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Robust Reactor Power Control System Design by Genetic Algorithm

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Abstract

The H_∞ robust controller for the reactor power control system is designed by use of the mixed weight sensitivity. The system is configured into the typical two-port model with which the weight functions are augmented. Since the solution depends on the weighting functions and the problem is of non-convex, the genetic algorithm is used to determine the weighting functions. The cost function applied in the genetic algorithm permits the direct control of the power tracking performances. In addition, the actual operating constraints such as rod velocity and acceleration can be treated as design parameters. Compared with the conventional approach, the controller designed by the genetic algorithm results in the better performances with the realistic constraints. Also, it is found that the genetic algorithm could be used as an effective tool in the robust design.

1. Introduction

In the process of designing the control system, the most important one is to define the plant to be controlled. But the exact modeling of the plant is impossible in reality. The plant modeling includes the linearization of the non-linearity as well as the approximations during the mathematical description of the plant. In addition, the designed system is apt to change due to the various operating conditions, set point drift and equipment aging so on. The actual system should work as intended under the real circumstance even though it is designed on the basis of inexact plant. Therefore, the ultimate purpose of the control system design is the robustness rather than the stability[1]. This robustness problem has been one of main issues in recent years, and many methods are developed for the robust design. Among these the H_∞ paradigm provides the synthetic method by which the size of the uncertainty is measured quantitatively by the infinity norm. However, although this method has been found to be a useful approach, the design process is not so easy. The system should be in the H_∞ space, that is, in the space of stability and properness. Further, since the problem of the H_∞ is non-convexing problem, the optimization process is not easy. All these properties result in the messy numerical processes and many iterations as well as the dependency on designer's discretion.

This motivates us to develop a new method in the frame of H_∞ paradigm. Since the H_∞ control is made in the frequency domain, the classical loop shaping can be used. And for the appropriate loop shaping, the frequency dependent weighting functions are introduced. Since these weighting functions are key parameters of the system design, numerous iterations are necessary and the result does not guarantee the best solution. On the contrary, our method eliminates these problems. Once the cost function being given, the algorithm finds the optimal results without designer's interference. We applied this algorithm to the reactor power control system and it shows the efficiencies both in design process and in accuracy of results.

2. System Configuration

The reactor dynamics are described in the linear state variable equations as below.

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \quad y = \mathbf{Cx} + \mathbf{Du} \tag{1}$$

The reactor dynamics of Eq. (1) is derived with the assumptions of one delayed neutron group, small perturbations for linearization, and constant control rod worth. The details are fully described in Ref.[2] and [3]. The plant has five variables of power, precursor density, coolant temperature, fuel temperature and reactivity. The system matrix **A** is the function of nuclear and thermal hydraulic properties which vary with the power level, and is subject to change during the transient. Since this plant has uncertainties, it is doubtful whether the control system designed with this erroneous plant will work with enough stability and performance. Hence it is desirable to build the control system with robust design. The overall system with uncertainty can be configured as outlined in Fig. 1. In this configuration all the uncertainties acting on the plant are treated as one multiplicative uncertainty and the perturbations of the uncertainty act on the system.

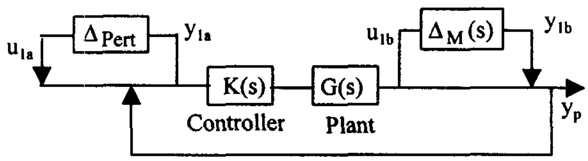

Figure 1. Unity Feedback Configuration with Uncertainties

The system of Fig. 1 can be converted into the canonical four port model, and the H_∞ control can be easily applied. The classical frequency domain analysis indicates that for the robustness of system against the measurement noise, the size of complementary sensitivity, **T**, should be small, and at the same time, the sensitivity, **S**, should be small to eliminate the system disturbance. But since **S**+**T**=**I**, the trade-offs between **S** and **T** are unavoidable. However, since the measurement noise usually has high frequencies and the disturbance is of low frequencies, the trade-off is made in the frequency region in such a manner that we lower the **S** at low frequency region, and lower the **T** at high frequency region, at the expense of each counterpart. This is the concept of the loop shaping, which is very useful in designing the system under the uncertainties.

For the loop shaping, the frequency weighting functions of W_1 and W_3 are applied to the controller input and plant output, respectively. Figure 2 shows this configuration. For the stability condition, the weight functions should satisfy the following inequalities.

$$\| T(j\omega) W_3 \|_\infty \le 1, \; \| S(j\omega) W_1 \|_\infty \le 1, \text{ or}$$

$$\bar{\sigma}(T(j\omega)) \le W_3^{-1}\ ,\ \bar{\sigma}(S(j\omega)) \le W_1^{-1} \tag{2}$$

where $\bar{\sigma}$ denotes the singular value.

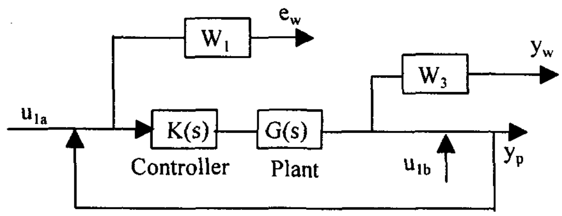

Figure 2. Unity Feedback Configuration Augmented with Weighting Functions

For the stability condition, the weight functions should satisfy the following inequalities.

$$\| T(j\omega) W_3 \|_\infty \le 1\ ,\ \| S(j\omega) W_1 \|_\infty \le 1,\ \text{or}\ \bar{\sigma}(T(j\omega)) \le W_3^{-1}\ ,\ \bar{\sigma}(S(j\omega)) \le W_1^{-1} \tag{3}$$

where $\bar{\sigma}$ denotes the singular value.

In the figure above, the transfer function between the input noise vector of $\mathbf{u} = (u_{1a}\ \ u_{1b})^T$ and $y = (e_w\ \ y_w)^T$ is

$$\mathbf{T}_{yu} = \begin{pmatrix} W_1 S \\ W_3 T \end{pmatrix} (\mathbf{I}\ \ -\mathbf{I})\ ,\ T = \frac{GK}{1+GK},\ S = \frac{1}{1+GK}\ . \tag{4}$$

For the tracking system, the noise signal of Fig. 2 can be replaced by the command input. Then the system becomes the typical two port model of Fig. 3 to which the H_∞ technique can be applied directly.

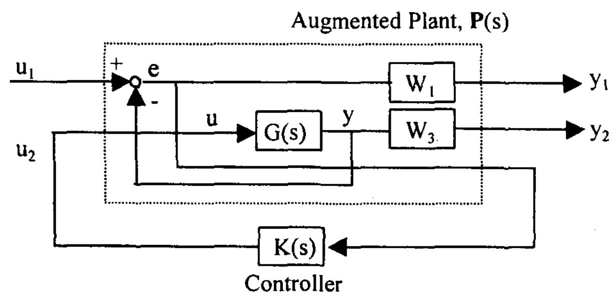

Figure 3. Two Port Model with Augmented Plant

In this configuration, the augmented plant P(s) is the MIMO with the inputs of command signal and control effort, and with the outputs of weighted error and weighted system output and is described by the following state equations.

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2\mathrm{u} \\ \mathbf{z} &= \mathbf{C}_1\mathbf{x} + \mathbf{D}_{11}\mathbf{w} + \mathbf{D}_{12}\mathrm{u} \\ \mathrm{y} &= \mathbf{C}_2\mathbf{x} + \mathbf{D}_{21}\mathbf{w} + \mathbf{D}_{22}\mathrm{u} \end{aligned} \tag{5}$$

The system matrices **A** and $\mathbf{B}_2$ are the same as the plant G(s), and the matrices related to the output include the weighting functions. The overall system is SIMO since the external signal is u_1 only, and is expressed in the transfer function of

$$\mathbf{T}_{yu} = \frac{(y_1 \;\; y_2)^T}{u_1} = \begin{pmatrix} W_1 S \\ W_3 T \end{pmatrix} \tag{6}$$

The control target is to minimize the infinite norm of the closed loop transfer function by selecting proper weighting functions.

3. Determination of Weighting Functions by GA

The key parameters of the problem posed as Fig.3 are the weighting functions. From Eq. (2), it can be known that the weight function $1/W_3$ has the form of lag, while $1/W_1$ has the form of lead. The high ordered weighting functions yield good results, but the order of the controller is increased with the heavy burden of numerical calculation. Therefore, the first order weighting functions of Eq. (7) are applied in this problem.

$$\frac{1}{W_1} = \gamma_1 \frac{s + 0.000627}{s + 0.06238}, \quad \frac{1}{W_3} = \gamma_3 \frac{0.06175}{s + 0.06238} \tag{7}$$

Even in the case of the first order, numerous parameters, including the shift factors of γ_1 and γ_3, should be determined and this is not an easy work. For instance, the numerical values in Eq. (7) are determined through trial and errors using the messy algorithms such as Yulewalk. In addition, further problems may arise during the calculation. Since the H_∞ paradigm consists of solving two sets of Riccati equations associated with the Hamiltonian system matrix, the existence of the solution depends on the rank conditions which, in turn, are affected by the weighting functions. Above all, there still is the fundamental question, that is, there can be another weight functions which yield the better solution. This leads to the concept of optimization, and various methods can be used. But, as indicated above, the H_∞ problem is of non-convex and the conventional approaches are ineffective.

These problems motivate the use of the genetic algorithm (GA), which has been proved useful in a variety of search and optimization fields. The GA emulates the biological evolutionary theories to solve the optimization problems. With the three major operators of reproduction, crossover and mutation which are analogous to the biological process in genetics, it searches the optimal design parameters of the problem. Since the GA is the direct searching method independent of the coupling between parameters, it provides more flexibility, particularly for the strongly coupled or stiff systems. Also it is a smart algorithm for the multi nodal problems because of its capability of concurrent multi point search.

The parameters to be searched in this problem are γ_1 and γ_3 of Eq. (7). In addition, for the consideration of the control input, the weighting constant γ_2 is applied to the u of Fig. 3. Then the number of the augmented plant output is three and the output vector **z** of Eq. (5) has three elements. This weighting constant is necessary also to satisfy the rank conditions.

In the reactor operation, the abrupt power transient is not desirable because of the nuclear characteristics. Hence there is a limitation on the maximum control speed. In terms of control, this indicates the trade off between the tracking error and control effort. This trade off can easily be made by defining the cost function as desired. Besides the power tracking and rod speed, the acceleration is also taken into account in this problem. The cost function, J, applied to the problem is as below.

$$J = \prod_i \mathrm{Cost}(i) \qquad (8)$$

where cost(1) = power deviation, cost(2) = rod speed and cost(3) = rod acceleration.
Of course, the cost function can take another form. In the cost function above, a penalty is given to the velocity and acceleration, if they exceed the maximum values. The maximum velocity and acceleration is supposed as 2 cm/sec, and 0.5 cm/sec^2, respectively.

Figures 4 through 6 show the power tracking, rod velocity and rod acceleration curves, respectively, when the power is step increased from the steady power of 90% to 100%. In each figure, the results of Case A in which $\gamma_1 = 1$, $\gamma_2 = 2$ and $\gamma_3 = 3$, that is determined by discretion, are described together

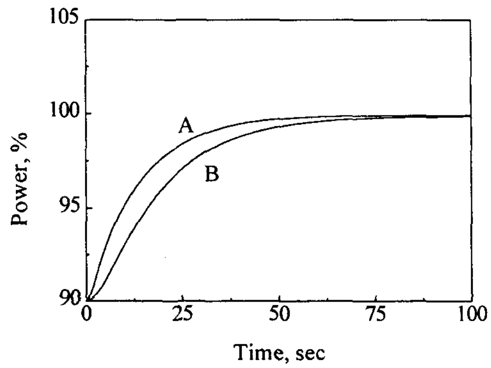

Figure 4. Power Transients

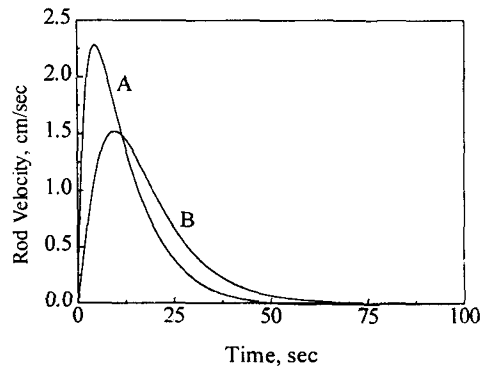

Figure 5. Control Rod Speeds

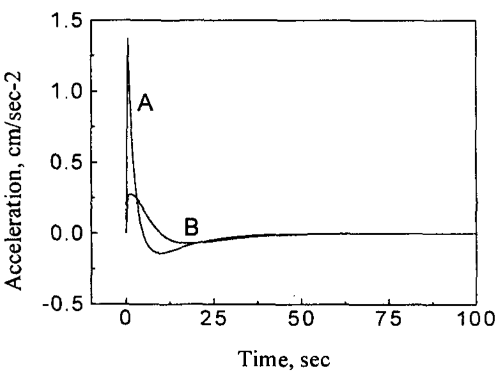

Figure 6. Control Rod Accelerations

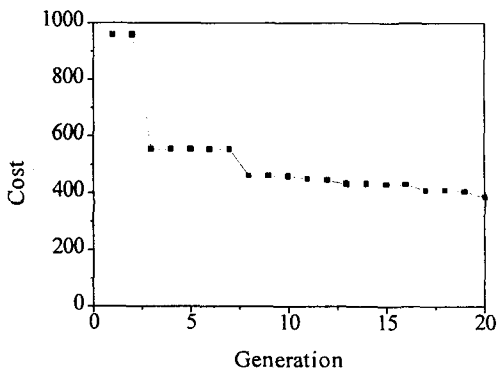

Figure 7. Cost of GA

with those of the GA (Case B). The GA gives the values of $\gamma_1 = 0.0095$, $\gamma_2 = 0.014$, and $\gamma_3 = 0.032$ after 20 generations. Figure 4 shows the power tracking. Both cases show no overshooting. Although the speed of Case B is somewhat slower than that of Case A, the tracking performances are similar each other. However, there is a great difference in the rod velocity as shown in Fig. 5. The maximum rod velocity specified in the FSAR is about 2 cm/sec. In Case A, the maximum rod velocity exceeds this limit value. But the maximum speed of the GA design is about 1.5 cm/sec, which is much less than the limit value. Figure 6 shows the accelerations of the rod movement for both cases, and the acceleration of the GA design is much less than that of Case A, which indicates that the mild rod movement. This is desirable both for the nuclear and actuator characteristics. In summary, the GA design yields the much improved system characteristics. The cost of the GA, as defined by Eq. (7) is shown in Fig. 7. With the progress of generation, the best cost of all the visited solutions decreases monotonically. The cost decreases drastically at the initial stage. As the generation goes on, it decreases with a small amount. This convergence efficiency can be improved further by the modified algorithm as we already proposed in Ref. [4].

4. Conclusion

In designing the control system, the robustness should be taken into account because of the various intrinsic uncertainties. The actual system should have the performance under the real circumstances. But the detail robust design procedure has many difficulties in that the models should satisfy mathematical conditions for the existence of the stable and proper controller. The robust control is made in the frame of frequency domain, and weighting functions are introduced to achieve the desirable loop shaping. But the determination of the proper weighting function is very difficult and is usually dependent on the discretion. To avoid these difficulties, the genetic algorithm is used. With the flexible cost functions which can reflect the actual constraints, the weighting functions are determined automatically in such a way to optimize the problem. This is particularly efficient for the non convexing problems such as optimal robust control. The results show that the reactor power control system designed by the genetic algorithm has good performances with relatively milder rod movements.

References

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