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INTERACTIONS OF A HORIZONTAL FLEXIBLE MEMBRANE WITH OBLIQUE INCIDENT WAVES

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ABSTRACT

The interaction of oblique monochromatic incident waves with a horizontal flexible membrane is investigated in the context of two-dimensional linear hydro-elastic theory. First, analytic diffraction and radiation solutions for a submerged impermeable horizontal membrane are obtained. Second, the theoretical prediction was compared with a series of experiments conducted in a two-dimensional wave tank at Texas A&M University. The measured reflection and transmission coefficients reasonably follow the trend of predicted values. Using the developed computer program, the performance of surface-mounted or submerged horizontal membrane wave barriers is tested with various system parameters and wave characteristics. It is found that the properly designed horizontal flexible membrane can be an effective wave barrier and its efficiency can be further improved using a porous material.

1. MATHEMATICAL FORMULATION AND ANALYTIC SOLUTIONS

We consider the interaction of a horizontal membrane wave barrier with monochromatic oblique incident waves. Cartesian axes are chosen with the x-axis along the mean free surface and y-axis pointing vertically upwards. The water depth is denoted by h and the submergence depth of the membrane by d. It is assumed that both ends of the membrane are fixed at $x = \pm a$, and a uniform tension T is applied on the membrane in the x direction (see Fig.1). It is also assumed that the fluid is incompressible and inviscid, and the wave and membrane motions are small so that linear potential theory can be used. The fluid particle velocity can then be described by the gradient of a velocity potential $\Phi(x, y, z, t)$. Assuming harmonic motion of frequency ω, the velocity potential can be written as $\Phi(x,y,z,t) = \Re[\phi(x,y)e^{ik_z z - i\omega t}]$, where $k_z = k_1 \sin\theta$ is the z-component wave number and θ is the heading of incident waves with respect to the x axis. Similarly, the vertical displacement of membrane can be written as

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$$\zeta(x,z,t) = \Re[\xi(x)e^{ik_z z - i\omega t}], \tag{1}$$

where $\xi(x)$ is the complex displacement of membrane.

The velocity potential ϕ satisfies the Helmholz equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - k_1^2 \sin^2 \theta\phi = 0 \ \textit{in the fluid}, \tag{2}$$

with the following boundary conditions

$$\frac{\partial \phi}{\partial y} - \nu\phi = 0 \ \ on \ \ y = 0. \ \ (\nu = \frac{\omega^2}{g}) \tag{3}$$

$$\frac{\partial \phi}{\partial y} = 0 \ on \ \ y = -h. \tag{4}$$

$$\lim_{|x| \to \infty} (\frac{\partial \phi}{\partial x} \pm ik_1 \cos \theta\phi) = 0. \tag{5}$$

$$\frac{\partial \phi}{\partial y} = -i\omega\xi \ \ \ on \ \ y = -d, -a \le x \le a. \tag{6}$$

The complex displacement of membrane can be expanded in terms of a set of natural modes of the membrane:

$$\xi(x) = \sum_{l=1}^{\infty} \varsigma_l f_l(x), \tag{7}$$

where ς_l is the unknown complex modal amplitude corresponding to the l th mode. The modal functions and eigenvalues of the membrane satisfying the membrane equation and the end condition are given by

$$f_l(x) = \begin{cases} f_l^S(x) = \cos\dfrac{\lambda_l^S x}{a}, \ \ \lambda_l^S = \dfrac{[2(l-1)+1]\pi}{2} \ \ (l = 1,2,3,\ldots), \\ f_l^A(x) = \sin\dfrac{\lambda_l^A x}{a}, \ \lambda_l^A = l\pi \ \ (l = 1,2,3,\ldots), \end{cases} \tag{8}$$

where the superscripts S and A denote symmetric and asymmetric modes about $x = 0$, respectively. The modal functions given in equation (8) are orthogonal to each other in the interval $[-a, a]$:

$$\int_{-a}^{a} f_i(x) f_j(x) \ dx = \begin{cases} a & i = j \\ 0 & i \neq j. \end{cases} \tag{9}$$

Including all the flexible membrane modes, the complex potential $\phi(x, y)$ can be expressed in the form

$$\phi(x,y)=\phi_D(x,y)+\sum_{l=1}^{\infty}\varsigma_l\phi_{lR}(x,y),$$
$$\phi_D(x,y)=\phi_I(x,y)+\phi_S(x,y), \tag{10}$$
where ϕ_I, ϕ_D are the incident and diffraction potential and ϕ_S, ϕ_{lR} denote the scattering and radiation potential, respectively.

Neglecting viscous (or material) damping, the motion of membrane is governed by the inhomogeneous one-dimensional wave equation as follows:

$$T\frac{d^2\xi}{dx^2}+m\omega^2\xi=-i\rho\omega[\phi^{(3)}(x,-d)-\phi^{(2)}(x,-d)], \tag{11}$$

where T, ρ, and m are the membrane tension, fluid density, and membrane mass per unit length, respectively. Region (I) is defined by $x\le -a, -h<y<0$, region (II) by $|x|\le a, -d<y<0$ and region (III) by $|x|\le a, -h<y<-d$. Substituting $\phi(x,y)=\phi_D(x,y)+\sum_{j=1}^{\infty}\varsigma_j\phi_{jR}(x,y)$, $\xi(x)=\sum_{j=1}^{\infty}\varsigma_j f_j(x)$ into (11) yields

$$\sum_{j=1}^{\infty}\varsigma_j\{-T\frac{d^2 f_j(x)}{dx^2}-m\omega^2 f_j(x)-p_{jR}(x)\}=p_D(x), \tag{12}$$

where

$$p_{jR}(x)=i\rho\omega[\phi_{jR}^{(3)}(x,-d)-\phi_{jR}^{(2)}(x,-d)],$$
$$p_D(x)=i\rho\omega[\phi_D^{(3)}(x,-d)-\phi_D^{(2)}(x,-d)].$$

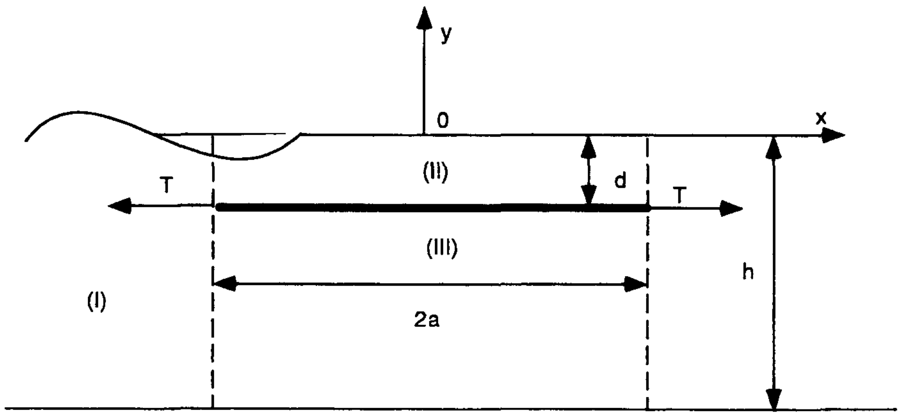

Fig.1 Definition sketch for horizontal impermeable flexible membrane

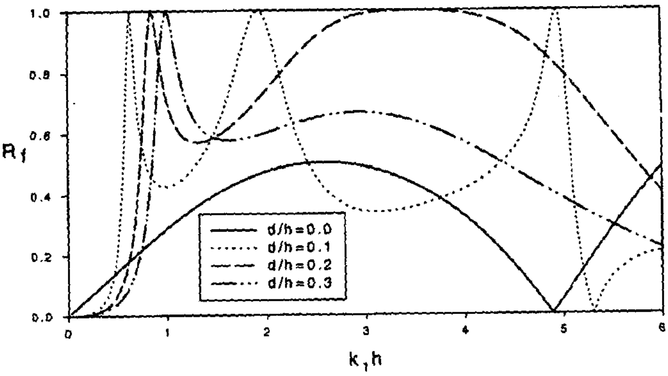

Fig.2 Reflection coefficients of a submerged impermeable membrane breakwater as function of submergence depth d/h and wavenumber k_1h for $a/h = 0.5, T/\rho g h^2 = 0.1$, and $\theta = 0^o$

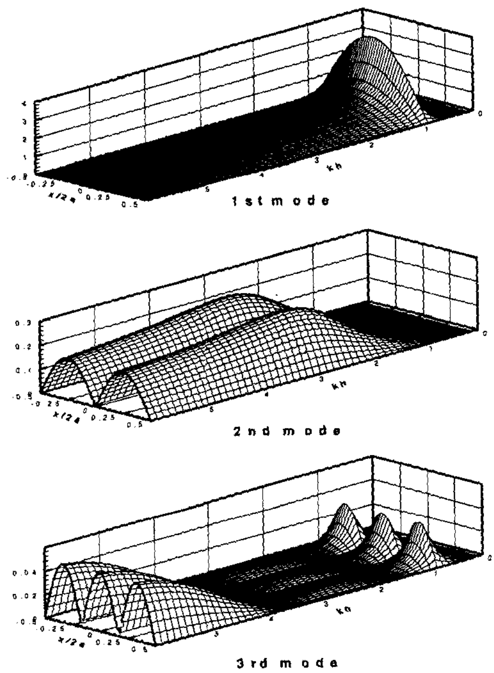

Fig.3 Modal response amplitude as function of wavenumber k_1h and horizontal coordinate $x/2a$ for $d/h = 0.2, a/h = 0.5, T/\rho g h^2 = 0.1$, and $\theta = 0^o$