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Wave Reflection over an Arbitrarily Varying Topography

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1. Introduction

As wind waves generated in deep water approach nearshore zone, they experience various physical phenomena caused by bathymetric variations, nonlinear interactions among different wave components and interferences with man-made coastal structures. Among these, the bathymetric variations may play a significant role in the change of wave climate. The accurate calculation of reflection and transmission coefficients of incident waves over a bottom topography is indispensible for the proper and economical design of coastal structures.

The propagation of monochromatic surface waves over a submerged object or sand bars has been frequently and widely investigated through numerical and experimental studies because of its practical applications in the coastal morphology and submerged breakwaters. Miles (1967) used the variational principle to calculate the reflection and transmission coefficients for the case of a step discontinuity between two finite depths and Devillard *et al.* (1988) used Miles' approach for the case of the successive steps. Kirby and Dalrymple (1983) calculated the reflection and transmission coefficients of monochromatic waves over a submerged trench using an eigenfunction expansion method. Davies and Heathershaw (1984) conducted experiments on the propagation of monochromatic waves over a sinusoidally varying topography to study the resonant Bragg reflection. Recently, Rey *et al.* (1992) performed a series of experiments for the propagation of linear and weakly nonlinear waves over a rectangular submerged bar. They demonstrated the importance of the evanescent modes by examining the experimental behavior of the fundamental wave amplitude over the submerged bed. More recently, the models developed by including higher-order bottom effects in the refraction-diffraction model are known to predict accurately the transformation of waves over rapidly varying topography such as steep slope or ripple beds (Massel, 1993; Suh *et al.*, 1997; Lee and Park, 1997).

In this study, the reflection of monochromatic waves over an arbitrarily varying topography is studied using the eigenfuncion expansion method. In the following section, a brief description of

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the eigenfunction expansion method is given. Then, the developed theoretical model is tested with several examples with different bottom topographies. Finally, concluding remarks are presented.

2. Eigenfunction Expansion Method

The bottom topography is divided into a finite number of small steps. The variation of the bottom topography is limited to the x-direction and thus, the wave number in the y-direction remains constant throughout the study. The velocity potentials satisfying the Laplace equation and the boundary conditions in each step can be written as:

$$\Phi_m^{\pm} = \left[A_m^{\pm} e^{\pm i l_m x} \cosh k_m(h_m + z) + \sum_{n=1}^{\infty} B_{m,n}^{\pm} e^{\pm \lambda_{m,n} x} \cos K_{m,n}(h_m + z) \right] e^{i(k_y y - \omega t)} \tag{1}$$

where $A_m^{\pm}$, $B_{m,n}^{\pm}$ are complex-valued amplitudes to be determined and the subscripts m and n represent the spatial region and evanescent mode to be considered, respectively. In equation (1), l_m and $\lambda_{m,n}$ are the x-directional wavenumbers of propagating and evanescent modes, respectively, which are determined from the relations given as:

$$l_m^2 = k_m^2 - k_y^2, \qquad \lambda_{m,n}^2 = K_{m,n}^2 + k_y^2 \tag{2}$$

in which k_y is the y-directional wavenumber. The wave numbers k_m and $K_{m,n}$ at the m-th region are determined from the dispersion relations given as:

$$\omega^2 = g k_m \tanh k_m h_m = -g K_{m,n} \tan K_{m,n} h_m \tag{3}$$

At the discontinuities $x = x_m$ between regions m and $m+1$, we have two appropriate boundary conditions given as:

$$\Phi_m = \Phi_{m+1}, \qquad \frac{\partial \Phi_m}{\partial x} = \frac{\partial \Phi_{m+1}}{\partial x} \qquad (x = x_m;\ -(h_m, h_{m+1})_{\min} \le z \le 0) \tag{4}$$

which ensures the continuity of pressure and x-directional flux, respectively. We can define the function $f_{m,n}$ which expresses vertical variation of the velocity potential as:

$$f_{m,n} = \begin{cases} \cosh k_m(h_m + z), & n = 0 \\ \cos K_{m,n}(h_m + z), & n \ge 1 \end{cases} \tag{5}$$

In the eigenfunction expansion method, two matching conditions given in equation (4) can be reexpressed by using the orthogonality of the function $f_{m,n}$ as:

$$\int_{-h_m}^{0} \Phi_m \Big|_{x=x_m} f_{m,n}\, dz = \int_{-h_{m+1}}^{0} \Phi_{m+1} \Big|_{x=x_m} f_{m,n}\, dz \qquad (n = 0, \cdots, N) \tag{6}$$

Table 1: A comparison of transmission and reflection coefficients ($k_1 h_1 = 0.75$, $\theta = 0°$)

Description	$N = 0$	$N = 2$	$N = 4$	$N = 8$	$N = 16$
forward - T	1.18775273	1.18261064	1.18229425	1.18207029	1.18200805
forward - R	0.20333157	0.22276964	0.22390794	0.22471002	0.22493236
backward - T	0.96031712	0.95649490	0.95614382	0.95598822	0.95592902
backward - R	0.07078085	0.11364253	0.11678659	0.11815288	0.11866856

$$\int_{-h_m}^{0} \left.\frac{\partial \Phi_m}{\partial x}\right|_{x=x_m} f_{m,n}\, dz = \int_{-h_{m+1}}^{0} \left.\frac{\partial \Phi_{m+1}}{\partial x}\right|_{x=x_m} f_{m,n}\, dz \quad (n = 0, \cdots, N) \tag{7}$$

The reflection and transmission coefficients are defined as:

$$R = |A_1^-|, \quad T = \frac{\cosh k_m h_m}{\cosh k_1 h_1} |A_m^+| \tag{8}$$

3. Waves over Double Steps

We first apply the developed model to forward and backward double steps to verify the model's accuracy and proper performance. By definition the depth increases as x increases in the forward double step ($h_1 < h_2 < h_3$), while it decreases in the backward double step ($h_1 > h_2 > h_3$). For simplicity, the water depths are given as $h_2 = 2h_1$ and $h_3 = 3h_1$ in the forward step, while they are given as $h_2 = h_1/2$ and $h_3 = h_1/3$ in the backward step. The width of the step is fixed as $w = 2h_3$ in the forward step and $w = 2h_1$ in the backward step.

The computed reflection and transmission coefficients are listed in Table 1 where N represents the number of evanescent modes considered in the model. The variation of the reflection coefficients between $N = 0$ and $N = 2$ clearly shows the role of evanescent modes. In general, the reflection and transmission coefficients decreases slightly as N increases. Fig. 1 shows the variations of reflection and transmission coefficients over forward and backward double steps. The effects of evanescent modes are not clear in the transmission coefficients but clear in the reflection coefficients. The conservation of energy in the diffracted wave field leads to the condition given as:

$$R^2 + \frac{n_3 k_1 \cos\theta_3}{n_1 k_3 \cos\theta_1} T^2 = 1 \tag{9}$$

in which

$$n_m = \frac{1}{2}\left[1 + \frac{2k_m h_m}{\sinh 2k_m h_m}\right] \tag{10}$$

It is noted that the coefficients listed in Table 1 satisfy equation (9) up to 10 significant figures. Fig. 2 displays the variations of reflection and transmission coefficients for the forward double step with different incident angles. The reflection coefficient increases as θ increases.

4. Waves over a Sinusoidally Varying Topography

The model is also used to calculate resonant and non-resonant reflection coefficients over a sinusoidally varying topography which was used in the experiments of Davies and Heathershaw (1984). The sinusoidal bottom topography is represented by a train of small steps. The depth is described as:

$$h(x) = \begin{cases} h_c, & x < 0 \\ h_c - b\sin(lx), & 0 \le x \le n\lambda \\ h_c, & x > n\lambda \end{cases} \tag{11}$$

where h_c is the constant water depth, b, λ, l, and n are the amplitude, wavelength, wavenumber, and number of ripples, respectively. A wavelength of the sinusoidal ripple is represented by 200 small steps.

In Fig. 3 the reflection coefficients with $N = 0$ and $N = 4$ are obtained for the case of ripples with $h_c = 15.6$ cm, $b = 5$ cm, $\lambda = 100$ cm, and $n = 4$. The agreement between the experimental data and obtained solutions is excellent. A significant wave reflection occurs near the ratio of wavenumbers $2k/l = 1$, which is the so-called Bragg reflection. The effects of evanescent modes are not remarkable except the wavenumber ratios around $2k/l = 2$.

The model is also used to calculate the reflection coefficients for obliquely incident waves. Fig. 4 shows the variations of the reflection coefficients for different incident angles with the same parameters used in Fig. 3. For the case of $\theta = 30°$, the reflection coefficient is shifted to the higher wavenumber and the magnitude of reflection is lower compared to the case of $\theta = 0°$.

5. Concluding Remarks

In this study we develop a theoretical model describing the reflection of monochromatic waves by abrupt depth changes using the eigenfunction expansion method. The developed model is applied to different topographies. The effects of the evanescent modes as well as the propagating mode are included in the model. Furthermore, the incident angle of monochromatic waves is variable. The developed model can be used as a tool for designing submerged breakwaters which protects coastal structures and prevents unwanted beach erosion by reflecting a significant amount of incident wave energy.

6. References

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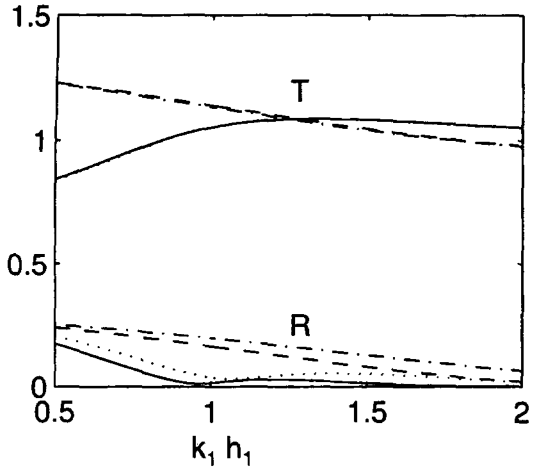

Fig. 1: Variation of reflection and transmission coefficients over double steps ($\theta = 0°$); —— = backward (N=0), - - - - = backward (N=16), – – – = forward (N=0), – . – . = forward (N=16).

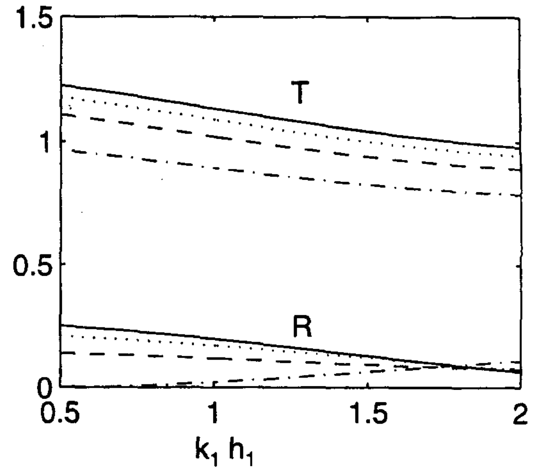

Fig. 2: Variation of reflection and transmission coefficients over forward double steps (N=16); —— = ($\theta = 0°$), - - - - = ($\theta = 30°$), – – – = ($\theta = 45°$), – . – . = ($\theta = 60°$).

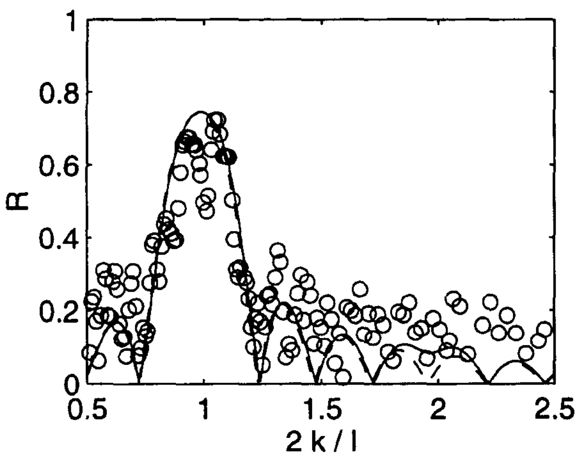

Fig. 3: Variation of reflection coefficients over a sinusoidally varying topography; o = experimental data, —— = present study (N=0), – – – = present study (N=2).

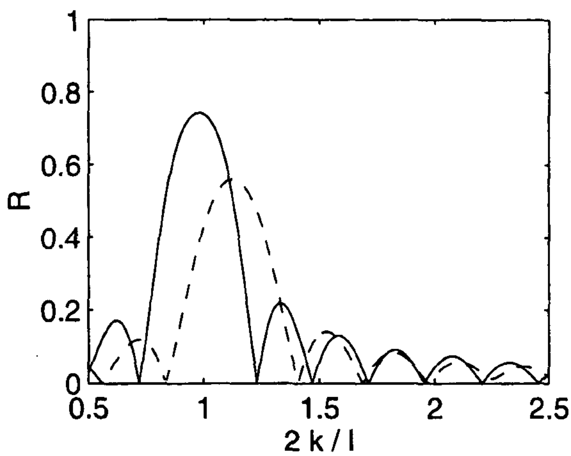

Fig. 4: Variation of reflection coefficients over a sinusoidally varying topography; —— = ($\theta = 0°$), – – – = ($\theta = 30°$).