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A Study on the Instability Criterion for the Stratified Flow in Horizontal Pipe at Cocurrent Flow Conditions

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Abstract

This paper presents a theoretical approach of the instability criterion from stratified to nonstratified flow in horizontal pipe at cocurrent flow conditions. The new theoretical instability criterion for the stratified and nonstratified flow transition in horizontal pipe has been developed by hyperbolic equations in two-phase flow. Critical flow condition criterion and onset of slugging at cocurrent flow condition correspond to zero and imaginary characteristics which occur when the hyperbolicity of a stratified two-phase flow is broken, respectively. Through comparison between results predicted by the present theory and the Kukita et al.[1] experimental data of pipes, it is shown that they are in good agreement with data.

1. Introduction

Numerous incidents of water-hammer have been encountered in various nuclear power plant systems. When water-hammer is combined with system structure-equipment interactions, aging effect, corrosion etc., water-hammer has the potential to aggravate many safety related problems in nuclear power plants. Particularly, Condensation Induced Water-Hammer (CIWH) in a horizontal pipe or in an inclined pipe is the most dangerous form of water-hammer and its diagnosis is extremely difficult because of complex nature of the underlying phenomena that occur at the two phase flow interface. A number of comprehensive studies on the phenomena associated with water-hammer events were reported.

This study is interested in a specific topic, the flow transition criterion, we called onset of slugging criterion or critical flow condition criterion in a horizontal pipe, among them. These flow transition criteria are important for the analysis of the two-phase flow and heat transfer in nuclear power plants. And these flow transition criteria are one of the key water-hammer initiation mechanism which is essential in the analysis of Condensation Induced Water-Hammer.

In this regard, the present study deals with a stratified two-phase flow in pipe. And two-phase flow instability analysis based on wave equations, i.e., hyperbolic equations, and general governing equations such as mass conservation equation and momentum conservation equation for the inclined pipe conditions, is presented and also compared with experimental data.

2. Analysis of Instability Criterion

Consider a stratified two-phase flow in an inclined pipe. To simplify the analysis, the governing equation, which describe the interaction between the liquid and gas phase, are based on the following assumptions.

- The flow is steady, stratified, incompressible and inviscid flow.
- The thermodynamic equilibrium between phases. ($u_g \neq u_f$)

The applicable governing equations of continuity and momentum balance equations for the stratified two-phase flow are addressed. The one dimensional averaging local instantaneous two-fluid model can be obtained as

Liquid phase mass

$$\frac{\partial}{\partial t}(\alpha_f)+\frac{\partial}{\partial z}(\alpha_f\ u_f)=-\Gamma\ /\ \rho_f \qquad (1)$$

Gas phase mass

$$\frac{\partial}{\partial t}(\alpha_g)+\frac{\partial}{\partial z}(\alpha_g\ u_g)=\Gamma\ /\ \rho_g \qquad (2)$$

We obtained as the following combined momentum equation[1]:

Combined momentum equation

$$\alpha_f\alpha_g\left[\rho_f\frac{\partial u_f}{\partial t}-\rho_g\frac{\partial u_g}{\partial t}\right]+\alpha_f\alpha_g\left[\rho_f u_f\frac{\partial u_f}{\partial z}-\rho_g u_g\frac{\partial u_g}{\partial z}\right]$$
$$-\alpha_f\alpha_g((\frac{\partial\Delta P}{\partial\alpha_g})+\frac{\Delta\rho g D\cos\beta}{\sin\gamma}\frac{\pi}{4})\frac{\partial\alpha_g}{\partial z}=$$
$$\alpha_f\alpha_g\Delta\rho\, g\sin\beta-\tau_f\frac{S_f}{A}\alpha_g+\tau_g\frac{S_g}{A}\alpha_f+\tau_i\frac{S_i}{A}(\alpha_g+\alpha_f)$$
$$-\alpha_g\Gamma(u_{fi}-u_f)-\alpha_f\Gamma(u_{gi}-u_g) \qquad (3)$$

where

$$\tau_f=\frac{f_f}{2}\rho_f|u_f|u_f,\quad \tau_g=\frac{f_g}{2}\rho_g|u_g|u_g,\quad \tau_i=\frac{f_i}{2}\rho_g|u_r|u_r,\quad \Delta\rho=\rho_f-\rho_g$$
$$\Gamma=\frac{Q_w}{i_{fg}}$$

In order to proceed with the analysis towards the instability, such as onset of slugging or flooding, etc., Eq.(1), Eq.(2) and Eq.(3) are transformed into first linearized matrix forms, in which are as follows:

$$\underline{A}\frac{\partial\underline{X}}{\partial t}+\underline{B}\frac{\partial\underline{X}}{\partial z}=\underline{C} \qquad (4)$$

If we introduced characteristic value, λ, into Eq.(14), this equation is divided into the following simultaneous ordinary difference equations:

Then Eq.(4) transformed into following form:

$$(\underline{\underline{A}}\lambda+\underline{\underline{B}})\frac{\partial \underline{X}}{\partial \xi}=\underline{C} \qquad (5)$$

For this hyperbolic equation, the characteristics, λ, must be real and non zero. To find the condition under which Eq.(14) becomes singular point, Eq.(5) is changed into;

$$\frac{\partial \underline{X}}{\partial \xi}=(\underline{\underline{A}}\lambda+\underline{\underline{B}})^{-1}\underline{C}=\frac{N_i}{\Delta} \qquad (6)$$

and N_i creates a determinant obtained from $A_{ij}\lambda+B_j$ by replacing the i-th column of C_j.

The phase space of Eq.(6) is constructed of the $n+1$ dimension where n is the number of variables and has three distinctive points of regular points, turning point, and singular point. If $\Delta\neq 0$, the points in the space are regular points. However, this condition depends on the characteristics, λ, and Eq.(6) shows that every point in the phase space seems to be irregular. That is to say, the real irregular point occurs when both conditions of $\Delta=0$ and hyperbolicity breaking are satisfied. The hyperbolic equations are broken when the characteristics of the equations become imaginary and zero.

The characteristic values, λ, obtained from the characteristic equation are all distinct and real for a hyperbolic equation. According to the requirement of well-posedness in linear partial difference equations of a form like Eq.(4) is same as that of the hyperbolic equation, which guarantees the stability of their solutions. When λ becomes zero or imaginary, critical or instabilities occurs. Therefore, mathematically, the condition of hyperbolicity breaking, such as critical flow condition or flooding and slugging instability conditions, for the stratified two-phase flow system of a form like Eq.(4) represents the criticality conditions and neutral stability conditions.

The mathematical background can be summarized as following: The singular points occur when both conditions (1) $\Delta=0$ and the hyperbolicity breaking are satisfied and (2) N_i=0. The conditions of hyperbolicity breaking, on the other hand, are (1) inflected nodes and (2) parallel lines.

As an example of the above claim, many instability analyses, such as onset of slugging, are based on the inflected nodes, which is one of the conditions of hyperbolicity breaking. And choked-flow or critical flow analyses are based on the parallel lines, which is the hyperbolicity breaking condition in hyperbolic equation domain.

The solution of the characteristic's equation is

$$\Delta = det\left|\underline{\underline{A}}\lambda + \underline{\underline{B}}\right| = 0 \qquad (7)$$

where

$$\left|\underline{\underline{A}}\lambda+\underline{\underline{B}}\right|=\begin{bmatrix} -\alpha_g\alpha_f\rho_g(u_g+\lambda) & \alpha_g\alpha_f\rho_f(u_f+\lambda) & -F \\ \alpha_g & 0 & (u_g+\lambda) \\ 0 & \alpha_f & -(u_f+\lambda) \end{bmatrix}$$

$$=\alpha_f\rho_g(\lambda+u_g)^2+\alpha_g\rho_f(\lambda+u_f)^2$$

$$-\alpha_f\alpha_g((\frac{\partial \Delta P}{\partial \alpha_g})+\frac{\Delta\rho g D\cos\beta}{\sin\gamma}\frac{\pi}{4})=0 \qquad (8)$$

and

$$N_{\alpha_g} = \alpha_f \alpha_g (\rho_f - \rho_g) g \sin\beta - \tau_f \frac{S_f}{A}\alpha_g + \tau_g \frac{S_g}{A}\alpha_f + \tau_i \frac{S_i}{A}(\alpha_g + \alpha_f) + \Gamma\left[\alpha_f(\lambda + u_g) + \alpha_g(\lambda + u_f)\right] \quad (9)$$

Solving Eq.(8) for λ, we obtain the characteristic values as:

$$\lambda = -p \pm \sqrt{p^2 - q} \quad (10)$$

where

$$p = \frac{\alpha_f \rho_g u_g + \alpha_g \rho_f u_f}{\alpha_f \rho_g + \alpha_g \rho_f} \quad (10a)$$

$$q = \frac{\alpha_f \rho_g u_g^2 + \alpha_g \rho_f u_f^2 - F}{\alpha_f \rho_g + \alpha_g \rho_f} \quad (10b)$$

and

$$F = \alpha_f \alpha_g \left(\left(\frac{\partial \Delta P}{\partial \alpha_g}\right) + \frac{\Delta\rho g D \cos\beta}{\sin\gamma}\frac{\pi}{4}\right) \quad (10c)$$

Now, the neutral stability is obtained by setting the square root part of equation Eq.(10) to be zero :

$$(u_g - u_f)^2 = \left[\left(\frac{\partial \Delta P}{\partial \alpha_g}\right) + \frac{\Delta\rho g D \cos\beta}{\sin\gamma}\frac{\pi}{4}\right]\left(\frac{\alpha_f \rho_g + \alpha_g \rho_f}{\rho_f \rho_g}\right) \quad (11)$$

Eq. (11) is a primitive form of the onset of Kelvin-Helmholz instability criterion that results from the analysis of a singular point and neutral stability conditions.

In this regard the present study deals with a large amplitude wave model. It is known that we should consider the effect of an irregular wave or chaotic wave and a finite amplitude wave at low void fraction region. It should be noted that we need the condition of the empirical factor. It can be recognized that the coefficient used by Taitel & Dukler constant satisfies all physical conditions. To incorporate the above physical conditions and introduce the dimensionless parameter into the present model can be modified as follows:

$$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{1-\alpha_g} = \left(1 - \frac{H_f}{D}\right)\sqrt{\frac{\alpha_g \cos\beta}{\sin\gamma}\frac{\pi}{4}} \quad (12)$$

The critical flow condition can be obtained by setting q for Eq (10).to zero.

Using Eq.(10), following relations can be obtained:

$$\frac{j_g^{*2}}{\alpha_g^3} + \frac{j_f^{*2}}{(1-\alpha_g)} = \frac{\pi\cos\beta}{4\sin\gamma} = \frac{\pi D\cos\beta}{4S_i} \quad (13)$$

The present results for the critical flow condition criterion in horizontal condition ($\beta = 0^o$), Eq(13), can be reduced to the same form obtained by Gardner. In Fig 1, experimental data by Kukita et al.[3] are selected to justify the present onset of slugging and critical flow condition criterion for cocurrent flow in horizontal pipe with conatant gas flow rates. Fig 1 shows, the dashed line and straight line, which represent the new onset of criterion from Eq.(12) and critical flow condition criterion from Eq(13), respectively. In Fig 1, at the subcritical region, the experimental data closely agree with the present onset of slugging criterion (dashed line), and then, at the supercritical region, experimental data closely agree with critical flow condition criterion (straight line). It is noted that when the flow was stratified and horizontal pipe conditions, the instabilities depend on a prior instability criterion.

3. The Criterion for the Water-Hammer Initiation

Bjorge and Griffith and Chun and Nam[4] have published the results of their investigation into the criterion for the lower boundary of condensation-induced water-hammer initiation, they proposed this criterion by the Taitel and Dukler's criterion for the stratified/ non-stratified flow transition in a horizontal circular pipe:

$$N_{TD} = (\frac{1-\alpha_g}{\alpha_g})(\frac{\rho_s}{\rho_l})(\frac{S_i}{g})(\frac{u_s^2}{A_l C_{TD}^2}) \geq 1 \qquad (14)$$

In the present work, the criterion for the lower boundary of condensation-induced water-hammer initiation may be obtained by the present model for the onset of slugging criterion, Eq.(12), in inclined pipe. It is noted by Bjorge and Griffith and Chun and Nam [4] that region where water-hammer is predicted to occur is bounded by the instability criterion. Considering the inclination angle to the horizontal, it can be:

$$N_{new}^{inclined} = (\frac{1-\alpha_g}{\alpha_g})(\frac{\rho_s}{\Delta\rho})(\frac{S_i}{g\cos\beta})(\frac{u_r^2}{A_l C_{new}^2}) \geq 1 \qquad (15)$$

where $C_{new} \cong (1-\frac{H_f}{D})$

The present results, Eq.(15), reduced to the same expression obtained by Bjorge and Griffith's criterion when the inclination angle is zero. Therefore, the present criterion, Eq.(15), can be used with more general and confidence to find the lower boundary criterion for the condensation-induced water-hammer initiation .

In Fig. 2, the comparison of present and Taitel and Dukler's model for the lower bounds of condensation-induced water-hammer initiation have applied to the water-hammer event of San Onofre Unit 1. Fig. 2 also show that the lower bounds for critical inlet flow rates obtained from the present model is slightly higher than that by Taitel and Dukler's model.

References

[1] M.H. Chun and C. K. Sung, Onset of Slugging Criterion based on Characteristics and Stability Analyses, Int. Comm. Heat Mass Transfer, V23 (1996) 473-484.
[2] G. C. Gardner, Cocurrent Flow of Air and Water from a Reservoir into a Short Horizontal pipe, Int. J Multiphase Flow V14 (1988) 375-388
[3] Y. Kukita et al., Hot Leg Flow, Characteristics during Two-Phase Natural Circulation in PWR, NURETH 4, Karlsruhe, Oct. (1989) 465-470.
[4] M.H. Chun and H.Y.Nam, Analysis of Water-Hammer Induced by Steam Water Counterflow in a Long Horizontal Pipe , Int. Comm. Heat Mass Transfer, V19 (1992) 507-518.

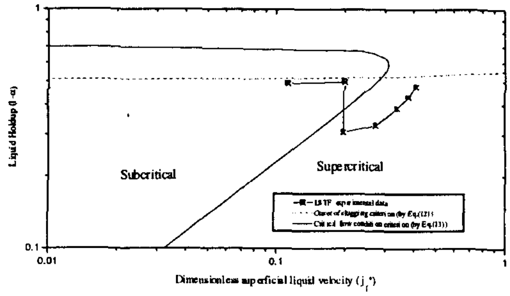

Fig. 1a Transition from Subcritical to Supercritical Flow during Increase in Loop Flow for 5% Core Power Test (j_g^*=0.15).

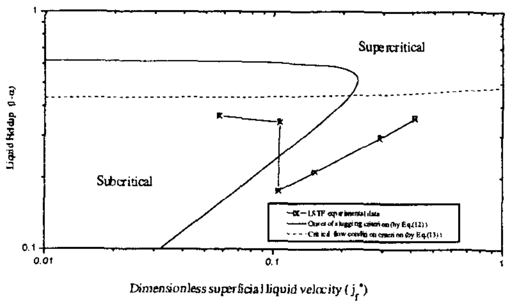

Fig. 1b Transition from Subcritical to Supercritical Flow during Increase in Loop Flow for 7% Core Power Test (j_g^*=0.21).

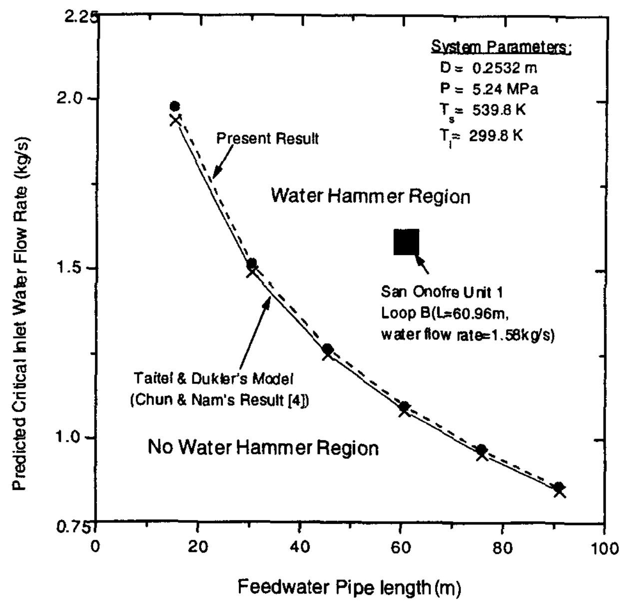

Fig 2. Comparison of Water-Hammer Region Lower Boundaries Predicted by Taitel & Dukler's Model to the Present Result for San Onofre Unit 1 Event