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Prediction of Dynamic Expected Time to System Failure

Deog-Yeon Oh and Chong Chul Lee

Korea Nuclear Fuel Co., Ltd.
150 Deogjin-dong, Yusong-gu, Taejon

Abstract

The mean time to failure (MTTF) expressing the mean value of the system life is a measure of system effectiveness. To estimate the remaining life of component and/or system, the dynamic mean time to failure concept is suggested. It is the time-dependent property depending on the status of components. The Kalman filter is used to estimate the reliability of components using the on-line information (directly measured sensor output or device-specific diagnostics in the intelligent sensor) in form of the numerical value (state factor). This factor considers the persistency of the fault condition and confidence level in measurement. If there is a complex system with many components, each calculated reliability's of components are combined, which results in the dynamic MTTF of system. The illustrative examples are discussed. The results show that the dynamic MTTF can well express the component and system failure behaviour whether any kinds of failure are occurred or not.

1. Introduction

A critical attribute determining system effectiveness is reliability, which is a measure of the system's ability to avoid failure. The reliability of a component can be quantitatively characterised by different measures: the probability of failure-free operation, the mean time to failure (MTTF), and other indexes. Of these, the MTTF is a very common and useful measure for the evaluation of systems. In reliability analysis [1&2], the time to failure or life length, say T, is usually treated as a random variable under specified environmental and operating conditions. The reliability of a component (or system) at time t, R(t), is defined as R(t) = P(T>t). It means that the reliability of a component equals the probability that the component does not fail during the interval [0,t]. Mean time to failure (MTTF) is the mean value of the life of a product, defined by

$$\text{MTTF} = \int_0^\infty R(t)dt \qquad (1)$$

Conventional MTTF calculations have relied only on the assumed life time model of components [3] or Markov modelling for dynamic redundant systems [4]. The MTTF concept means the weighted average of random variable with respect to time from the probabilistic point of view. Since this integrated numerical value removes any dependence on time, it is useful at the design stage but not suitable to predict the remaining life for the specified system with the on-line failure information. To overcome this restriction, an alternative definition of the time-dependent (dynamic) MTTF needs to be found. A dynamic MTTF can reflect the current likelihood of failure and the reduced life remaining due to the failure symptom. These symptoms may be directly observed by the sensor output. Or, especially, in an on-line processing sensor like SEVA [5&6], the current status of components can be continuously detected and applied to predict the dynamic MTTF. To calculate severity of the failure symptom, the on-line information should be expressed as a numerical value depending on failure severity. The Kalman filter is the very useful tool to estimate the probability of components with the on-line measurements. In reliability theory, it was used to estimate and predict the time dependent reliability in Refs. 7 and 8. N. Singh [8] made a reliability state model using the observable failure rates. However, since it is not easy to measure the failure rate, the state factor is suggested converting the on-line information into the numerical value in this work. Also, the method is developed to estimate the system reliability of components connected in parallel or series.

2. Methodology

If a component is put under stress conditions at some specified time, the time to failure may be considered as a continuous random variable, since identical components subjected to identical stress will fail at different and unpredictable times. There are two types of failure. The first is sudden, catastrophic failure. On the other hand, failure, for example in a steel beam, proceeds slowly over a long time. For both cases, failure behaviour of component may be represented by the proper probabilistic mathematical model, while it is difficult to predict the sudden failure due to the random disturbance. If the on-line information about the current status is available for a component, the symptom of the failure will be detected earlier and a prediction of remaining life will be more accurate. Using the measured status, the likelihood of failure can be expressed as a numerical value. These combinations of the probabilistic model and measured current status is used to estimate the component reliability. Since the remaining mean time to failure (dynamic MTTF) will also be a function of reliability, it can be treated as the dynamic quantity. Figure 1 shows the overall diagram to calculate the dynamic MTTF.

This study considers the recursive estimation (Kalman filter) of time-dependent reliability of components of a system using the multivariate state-space model, given that the component life time model and on-line component status are known. The state space model is used to estimate the reliability with the probabilistic model and the measurement equation is written as linear combinations of the system state variables. Also, they are indirectly measured using the on-line information in the intelligent sensors.

As a measure of the component status, the time-dependent state factor is considered. It is defined as the probability that the component will fail during the time t and t+Δt, i.e., $P(t<T\le t+\Delta t)$. This can be expressed as the probability density function f(t) as follows:

$$\xi(t) = \int_t^{t+\Delta t} f(t)dt \cong \Delta t\, f(\eta) \qquad (2)$$

where $t\le\eta\le t+\Delta t$.

The last expression is (for small Δt and supposing that f(t) is continuous) approximately equal to Δt f(t). Since the density function is given by the product of the failure rate (sometimes called hazard function) h(t) and reliability function R(t), it represents

$$\xi(t) = \Delta t\, h(t)\, R(t) \qquad (3)$$

Since R(t) has the values between 0 and 1 ($0\le R(t)\le 1$), the state factor, ξ_i, for a component i should have the values from 0 to Δt h(t) depending on the failure severity.

The state factor can be indirectly measured using the on-line information in the intelligent sensor/actuator. The state factor is dependent on the failure severity. There are a lot of parameters to estimate the failure probability. They are combined into one integrated factor using the weighted mean method. This gives the weighted average of the state factor as follows:

$$\xi(t) = \frac{\sum_{i=1}^{n} w_i y_i(t)}{\sum_{i=1}^{n} w_i} \qquad (4)$$

where w_i is the weighting value and $y_i(t)$ is failure severity. The weighting is determined by engineering judgement based on the importance of parameters. The failure severity will be explained in detail in section 3.1.

The state-space model is used for estimating and predicting the time-dependent state factor and reliability using the on-line component status from the SEVA diagnosis.

In general, the state factor vector (l-dimensional) at time t, $\xi(t)$ is expressed as

$$\xi(t) = H(t)\,R(t) + v(t) \tag{5}$$

where H(t) is a lxl diagonal matrix defined by $\Delta t\, h(t)$ and v(t) is a vector of random noise quantities (zero mean, covariance matrix S). In this case, it is equivalent to the measurement uncertainty.

At discrete points in time k, it can be written

$$\xi(k) = H(k)R(k) + v(k) \tag{6}$$

The reliability vector R(t) equations are

$$R(t) = A(t)R(t-1) + w(t) \tag{7}$$

where A(t) is a lxl system matrix.

In the discrete form, it is

$$R(k+1) = \Phi(k+1,k)\,R(k) + w(k) \tag{8}$$

where $\Phi(k+1,k)$ is the state transition matrix, given by $e^{A\Delta T}$, and w(k) is a zero-mean white sequence vector of covariance Q.

The Kalman filter [9] is a computational algorithm to minimise the estimation error in a well-defined statistical sense by utilising measurement data plus prior knowledge about the system. It is used to generate optimal estimates of the state and output. It minimises the performance index $E[(R-\hat{R})^T(R-\hat{R})]$, where the superscript caret means the estimated value.

Across a measurement, the state and error covariance are updated:

$$\hat{R}(k,k) = \hat{R}(k,k-1) + K(k)\,[\xi(k) - H(k)\,\hat{R}(k,k-1)] \tag{9}$$

$$P(k,k) = [I - K(k)H(k)]\,P(k,k-1) \tag{10}$$

where P(k,k) is the error covariance matrix and I is the identity matrix.

The optimal Kalman gain K(k) is calculated

$$K(k) = P(k,k-1)H^T(k)[H(k)P(k,k-1)H^T(k)+S(k)]^{-1} \tag{11}$$

The extrapolation of these quantities between measurements is

$$\hat{R}(k+1,k) = \Phi(k+1,k)\,\hat{R}(k,k) \tag{12}$$

$$P(k+1,k) = \Phi(k+1,k)\,P(k,k)\,\Phi^T(k+1,k) + Q(k) \tag{13}$$

The estimated state factor (vector), $\hat{\xi}(k)$, is calculated as

$$\hat{\xi}(k) = H(k)\,\hat{R}(k,k-1) \tag{14}$$

Figure 2 shows a time diagram of the various quantities involved in the discrete optimal filter equations.

The reliability of a component i at time t is estimated and updated using the measured state factor in Kalman filter. Each component reliability's are used in the calculation of the system reliability and result in the dynamic mean time to failure. The system *usually consists of several components, often* connected in a complex way. The first step toward evaluating the time-dependent reliability of a system, R(t), consists of drawing a reliability block diagram [1] showing the combinations of components that must function for successful system operation. Many reliability block diagrams can be decomposed into series and parallel combinations of components. The component reliability can be determined individually in the above paragraph and then combined to gauge the reliability of the overall system.

The conventional MTTF (static MTTF) is divided into two parts as follows:

$$MTTF = \int_0^\infty R(t)dt = \int_0^t R(t)dt + \int_t^\infty R(t)dt \tag{17}$$

The dynamic MTTF at every time t can be defined as the remaining mean life from time t.

$$MTTF_d(t) = \int_t^\infty R(t)dt = MTTF - \int_0^t R(t)dt \tag{18}$$

If the estimated reliability is changed, the dynamic MTTF will also be in Eq. (18). As the component is operated well, the remaining life may be extended longer than the predicted life of the mathematical model. Meanwhile, it will be shortened if the component is likely to fail. Since the phenomena likely to fail or success will be repeated during operation, the dynamic MTTF is the time-dependent property. Virtually, it relies on the current reliability

3. Applications

3.1 Failure Severity to be Measured in Sensor

In the intelligent sensor, there are a lot of checks to detect the several failures. Based on these results, the failure severity for each failure will be determined and quantified in a numerical value. It is assumed that the failure severity values are linear function of input as shown in Figure 3. This is the general form to calculate the severity output for out-of range variables. Usually, it is inversely proportional to the failure input since the larger the severity, the less reliable the component is. The input x is the time-dependent information given by sensor output or the intelligent sensor, which may be the direct or indirect measured value. All dependent values should have the values between 0 and 1. The severity function is given by

$$\begin{aligned} f(x) &= 1 & x \le x_1 \\ &= -(x-x_1)/(x_2-x_1)+1 & x_1 < x \le x_2 \\ &= 0 & x > x_2 \end{aligned} \qquad (21)$$

If x_1 is equal to x_2 for a special case, the severity function has the value 0 or 1, i.e., step function.

Sometimes, if the special fault indicator or value is only monitored, it may have the delta function as follows:

$$\begin{aligned} f(x) &= 1 - \delta(x_i), & x = x_i \\ &= 1 & \text{elsewhere} \end{aligned} \qquad (22)$$

In this study, the inputs used to determine the failure severity are its persistence of the fault condition (frequency of occurrence), and confidence level in measurement (degree of failure) to prevent false indication owing to the abrupt noises. Persistency represents the consecutive number of failure likelihood, which is counted if the measured value exceeds the threshold, i.e., the component is likely to fail.

3.2 Simplified Thermocouple System

The self-validation sensor (SEVA) has been developed in Oxford University [5&6]. This sensor employs self-diagnostics. In SEVA sensor, the diagnostic state machine processes the information available in the device domain to generate a diagnosis, which is used to drive the validity indices calculation, including the device status. The state information may be obtained from this diagnostic result. The degree of failure is determined based on the similar criteria used to judge the fault indicator. The failure occurrence is incremented if the variable exceeds the threshold. The SEVA thermocouple [6] is consisted of the sensing part (transducer) and transmitter (signal conditioning and processing unit, etc.). It has two separate channels to measure temperature. As shown in Figure 4, this sensor system can be simplified into the components corresponding to the failure cause, i.e., two redundant thermal wells, two connection cables and one power supply.

If all component failure behaviours are represented by the exponential distribution, the system reliability and the dynamic MTTF at time t are written as

$$R(t) = e^{-(\lambda_1+\lambda_2+\lambda_5)t} + e^{-(\lambda_3+\lambda_4+\lambda_5)t} - e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}$$

$$MTTF_d(t) = e^{-(\lambda_1+\lambda_2+\lambda_5)t}/(\lambda_1+\lambda_2+\lambda_5)+e^{-(\lambda_3+\lambda_4+\lambda_5)t}/(\lambda_3+\lambda_4+\lambda_5)-e^{-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)t}/(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)$$

There are two kinds of simulations; one is the failure of component 5 (power supply) causing the system failure, but another is the failures of components 1 and 2 without system failure. To simulate the loss of device power fault in component 5, the inputs in the fault revealing gain become zero from

20 seconds. It results in the decrease of state factor as shown in Figure 5. If the failure symptom is continued, it will be drop to zero making the system reliability and dynamic MTTF decreased abruptly. Since the power supply does not have the redundant channel in the sensor system, the failure of power supply is serious and leads to the system failure.

In this simulation, both the open circuit failure in component 2 and thermal contact failure in component 1 are simultaneously occurred at 20 seconds. Following the failures, the state factors are sharply decreased. However, the reliability of overall sensor system is nearly not influenced since the redundant path is operating well, while the dynamic MTTF is shortened as soon as one redundant path is failed and reaches as much as a third of initial value without failure. It emphasizes the importance of redundant system in case of component failures. Figure 6 shows those variation of the dynamic MTTF.

4. Conclusions

Up to now, the life of component or system made in factory has been predicted by the mean time to failure (MTTF). However, if they are likely to fail, it is difficult to know the remaining life of component or how the MTTF is reduced. Also, the conventional MTTF value is not able to explain how much the life of system is affected by the component failure for a system composed of multi-components. They should be treated as the time-dependent problems. So, the dynamic MTTF concept was newly defined in this study. The on-line information is the pre-requirement for this sake, which is converted to the numerical value of state factor. To combine this factor with the probabilistic model, the Kalman filter was used because it is one of the most useful tools to estimate the state with measurements. For both normal and intelligent sensors, there are several simulations to test the algorithm. As a result, it was proved that this method could predict the reliability and remaining life of component and/or system when they had the failure symptom. It means that the dynamic MTTF can well express the component and system failure behaviour whether any kinds of failure are occurred. Also, it is very helpful to have a proper repair and maintenance time, resulting in the economical savings.

5. References

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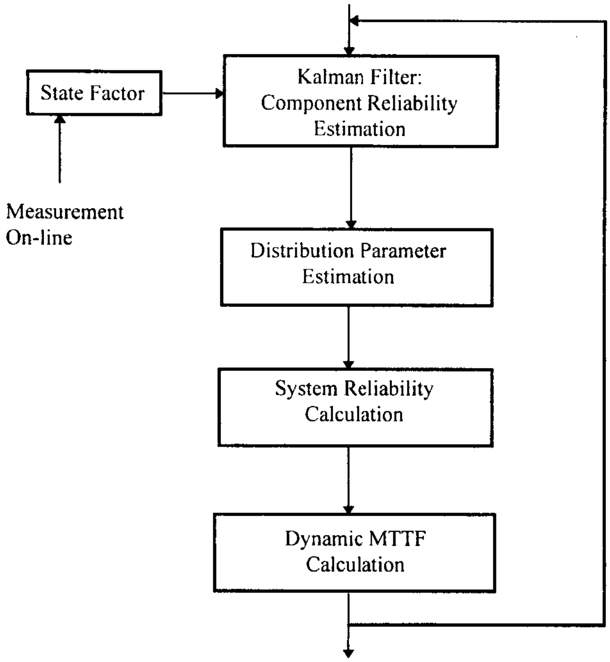

Figure 1 Overall Diagram for Dynamic MTTF Calculation

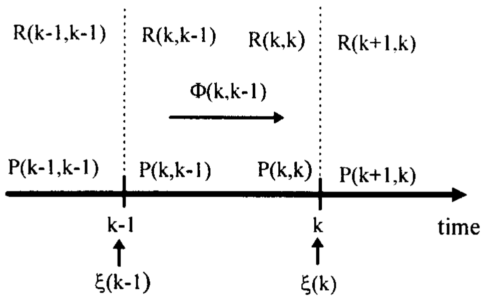

Figure 2 Discrete Kalman Filter Timing Diagram

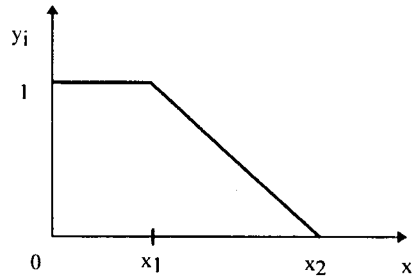

Figure 3 General Failure Severity Function

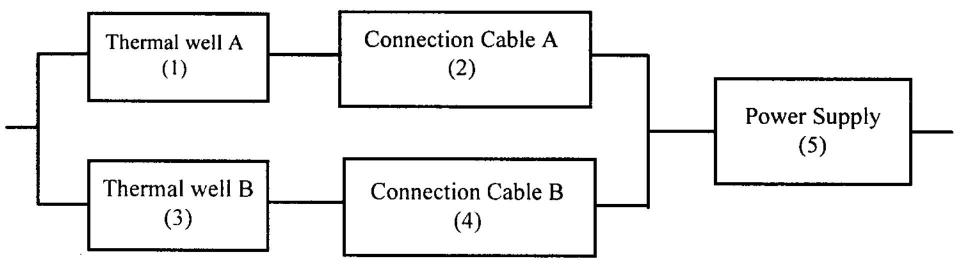

Figure 4 Simplified Thermocouple System in SEVA

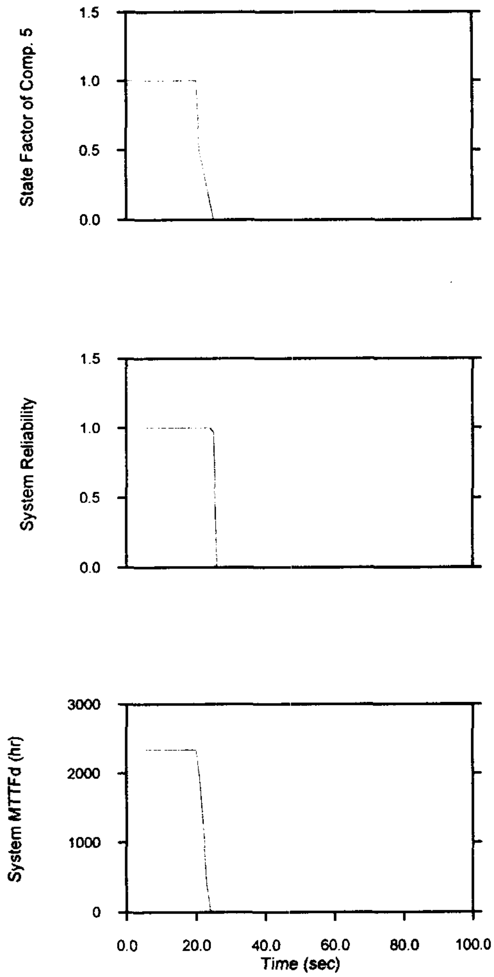

Figure 5 Simulation Results vs Time with the Power Supply Failure

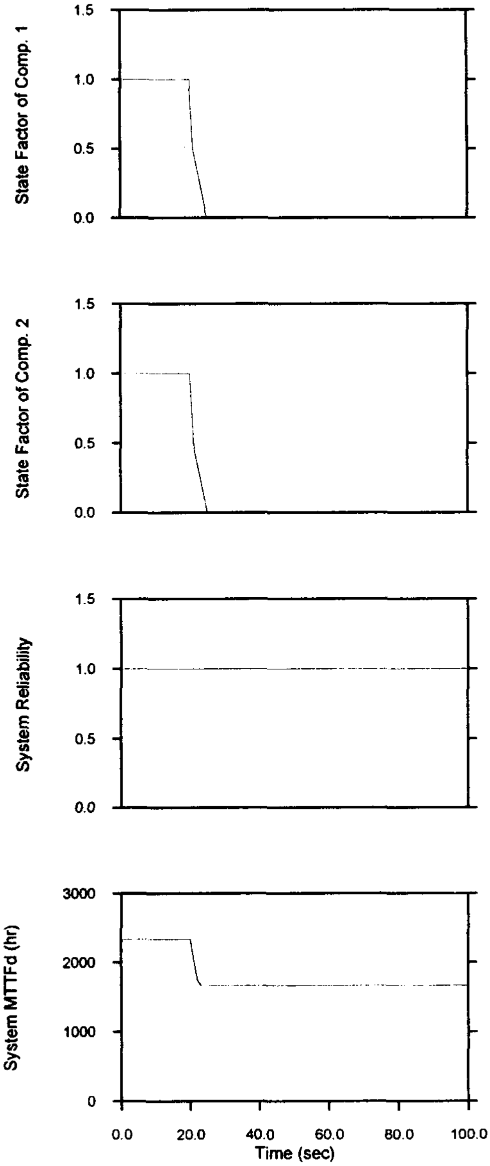

Figure 6 Simulation Results vs Time with the Multi-component Failure