

# 비선형 PSS 을 위한 NFL-FOO/SMC 의 설계 : Part A

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## NFL-FOO/SMC Design for Nonlinear PSS : Part A

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**[Abstract]** In this paper, the proposed controller is obtained by combining the nonlinear feedback linearization-sliding mode controller (NFL-SMC) with the full order observer (FOO) and eliminates the need to measure all the state variables in the conventional NFL-SMC.

**Keywords :** nonlinear feedback linearization-full order observer/sliding mode controller, power system stabilizer

### 1. Introduction

To solve the problem associated with the unmeasurable state variables in the conventional SMC [1-17] and to obtain the smooth control as the linearized controller for a linear system (or to cancel the nonlinearity for the nonlinear system), the nonlinear feedback linearization-full order observer/sliding mode control (NFL-FOO/SMC) is proposed in this paper. The proposed controller is obtained by combining the nonlinear feedback linearization-based sliding mode control (NFL-SMC) with the full order observer (FOO) [18] and eliminates the need to measure all the state variables in conventional SMC. The effectiveness of the proposed controller is verified by the nonlinear time-domain simulations in case of 3-cycle line-ground fault and in case of parameter variations (20% over-estimations) for AVR gain  $K_A$  and for inertia moment  $M$ .

### 2. Nonlinear power system model

The d-axis current and the q-axis current are [19]

$$i_d(t) = c_1 e_q'(t) - c_2 (R_2 \sin \delta(t) + X_1 \cos \delta(t)) \quad (1)$$

$$i_q(t) = c_3 e_q'(t) - c_4 (-X_1 \sin \delta(t) + R_1 \cos \delta(t)) \quad (2)$$

$$c_1 = \frac{(C_1 X_1 - C_2 R_2)}{(R_1 R_2 + X_1 X_2)}, \quad c_2 = \frac{V_{inf}}{(R_1 R_2 + X_1 X_2)}$$

$$c_3 = \frac{(C_1 R_1 + C_2 X_2)}{(R_1 R_2 + X_1 X_2)}, \quad c_4 = \frac{V_{inf}}{(R_1 R_2 + X_1 X_2)}$$

$$Z_1 = R_1 + jX_1,$$

$$Z_2 = R_2 + jX_2$$

$$Y = G + jB,$$

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$1 + Z_1 Y = C_1 + jC_2,$$

$$C_1 := 1 + RG - XB$$

$$C_2 := XG + RB,$$

$$R_1 := R - C_2 x_d'$$

$$R_2 := R - C_2 x_q,$$

$$X_1 := X + C_1 x_q$$

$$X_2 := X + C_1 x_d'$$

$$v_d(t) = x_q i_q(t) \quad (3)$$

$$v_q(t) = e_q'(t) - x_d' i_d(t) \quad (4)$$

$$v_d^2(t) = v_d^2(t) + v_q^2(t) \quad (5)$$

$$T_e(t) \approx P_e(t) = i_d(t) v_d(t) + i_q(t) v_q(t) \\ = e_q'(t) i_q(t) + (x_q - x_d') i_d(t) i_q(t) \quad (6)$$

where  $i_d(t)$  is the d-axis current,  $i_q(t)$  is the q-axis current,  $P_e(t)$  is the electrical power,  $e_q'(t)$  is the q-axis transient voltage,  $\delta(t)$  is the torque angle,  $V_{inf}$  is the infinite bus voltage,  $x_d$  is the d-axis reactance,  $x_q$  is the q-axis reactance, and  $x_d'$  is the d-axis transient reactance.

The nonlinear 4-th order state equations are [19]

$$\dot{\omega}(t) = \frac{1}{M} T_m - \frac{1}{M} T_e(t) \quad (7)$$

$$\dot{\delta}(t) = \omega_o(\omega(t) - 1) \quad (8)$$

$$\dot{e}_q'(t) = -\frac{1}{T_{do}'} e_q'(t) - \frac{(x_d - x_d')}{T_{do}'} i_d(t) + \frac{1}{T_{do}'} e_{fd}(t) \quad (9)$$

$$\dot{e}_{fd}(t) = -\frac{1}{T_A} e_{fd}(t) + \frac{K_A}{T_A} (V_{ref} - v_T(t) + u_E(t)) \quad (10)$$

$$e_{fd \min} \leq e_{fd} \leq e_{fd \max} \quad \text{and} \quad u_{E \min} \leq u_E \leq u_{E \max} \quad (11)$$

$e_{fd \max} = 6.0$ ,  $e_{fd \min} = -6.0$ , and  $u_{E \max} = +0.2$ ,  $u_{E \min} = -0.2$  where  $\omega(t)$  is the angular velocity,  $e_{fd}(t)$  is the exciter output voltage,  $T_m$  is the mechanical torque,  $T_e$  is the electrical torque,  $T_A$  is the voltage regulator time constant,  $K_A$  is the voltage regulator gain,  $T'_{do}$  is the d-axis transient open circuit time constant,  $M$  is the inertia coefficient,  $\omega_s$  is the synchronous angular velocity,  $V_{ref}$  is the regulator reference voltage,  $v_r$  is the generator terminal voltage, and  $u_E$  is the supplementary excitation control input.

### 3. NFL-FOO/SMC design

The full-state feedback equation and the output equation based on NFL are

$$z(t) = T(x(t)) \quad (12)$$

$$\dot{z}(t) = Az(t) + Bu(t) \quad (13)$$

$$y(t) = Cz(t) \quad (14)$$

The observer state equation based on NFL is [18]

$$\begin{aligned} \dot{\hat{z}}(t) &= A\hat{z}(t) + Bu(t) + L(y(t) - C\hat{z}(t)) \\ &= (A - LC)\hat{z}(t) + Bu(t) + Ly(t) \end{aligned} \quad (15)$$

$$L = PC^T R^{-1} \quad (16)$$

$$AP + PA^T - PC^T R^{-1} CP + Q = 0 \quad (17)$$

where  $Q$  and  $R$  are positive definite matrices.

The estimated control input vector is

$$u_{NFL-FOO/LQR} = -K_{LQR} \hat{z}(t) \quad (18)$$

$$K_{LQR} = R^{-1} B^T P \quad (19)$$

$$PA + A^T P - PBR^{-1} B^T P + Q = 0 \quad (20)$$

The closed loop system can be expressed

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & -BK_{LQR} \\ LC & A - BK_{LQR} - LC \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} \quad (21)$$

$$[y(t)] = [C \ 0] \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} \quad (22)$$

The sliding surface vector and the differential sliding surface vector can be expressed as

$$\sigma(z(t)) = G^T z(t) \quad (23)$$

$$\dot{\sigma}(z(t)) = G^T \dot{z}(t) \quad (24)$$

where  $G^T$  is the sliding surface gain [1-4,10].

The Lyapunov's function is chosen by

$$V(z(t)) = \sigma^2(z(t)) / 2 \quad (25)$$

The time derivative of  $V(z(t))$  can be expressed as

$$\dot{V}(z(t)) = \sigma(z(t)) \dot{\sigma}(z(t)) \quad (26)$$

$$\begin{aligned} &= G^T z(t) G^T \dot{z}(t) \\ &= G^T z(t) G^T [Az(t) + Bu_{NFL-SMC}(t)] \leq 0 \end{aligned} \quad (27)$$

The control inputs with switching function are

$$u_{NFL-SMC}^+(t) \geq -(G^T B)^{-1} (G^T A) z(t) \quad \text{for } G^T z(t) > 0 \quad (28)$$

$$u_{NFL-SMC}^-(t) \leq -(G^T B)^{-1} (G^T A) z(t) \quad \text{for } G^T z(t) < 0 \quad (29)$$

The control input with sign function is formed as

$$u_{NFL-SMC}^{sig}(t) = -(G^T B)^{-1} [G^T A] z(t) \text{ sign}(\sigma(z(t))) \quad (30)$$

The above equation (30) can be reformed as

$$u_{NFL-SMC}^{sig}(t) = -K_{SMC} z(t) \text{ sign}(\sigma(z(t))) \quad (31)$$

$$K_{SMC} := (G^T B)^{-1} [G^T A] \quad (32)$$

Finally, the estimated control input vector is

$$u_{NFL-FOO/SMC}^{sig}(t) = -K_{SMC} \hat{z}(t) \text{ sign}(\sigma(\hat{z}(t))) \quad (33)$$

The closed loop system can be expressed as

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\hat{z}}(t) \end{bmatrix} = \begin{bmatrix} A & -BK_{SMC} \text{ sign}(\sigma(\hat{z}(t))) \\ LC & A - LC - BK_{SMC} \text{ sign}(\sigma(\hat{z}(t))) \end{bmatrix} \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} \quad (34)$$

$$[y(t)] = [C \ 0] \begin{bmatrix} z(t) \\ \hat{z}(t) \end{bmatrix} \quad (35)$$

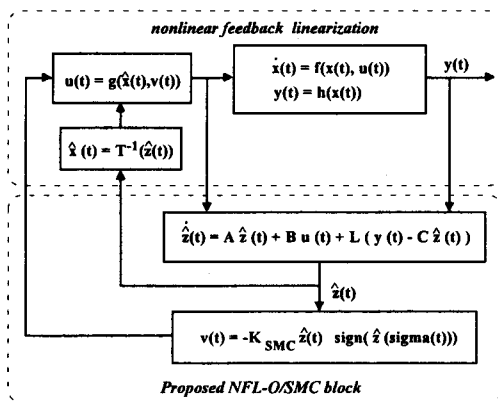


Fig. 1. Proposed NFL-FOO/SMC.

### 4. Simulation

The proposed NFL-FOO/SMC-PSS in Fig. 2 exhibits better damping properties.

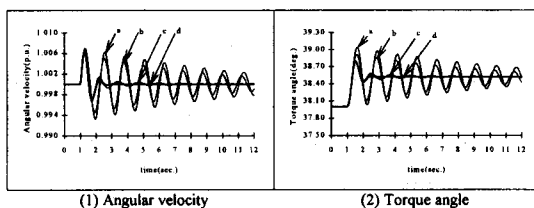


Fig. 2. Normal load operation. (a: no control b: conventional PSS c: NFL-FOO/LQR-PSS d: proposed NFL-FOO/SMC-PSS)

#### 2. Parameter variation test

The proposed NFL-FOO/SMC-PSS in Fig. 3 (2)

and in Fig. 4 (2) exhibits better damping properties and is less sensitive to variations of AVR gain as compared to the NFL-FOO/LQR-PSS in Fig. 3 (1) and in Fig. 4 (1).

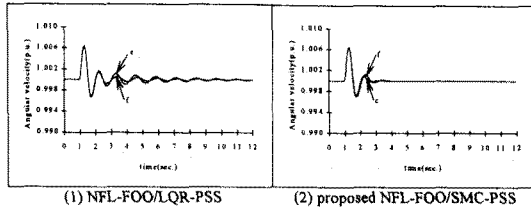


Fig. 3. Angular velocity waveforms for parameter variation of AVR gain. (e : normal f : parameter variation)

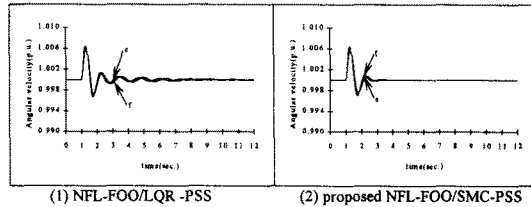


Fig. 4. Angular velocity waveforms under parameter variation of inertia moment. (e : normal f : parameter variation)

## 5. Conclusion

The effectiveness of the proposed controller has been verified by the nonlinear time-domain simulations in case of 3-cycle line-ground fault and in case of parameter variations for AVR gain  $K_A$  and for inertia moment  $M$ .

## Appendix

### A.1 Preliminary for NFLC [20]

Let us consider the general nonlinear system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \quad (A.1)$$

$$y(t) = h(x(t)) \quad (A.2)$$

The linearizing diffeomorphism is given by

$$z(t) = T(x(t)) = [h \quad L_f h \quad L_f^2 h \quad L_f^3 h \dots]^T \\ := [z_1(t) \quad z_2(t) \quad z_3(t) \quad z_4(t) \dots]^T \quad (A.3)$$

The state space forms based on NFL are

$$\frac{dz(t)}{dt} = Az(t) + Bu(t) \quad (A.4)$$

$$y(t) = Cz(t) \quad (A.5)$$

The derivatives of the output are

$$y(t) = L_f^k h(x(t)), \quad \frac{dy(t)}{dt} = L_f h(x(t)) + L_x h(x(t))u(t)$$

.....

$$\frac{d^r y(t)}{dt^r} = L_f^r h(x(t)) + L_x L_f^{r-1} h(x(t))u(t) \quad (A.6)$$

Remark : The above equations (A.1) and (A.2) are said to have relative degree  $r$  at a point  $x^o$  if (i)  $L_x L_f^k h(x) = 0$  for all  $x$  in a neighborhood of  $x^o$  and all  $k < r-1$  and (ii)  $L_x L_f^{r-1} h(x^o) \neq 0$ .

The control input vector based on NFL is

$$u(t) = g(x(t), v(t)) = -\frac{L_f^r h}{L_x L_f^{r-1} h} + \frac{1}{L_x L_f^{r-1} h} v(t) \quad (A.7)$$

where  $v(t) = \frac{d^r y(t)}{dt^r}$  has a linear relation.

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