced-order observer is represented by

$$\dot{\hat{q}}(t) = A_{22}\hat{q}(t) + A_{21}y(t) + B_2u(t) + L(\dot{y}(t) - A_{11}y(t) - B_1u(t) - A_{12}\hat{q}(t)) \tag{20}$$

Define w and $\dot{w}$, we get

$$w(t) := \hat{q}(t) - Ly(t) \tag{21}$$

$$\dot{w}(t) := \dot{\hat{q}}(t) - L\dot{y}(t) \tag{22}$$

The final form of the reduced-order observer is

$$\dot{w}(t) = (A_{22} - LA_{12})w(t) + [(A_{22} - LA_{12})L$$

$$+A_{21} - LA_{11}]y(t) + (B_2 - LB_1)u(t) \quad (23)$$

$$\hat{z}(t) = Mw(t) + (P + ML)y(t) \quad (24)$$

The differential form of the equation (24) is

$$\dot{\hat{z}}(t) = M\dot{w}(t) + (P + ML)\dot{y}(t) \quad (25)$$

The equations (23) and (24) are simplified as [1]

$$\dot{w}(t) = Fw(t) + Gy(t) + Hu(t) \quad (26)$$

$$\hat{z}(t) = Mw(t) + Ny(t) \quad (27)$$

where, $F := (A_{22} - LA_{12})$ (28)

$$G := [(A_{22} - LA_{12})L + A_{21} - LA_{11}] \quad (29)$$

$$H := (B_2 - LB_1) \quad (30)$$

$$N := (P + ML) \quad (31)$$

Theorem 1: Consider the state equations based on NFL for the regulation problem

$$\dot{z} = Az + B\hat{u}^{sign}_{ROO/SMC} \text{ and } y = Cz$$

Consider the reduced order observer state equation based on NFL

$$q = Tz \text{ and } \hat{q} = T\hat{z}$$

$$\dot{q} = A_{22}q + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC}$$

$$\dot{\hat{q}} = A_{22}\hat{q} + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC} + Ly - LC\hat{z}$$

The equal controller gain K^{equal}_{RSMC} and observer gain L *may be selected separately* for desired closed-loop behavior.

Proof. Let us consider the estimation error and differential estimation error

$$e = q - \hat{q}$$

$$\dot{e} = \dot{q} - \dot{\hat{q}} = A_{22}q + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC} - \big(A_{22}\hat{q} + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC} + L(\dot{y} - A_{11}y - B_1\hat{u}^{sign}_{ROO/SMC} - A_{12}\hat{q})\big)$$

$$= A_{22}q - A_{22}\hat{q} - LA_{12}q + LA_{12}\hat{q} = (A_{22} - LA_{12})(q - \hat{q})$$

$$= (A_{22} - LA_{12})e$$

$$\dot{z} = Az + B\hat{u}^{sign}_{ROO/SMC} = Az + B(-K^{equal}_{RSMC}\hat{z})$$

Let $\hat{z} = z - Me$

$$\dot{z} = Az - BK^{equal}_{RSMC}(z - Me) = (A - BK^{equal}_{RSMC})z - BK^{equal}_{RSMC}Me$$

The complete closed loop dynamics is

$$\begin{bmatrix}\dot{z}\\ \dot{e}\end{bmatrix} = \begin{bmatrix} A - BK^{equal}_{RSMC} & -BK^{equal}_{RSMC}M \\ 0 & A_{22} - LA_{12}\end{bmatrix}\begin{bmatrix} z \\ e\end{bmatrix}$$

The characteristic values of the system is

$$\Delta(s) = \det\begin{pmatrix} sI - A + BK^{equal}_{RSMC} & -BK^{equal}_{RSMC}M \\ 0 & sI - (A_{22} - LA_{12})\end{pmatrix}$$

$$= |sI - (A - BK^{equal}_{RSMC})| \cdot |sI - (A_{22} - LA_{12})|$$

Thus, the *separation principle* is satisfied.
This completes the proof of this theorem. □

The Lyapunov's function candidate is chosen by

$$q(t) = Tz(t)$$

$$V(q(t)) = \sigma^2(q(t))/2 \quad (32)$$

The time derivative of $V(q(t))$ can be expressed as

$$\dot{V}(q(t)) = \sigma(q(t))\dot{\sigma}(q(t)) = G^T_{SS}q(t)G^T_{SS}\dot{q}(t)$$

$$= G^T_{SS}q(t)G^T_{SS}[A_{22}q(t) + A_{21}y(t) + B_2u_{RSMC}(t)] \le 0 \quad (33)$$

The control inputs with switching function are

$$u^+_{RSMC}(t) \ge -(G^T_{SS}B_2)^{-1}G^T_{SS}(A_{22}q(t) + A_{21}y(t))$$

$$\text{for } G^T_{SS}q(t) > 0 \quad (34)$$

$$u^-_{RSMC}(t) \le -(G^T_{SS}B_2)^{-1}G^T_{SS}(A_{22}q(t) + A_{21}y(t))$$

$$\text{for } G^T_{SS}q(t) < 0 \quad (35)$$

The control input with sign function is formed as

$$u^{sign}_{RSMC}(t) = -(G^T_{SS}B_2)^{-1}G^T_{SS}(A_{22}q(t) + A_{21}y(t))sign(\sigma(q(t))) \quad (36)$$

The above control input can be simplified as

$$u^{sign}_{RSMC}(t) = -(K_{RSMC1}q(t) + K_{RSMC2}y(t))sign(\sigma(q(t))) \quad (37)$$

where, $K_{RSMC1} := (G^T_{SS}B_2)^{-1}G^T_{SS}A_{22}$ (38)

$$K_{RSMC2} := (G^T_{SS}B_2)^{-1}G^T_{SS}A_{21} \quad (39)$$

Finally, the *estimated control input vector* is

$$\hat{u}^{sign}_{ROO/SMC}(t) = -(K_{RSMC1}\hat{q}(t) + K_{RSMC2}y(t))sign(\sigma(\hat{q}(t))) \quad (40)$$

subject to $sign(\sigma(\hat{q}(t))) = 1$ for $\sigma(\hat{q}(t)) > 0$

$sign(\sigma(\hat{q}(t))) = 0$ for $\sigma(\hat{q}(t)) = 0$

$sign(\sigma(\hat{q}(t))) = -1$ for $\sigma(\hat{q}(t)) < 0$

Theorem 2: Consider the reduced order system

$$q = Tz \text{ and } \hat{q} = T\hat{z}$$

$$\dot{q} = A_{22}q + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC}$$

$$\dot{\hat{q}} = A_{22}\hat{q} + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC} + Ly - LC\hat{z}$$

The estimated sliding mode reduced order control law with sign function based on NFL is guaranteed an *asymptotically stable* for system (2)

$$\hat{u}^{sign}_{ROO/SMC} = -(K_{RSMC1}\hat{q} + K_{RSMC2}y)sign(\sigma(\hat{q}))$$

subject to $sign(\sigma(\hat{q})) = 1$ for $\sigma(\hat{q}) > 0$

$sign(\sigma(\hat{q})) = 0$ for $\sigma(\hat{q}) = 0$

$sign(\sigma(\hat{q})) = -1$ for $\sigma(\hat{q}) < 0$

Proof. Let us define the estimation error and the differential estimation error

$$e = q - \hat{q}$$

$$\dot{e} = \dot{q} - \dot{\hat{q}} = A_{22}q + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC} - \big(A_{22}\hat{q} + A_{21}y + B_2\hat{u}^{sign}_{ROO/SMC} + L(\dot{y} - A_{11}y - B_1\hat{u}^{sign}_{ROO/SMC} - A_{12}\hat{q})\big)$$

$$= A_{22}q - A_{22}\hat{q} - LA_{12}q + LA_{12}\hat{q} = (A_{22} - LA_{12})(q - \hat{q})$$

$$= (A_{22} - LA_{12})e$$

Lyapunov's function candidate is chosen by

$$V = \frac{1}{2}\sigma^T\sigma + \frac{1}{2}e^T e$$

The derivative is formed as

$$\dot{V} = \sigma^T\dot{\sigma} + e^T\dot{e} = \sigma^T\left(G_{SS}^T\dot{\hat{q}}\right) + e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T\left(A_{22}\hat{q} + A_{21}y + B_2\hat{u}_{ROO/SMC}^{sign} + L(y - C\hat{z})\right)$$

$$+e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y + \sigma^T G_{SS}^T B_2\hat{u}_{ROO/SMC}^{sign}$$

$$+\sigma^T G_{SS}^T Ly - \sigma^T G_{SS}^T LC\hat{z} + e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y$$

$$-\sigma^T G_{SS}^T B_2(K_{RSMC1}\hat{q} + K_{RSMC2}y)sign(\sigma(\hat{q}))$$

$$+\sigma^T G_{SS}^T Ly - \sigma^T G_{SS}^T LC\hat{z} + e^T(A_{22} - LA_{12})e$$

Let $K_{RSMC1} := (G_{SS}^T B_2)^{-1} G_{SS}^T A_{22}$ and $K_{RSMC2} := (G_{SS}^T B_2)^{-1} G_{SS}^T A_{21}$

$$\dot{V} = \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y$$

$$-\sigma^T G_{SS}^T B_2\left((G_{SS}^T B_2)^{-1} G_{SS}^T A_{22}\hat{q} + (G_{SS}^T B_2)^{-1} G_{SS}^T A_{21}y\right)sign(\sigma(\hat{q}))$$

$$+\sigma^T G_{SS}^T Ly - \sigma^T G_{SS}^T LC\hat{z} + e^T(A_{22} - LA_{12})e$$

Let $G_{SS}^T B_2(G_{SS}^T B_2)^{-1} = I$,

$$\dot{V} = \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y - \sigma^T G_{SS}^T A_{22}\hat{q}sign(\sigma(\hat{q}))$$

$$-G_{SS}^T A_{21}ysign(\sigma(\hat{q})) + \sigma^T G_{SS}^T Ly - \sigma^T G_{SS}^T LC\hat{z}$$

$$+e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y + \sigma^T G_{SS}^T Ly$$

$$-\sigma^T G_{SS}^T A_{22}\hat{q}sign(\sigma(\hat{q})) - \sigma^T G_{SS}^T A_{21}ysign(\sigma(\hat{q}))$$

$$-\sigma^T G_{SS}^T LC\hat{z} + e^T(A_{22} - LA_{12})e$$

Let $\hat{z} := Mw + Ny$, $N := P + ML$, $CP = I$, and $CM = 0$

$$\dot{V} = \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y + \sigma^T G_{SS}^T Ly$$

$$-\sigma^T G_{SS}^T A_{22}\hat{q}sign(\sigma(\hat{q})) - \sigma^T G_{SS}^T A_{21}ysign(\sigma(\hat{q}))$$

$$-\sigma^T G_{SS}^T LCMw - \sigma^T G_{SS}^T LCNy + e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y + \sigma^T G_{SS}^T Ly$$

$$-\sigma^T G_{SS}^T A_{22}\hat{q}sign(\sigma(\hat{q})) - \sigma^T G_{SS}^T A_{21}ysign(\sigma(\hat{q}))$$

$$-\sigma^T G_{SS}^T LC(P + ML)y + e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T A_{22}\hat{q} + \sigma^T G_{SS}^T A_{21}y + \sigma^T G_{SS}^T Ly$$

$$-\sigma^T G_{SS}^T A_{22}\hat{q}sign(\sigma(\hat{q})) - \sigma^T G_{SS}^T A_{21}ysign(\sigma(\hat{q}))$$

$$-\sigma^T G_{SS}^T Ly + e^T(A_{22} - LA_{12})e$$

$$= \sigma^T G_{SS}^T A_{22}\hat{q} - \sigma^T G_{SS}^T A_{22}\hat{q}sign(\sigma(\hat{q})) + \sigma^T G_{SS}^T A_{21}y$$

$$-\sigma^T G_{SS}^T A_{21}ysign(\sigma(\hat{q})) + e^T(A_{22} - LA_{12})e$$

If $(A_{22} - LA_{12})$ is stable, the error is $e \to 0$ as $t \to 0$.

$$\dot{V} = \sigma^T G_{SS}^T A_{22}\hat{q}\left(1 - sign(\sigma(\hat{q}))\right) + \sigma^T G_{SS}^T A_{21}y\left(1 - sign(\sigma(\hat{q}))\right) \le 0$$

subject to if $\sigma > 0$, $\dot{V} = 0$

if $\sigma = 0$, $\dot{V} = 0$

if $\sigma < 0$, $\dot{V} \le -2kG_{SS}^T A_{22}\hat{q} - 2kG_{SS}^T A_{21}y < 0$,

k is positive constant.

The above condition is satisfied on negative definite, and is *asymptotically stable.* This completes the proof of this theorem. □

3. Conclusion

A separation theorem and a stability proof for a nonlinear feedback linearization-reduced order observer/sliding mode controller (NFL-ROO/SMC) have been done.

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