NFL-FOO/SMC 의 안정도 증명 : Part 1

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Stability Proof of NFL-FOO/SMC : Part 1

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[Abstract] For a nonlinear feedback linearization-full order observer/sliding mode controller (NFL-FOO/SMC), the separation principle is derived, and the closed-loop stability is proved by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.
Keywords : nonlinear feedback linearization-full order observer/sliding mode controller, Lyapunov function, separation principle, stability proof

1. Introduction

To solve the problem associated with the full state feedback [1-17], and to cancel the nonlinearities for the nonlinear system, the nonlinear feedback linearization-observer/sliding mode controller (NFL-FOO/SMC) has been developed. By the separation principle, the proposed controller is obtained by combining the nonlinear feedback linearization-based sliding mode control (NFL-SMC) with the full order observer (FOO) [18]. In this paper, the separation principle is proved and the closed-loop stability is done by a Lyapunov function candidate using an addition form of the sliding surface vector and the estimation error.

2. NFL-FOO/SMC

Let us consider the general nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \tag{1}$$

$$y(t) = h(x(t)) \tag{2}$$

in which $f(x)$ and $g(x)$ are smooth vector fields, and $h(x)$ is a smooth function, defined on R^n.

The state equations based on nonlinear feedback linearization (NFL) [19] can be expressed as

$$z(t) = T(x(t)) \tag{3}$$

$$\dot{z}(t) = Az(t) + Bu(t) \tag{4}$$

$$y(t) = Cz(t) \tag{5}$$

where $x \in R^n$, $z \in R^n$, $u \in R^m$, $y \in R^p$, A is the system matrix, B is the control matrix, and C is the output matrix.

The observer based on NFL is to construct of the form [18]

$$\begin{aligned}\dot{\hat{z}}(t) &= A\hat{z}(t) + Bu(t) + L(y(t) - C\hat{z}(t)) \\ &= (A - LC)\hat{z}(t) + Bu(t) + Ly(t)\end{aligned} \tag{6}$$

$$L = PC^T R^{-1} \tag{7}$$

$$AP + PA^T - PC^T R^{-1} CP + Q = 0 \tag{8}$$

where $\hat{z} \in R^n$ is the estimated state, L is the $n \times m$ output injection matrix, P is the symmetric positive definite solution, and Q and R are positive definite matrices.

The estimated control input vector is defined as

$$u_{FL-O/LQR} = -K_{LQR}\hat{z}(t) \tag{9}$$

$$K_{LQR} = R^{-1} B^T P \tag{10}$$

$$PA + A^T P - PBR^{-1} B^T P + Q = 0 \tag{11}$$

where K_{LQR} is an optimal feedback gain.

The sliding surface vector and the differential sliding surface vector are expressed as

$$\sigma(z(t)) = G_{SS}^T z(t) = 0 \tag{12}$$

$$\dot{\sigma}(z(t)) = G_{SS}^T \dot{z}(t) = 0 \tag{13}$$

where G_{SS}^T is the sliding surface gain [1-4,10].

Substituting equation (4) into equation (13), we get

$$\begin{aligned}\dot{\sigma}(z(t)) &= G_{SS}^T \dot{z}(t) \\ &= G_{SS}^T (Az(t) + Bu_{SMC}^{equal}(t)) = 0\end{aligned} \tag{14}$$

An equal control input from equation (14) is obtained by

$$u_{SMC}^{equal}(t) = -\left(G_{SS}^{T}B\right)^{-1}\left(G_{SS}^{T}A\right)z(t) \quad \text{if} \quad \left(G_{SS}^{T}B\right)^{-1} \neq 0 \tag{15}$$

$$= -K_{SMC}^{equal}z(t) \tag{16}$$

The estimated equal sliding mode control input is

$$\hat{u}_{O/SMC}^{equal}(t) = -K_{SMC}^{equal}\hat{z}(t) \tag{17}$$

Theorem 1: Consider the state equations based on NFL for the regulation problem, and the observer problem based on NFL

$$\dot{z} = Az + B\hat{u}_{O/SMC}^{sign}$$

$$y = Cz$$

$$\dot{\hat{z}} = A\hat{z} + B\hat{u}_{O/SMC}^{sign} + L(y - C\hat{z})$$

Suppose that (A,C) is detectable and $(A-LC)$ is Hurwitz. The equal controller gain K_{SMC}^{equal} and observer gain L *may be selected separately* for desired closed-loop behavior.

Proof. To obtain the closed loop system from equations (4) and (17), we get

$$\dot{z} = Az + B\hat{u}_{O/SMC}^{equal}$$

$$= Az - BK_{SMC}^{equal}\hat{z} \tag{18}$$

Or, from equations (6) and (17), we get

$$\dot{\hat{z}} = A\hat{z} + B\hat{u}_{O/SMC}^{equal} + L(y - C\hat{z})$$

$$= (A - LC)\hat{z} + B\hat{u}_{O/SMC}^{equal} + Ly$$

$$= LCz + \left(A - LC - BK_{SMC}^{equal}\right)\hat{z} \tag{19}$$

From equations (18) and (19), the closed loop system can be described as

$$\begin{bmatrix}\dot{z}\\ \dot{\hat{z}}\end{bmatrix} = \begin{bmatrix}A & -BK_{SMC}^{equal}\\ LC & A - LC - BK_{SMC}^{equal}\end{bmatrix}\begin{bmatrix}z\\ \hat{z}\end{bmatrix} \tag{20}$$

$$[y] = \begin{bmatrix}C & 0\end{bmatrix}\begin{bmatrix}z\\ \hat{z}\end{bmatrix} \tag{21}$$

To show the separation property of the closed-loop system, the estimation error equation based on NFL are considered as

$$e = z - \hat{z} \tag{22}$$

$$\dot{e} = \dot{z} - \dot{\hat{z}} \tag{23}$$

$$= Az + B\hat{u}_{O/SMC}^{equal} - \left[A\hat{z} + B\hat{u}_{O/SMC}^{equal} + L(y - C\hat{z})\right]$$

$$= Az - (A - LC)\hat{z} - Ly$$

Let $\hat{z} = e + z$

$$\dot{e} = Az - (A - LC)(e + z) - Ly$$

$$= (A - LC)e \tag{25}$$

If the eigenvalues of $(A - LC)$ can be chosen arbitrarily, then the behavior of the error e can be controlled. Or, from equation (4), (17) and (22), we get

$$\dot{z} = Az + B\hat{u}_{O/SMC}^{equal}$$

$$= Az - BK_{SMC}^{equal}(e + z) \qquad \because \hat{z} = e + z$$

$$= \left(A - BK_{SMC}^{equal}\right)z - BK_{SMC}^{equal}e \tag{26}$$

From equations (25) and (26), the complete closed loop dynamics with a sliding mode; namely, those of the plant and those of the error is

$$\begin{bmatrix}\dot{z}\\ \dot{e}\end{bmatrix} = \begin{bmatrix}A - BK_{SMC}^{equal} & -BK_{SMC}^{equal}\\ 0 & A - LC\end{bmatrix}\begin{bmatrix}z\\ e\end{bmatrix} \tag{27}$$

The characteristic values is

$$\Delta(s) = \det\begin{pmatrix}sI - A + BK_{SMC}^{equal} & -BK_{SMC}^{equal}\\ 0 & sI - A + LC\end{pmatrix} \tag{28}$$

$$= \left|sI - \left(A - BK_{SMC}^{equal}\right)\right| \cdot \left|sI - (A - LC)\right| \tag{29}$$

Thus, the *separation principle* in equation (29) is satisfied.

This completes the proof of this theorem. □

The Lyapunov's function candidate is chosen by

$$V(z(t)) = \sigma^2(z(t))/2 \tag{30}$$

The time derivative of $V(z(t))$ can be expressed as

$$\dot{V}(z(t)) = \sigma(z(t))\dot{\sigma}(z(t))$$

$$= G_{SS}^{T}z(t)G_{SS}^{T}\dot{z}(t)$$

$$= G_{SS}^{T}z(t)G_{SS}^{T}\left[Az(t) + Bu_{SMC}(t)\right] \leq 0 \tag{31}$$

The control inputs with switching function are

$$u_{SMC}^{+}(t) \geq -\left(G_{SS}^{T}B\right)^{-1}\left(G_{SS}^{T}A\right)z(t) \quad \textit{for} \quad G_{SS}^{T}z(t) > 0 \tag{32}$$

$$u_{SMC}^{-}(t) \leq -\left(G_{SS}^{T}B\right)^{-1}\left(G_{SS}^{T}A\right)z(t) \quad \textit{for} \quad G_{SS}^{T}z(t) < 0 \tag{33}$$

The control input with sign function is

$$u_{SMC}^{sign}(t) = -\left(G_{SS}^{T}B\right)^{-1}\left[G_{SS}^{T}A\right]z(t)sign(\sigma(z(t))) \tag{34}$$

subject to $sign(\sigma(z(t))) = 1$ for $\sigma(z(t)) > 0$

$sign(\sigma(z(t))) = 0$ for $\sigma(z(t)) = 0$

$sign(\sigma(z(t))) = -1$ for $\sigma(z(t)) < 0$

The control input $u_{SMC}^{sign}(t)$ can be simplified as

$$u_{SMC}^{sign}(t) = -K_{SMC}z(t)sign(\sigma(z(t))) \tag{35}$$

$$K_{SMC} := \left(G_{SS}^{T}B\right)^{-1}\left[G_{SS}^{T}A\right] \tag{36}$$

where K_{SMC} is sliding mode controller gain.

Finally, the *estimated control input vector* is

$$\hat{u}_{O/SMC}^{sign}(t) = -K_{SMC}\hat{z}(t)sign(\sigma(\hat{z}(t))) \tag{37}$$

subject to $sign(\sigma(\hat{z}(t))) = 1$ for $\sigma(\hat{z}(t)) > 0$

$sign(\sigma(\hat{z}(t))) = 0$ for $\sigma(\hat{z}(t)) = 0$

$sign(\sigma(\hat{z}(t))) = -1$ for $\sigma(\hat{z}(t)) < 0$

Theorem 2: Consider the state equation and the observer equation based on NFL for the regulation problem

$$\dot{z} = Az + B\hat{u}_{O/SMC}^{sign}$$

$$y = Cz$$

$$\dot{\hat{z}} = A\hat{z} + B\hat{u}^{sign}_{O/SMC} + L(y - C\hat{z})$$

Suppose that (A,C) is detectable and $(A-LC)$ is Hurwitz. The estimated sliding mode control law with sign function based on NFL that keeps the system stable is guaranteed an *asymptotically stable* for the system (2)

$$\hat{u}^{sign}_{O/SMC} = -K_{SMC}\hat{z}sign(\sigma(\hat{z}))$$

subject to $sign(\sigma(\hat{z})) = 1$ for $\sigma(\hat{z}) > 0$

$sign(\sigma(\hat{z})) = 0$ for $\sigma(\hat{z}) = 0$

$sign(\sigma(\hat{z})) = -1$ for $\sigma(\hat{z}) < 0$

Proof. Let us define the estimation error equation and differential estimation error equation

$$\begin{aligned} e &= z - \hat{z} \\ \dot{e} &= \dot{z} - \dot{\hat{z}} \\ &= Az + B\hat{u}^{sign}_{O/SMC} - A\hat{z} - B\hat{u}^{sign}_{O/SMC} - LCz + LC\hat{z} \\ &= (A - LC)e \end{aligned}$$

Lyapunov's function candidate using the addition form of the sliding surface and the estimation error is chosen by

$$V = \frac{1}{2}\sigma^T\sigma + \frac{1}{2}e^T e$$

The derivative of a Lyapunov's function candidate is obtained by

$$\begin{aligned} \dot{V} &= \sigma^T\dot{\sigma} + e^T\dot{e} \\ &= \sigma^T\left(G^T\dot{\hat{z}}\right) + e^T(A - LC)e \\ &= \sigma^T\left(G^T\left(A\hat{z} + B\hat{u}^{sign}_{O/SMC} + L(y - C\hat{z})\right)\right) + e^T(A - LC)e \\ &= \sigma^T\left(G^T\left(A\hat{z} + B\left(-K^{equal}_{SMC}\hat{z}sign(\sigma(\hat{z}))\right) + LC(z - \hat{z})\right)\right) \\ &\quad + e^T(A - LC)e \\ &= \sigma^T\left(\left(G^T A - G^T BK^{equal}_{SMC} sign(\sigma(\hat{z}))\right)\hat{z} + G^T LCe\right) \\ &\quad + e^T(A - LC)e \end{aligned}$$

Let $K^{equal}_{SMC} := \left(G^T_{SS}B\right)^{-1} G^T_{SS}A$,

$$\begin{aligned} \dot{V} &= \sigma^T\left(\left(G^T_{SS}A - G^T_{SS}B\left(\left(G^T_{SS}B\right)^{-1} G^T_{SS}A\right)sign(\sigma(\hat{z}))\right)\hat{z}\right. \\ &\quad \left. + G^T_{SS}LCe\right) + e^T(A - LC)e \\ &= \sigma^T\left(G^T_{SS}\left(A\left(1 - sign(\sigma(\hat{z}))\right)\hat{z} + LCe\right)\right) + e^T(A - LC)e \end{aligned}$$

If $(A - LC)$ is stable, the estimation error is $e \to 0$ as $t \to 0$. $\dot{V} = \sigma^T G^T_{SS} A\left(1 - sign(\sigma(\hat{z}))\right)\hat{z} \le 0$

subject to if $\sigma > 0$, $\dot{V} = 0$

if $\sigma = 0$, $\dot{V} = 0$

if $\sigma < 0$, $\dot{V} \le -2kG^T_{SS}A\hat{z} < 0$,

k is positive constant.

The above condition is satisfied on negative definite, and is *asymptotically stable*.

This completes the proof of this theorem. □

3. Conclusion

A separation theorem and a stability proof of a nonlinear feedback linearization-full order observer/sliding mode power system stabilizer (FOO/SMC) for unmeasurable state variables have been presented in this paper.

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