

Approximate Method in Estimating Sensitivity Responses to Variations in Delayed Neutron Energy Spectra

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Abstract

Previous our numerical results in computing point kinetics equations show a possibility in developing approximations to estimate sensitivity responses of nuclear reactor. We recalculate sensitivity responses by maintaining the corrections with first order of sensitivity parameter. We present a method for computing sensitivity responses of nuclear reactor based on an approximation derived from point kinetics equations. Exploiting this approximation, we found that the first order approximation works to estimate variations in the time to reach peak power because of their linear dependence on a sensitivity parameter, and that there are errors in estimating the peak power in the first order approximation for larger sensitivity parameters. To confirm legitimacy of our approximation, these approximate results are compared with exact results obtained from our previous numerical study.

I. Introduction

It has long been recognized that delayed neutrons play an essential role in controlling nuclear reactors. Since fission and absorption reactions are almost balanced during a normal operation of a nuclear reactor, a static theory can be applied to reactor analyses. In a transient phase of a reactor, however, the populations of precursors and prompt neutrons are not in their quasi-equilibrium state so that the dynamical characteristics of a reactor determine time rates of population changes inside the reactor. In general the data related to delayed neutrons are calculated from other reactor variables or experimental results, since it is very difficult to measure directly the delayed-neutron precursor density that is essentially required to control reactors.

Recently a series of studies of sensitivity analyses based on point kinetic theory has been performed to estimate variations in dynamical responses of a nuclear reactor caused by uncertainties plagued in the delayed neutron data.^{1,2} In our previous paper, however, we have pointed out that the numerical procedure employed in Ref. [2] only works for a reactor accident model given there where the reactivity accident was assumed being occurred from an initial equilibrium state. We, however, had developed a numerical procedure to deal with a stiff differential equation involved in dynamical systems without demanding vanishing higher order derivatives of each population with respect to time for initial conditions. Our general numerical procedure enables us to investigate dynamical models for reactor accident and safety in transient phases.

In a field of reactor physics, mathematical formalism⁴ of sensitivity analysis has been well established by

introducing an inner product with adjoint systems, which enable one to take Frechet derivative with respect to sensitivity parameter for a purpose of optimizations in a parameter space. In a due course of our previous numerical study of point kinetics equations, however, we learned that there must be a method to way around tedious computational jobs that just require to change sensitivity parameters in the same mathematical equations. Our major motivation of this paper is to investigate reactor responses as sensitivity parameter changes.

In Sec. II we address the point kinetics equation with emphasis on its role in sensitivity theory. A reactor accident model and computational methods are described in Sec. III for sensitivity analyses. We make our conclusion after presenting our results and discussions in Sec. IV.

II. Sensitivity Analysis Based on Response Function

Point kinetics equation can be written in a set of first order differential equations

$$\frac{d}{dt} \vec{y}(t) = A(t) \vec{y}(t) + \vec{s}(t) \quad (1)$$

where t is a time variable, $\vec{y}(t)$ and $\vec{s}(t)$ represent the populations of each particles and the external neutron source, respectively, and initially the system has populations \vec{y}_0 . The coefficient matrix $A(t)$ is

$$A(t) \equiv \begin{pmatrix} l_{eff}^{-1} [(1 - \beta_{eff})k(t) - 1] & \lambda_1 & . & . & . & \lambda_4 \\ l_{eff}^{-1} \beta_{eff} k(t) & -\lambda_1 & 0 & . & . & 0 \\ . & 0 & -\lambda_2 & 0 & . & 0 \\ . & . & 0 & . & 0 & 0 \\ . & . & . & 0 & . & 0 \\ l_{eff}^{-1} \beta_{eff} k(t) & . & . & . & 0 & -\lambda_4 \end{pmatrix} \quad (2)$$

Since the general solution to Eq. (1) can be obtained from a sum of a solution of the homogeneous equation and a special solution of the inhomogeneous equation, we can write an analytic solution in an integration form as follows³

$$\vec{y}(t) = e^{\int_0^t A(\xi) d\xi} \left[\vec{y}_0 + \int_0^t e^{-\int_0^\tau A(\xi) d\xi} \vec{s}(\tau) d\tau \right] \quad (3)$$

The first term of Eq. (3) represents the time evolution of a dynamical system due to internal mechanisms, on the other hand, the second term of Eq. (3) can be regarded as a system's response caused by an external source.

If we limit our interests of sensitivity analysis in a dynamical system with no external sources, we can ignore the source term in right hand side of Eq. (1). Taking a variational derivative of Eq. (1) with respect to α , we can find equations for sensitivity analysis

$$\frac{\delta}{\delta \alpha} \left(\frac{d}{dt} \vec{y}(t; \alpha) \right) = \frac{\delta A(t; \alpha)}{\delta \alpha} \vec{y}(t; \alpha) + A(t; \alpha) \frac{\delta}{\delta \alpha} \vec{y}(t; \alpha) \quad (4)$$

Because independent variables are not affected by the sensitivity parameter α , the variational derivative is commute with the time derivative. A set of differential equations involving the first order of sensitivity parameter α reads as follows

$$\left(\frac{d}{dt} - A(t; \alpha) \right) \frac{\delta}{\delta \alpha} \vec{y}(t; \alpha) = \frac{\delta A(t; \alpha)}{\delta \alpha} \vec{y}(t; \alpha) \quad (5)$$

Thus the equation is very similar to the point kinetics equation with an external source if we regard the right hand side of Eq. (5) as an external source term.

Assuming that the initial conditions of the dynamical system are not influenced by sensitivity parameters, that is initially no sensitivity involved, the response sensitivity of particle populations can be written in an integration form,

$$\frac{\delta \vec{y}}{\delta \alpha} = e^{\int_0^t A(\xi, \alpha) d\xi} \left[\left(\frac{\delta \vec{y}}{\delta \alpha} \right)_0 + \int_0^t e^{-\int_0^\tau A(\xi, \alpha) d\xi} \frac{\delta A(\tau, \alpha)}{\delta \alpha} \vec{y}(\tau, \alpha) d\tau \right] \quad (6)$$

Practically it is much more convenient to do numerical computations with the differential form than its integral counterpart.

III. Numerical Calculations with Reactor Accident Model

Since the same reactor and accident model used in Ref. [3] are adapted for a purpose of comparing numerical results obtained in this study, we make a brief description of the model in this section. A fast reactor is assumed in its normal operation with 1250 MW thermal power. At $t = 0$, a reactivity accident is occurred and the reactor increases its temperature adiabatically by assuming no circulation of coolant in primary loops. Notice that we are interested in variations of events of the maximum reactor power in sensitivity parameters. We can make a computational model by shutting down the reactor abruptly at a certain time t when the reactor temperature reaches 573 K equivalent to accumulated thermal energy of 1061.55 MJ.

In our model, we assume that the reactivity is increased with ramp rate after the accident

$$k(t) = 1 + \Delta k t \quad (7)$$

We make a mathematical model² for uncertainties in delayed neutrons data by introducing sensitivity parameter $K_\beta = 1 + \alpha$ and time evolution constants

$$\beta_{eff}(t) = K_\beta (\beta_{eff})_0 + Q_\beta t \quad (8)$$

$$\beta_{ieff}(t) = K_\beta (\beta_{ieff})_0 + Q_{i\beta} t \quad (9)$$

where we assume that $Q_\beta = 6 Q_{i\beta}$.

By making use of Eq. (6) we write a first order variational derivative of populations with respect to α in a simple recursive form for evaluating numerical integration iteratively

$$\frac{\delta \vec{y}(t_f; 0)}{\delta \alpha} = \left(I + A(t_i; 0)(t_f - t_i) \right) \frac{\delta \vec{y}(t_i; 0)}{\delta \alpha} + \frac{\delta A(t_i; 0)}{\delta \alpha} A(t_i; 0) \vec{y}(t_i; 0) (t_f - t_i)^2 \quad (10)$$

Note that the subscripts i, f in Eq. (10) represent successive time sequences and their differences are chosen to be sufficiently small to reduce numerical errors. Variations in the time to reach peak power can be calculated from energy expression and the particle populations

$$\frac{\delta E}{\delta \alpha} \propto \frac{\delta}{\delta \alpha} \int_0^{t_m(\alpha)} n(\tau; \alpha) d\tau = 0 \quad (11)$$

$$\frac{\delta t_m(0)}{\delta \alpha} = - \frac{1}{n(t_m(0); 0)} \int_0^{t_m(0)} \frac{\delta n(\tau; 0)}{\delta \alpha} d\tau \quad (12)$$

From Eq. (12) we can calculate an approximate times to reach peak power for each sensitivity parameters

$$t_m(\alpha) = t_m(0) + \frac{\delta t_m(0)}{\delta \alpha} \delta \alpha \quad (13)$$

Combining Eq. (10) and Eq. (13), we can compute the peak power for each sensitivity parameters with variational derivative evaluated at $\alpha = 0$ explicitly.

IV. Conclusion and Discussions

The exact variations of the peak power and the time to reach the peak power are plotted in Fig. 1 and Fig. 2, respectively. The numerical result of Eq. (12), which must be equivalent to the slope of the plot shown in Fig. 2, is 7.09715×10^{-2} , since the integration part of Eq. (12) is turned out to be -9.32865×10^{14} . By exploiting this calculated slope, we summarized the time to reach peak power and deviations from their exact values in Table 3. Since the exact variations in the time to reach peak power show linearity in the sensitivity parameter, their counterparts obtained from linear approximation in the sensitivity parameter are turned out to be in small error bounds.

Sensitivity responses in nuclear reactor on uncertainties in delayed neutron data can be estimated with the first order approximation, since the time to reach the peak power depends on the first variational derivative of the peak power variations in the parameter space. The estimated variation on the time to reach peak power shows very good reliability even in cases for larger sensitivity parameters, since it depends linearly on the first order variational derivative at the peak power. It, however, should be cautious in estimating sensitivity responses on the peak power with larger sensitivity parameters where the higher order corrections are not small.

References

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Table 1 Reactor parameters used in numerical computation.

Initial reactor thermal power, P_o	1250 MW
Mean prompt neutron lifetime, l_{eff}	$0.346 \mu s$
Number of fission neutrons emitted, $\bar{\nu}_p$	2.89
Total effective delayed neutron fraction, $(\beta_{eff})_o$	3.6×10^{-3}
Conversion factor from joules to integrated fission, C	3.1×10^{10}
Mass of nuclear fuel	10.11 ton
Heat capacity of fuel, C_p	0.35 J/gK
Reactivity insertion rate, $\Delta k / s$	0.01~0.80
Time variation of β_{eff} , Q_β	5.0×10^{-4}

Table 2 Delayed neutron fractions and decay constants of each precursors.

Group (i)	$(\beta_{ieff})_o$	λ_i
1	8.3845×10^{-5}	0.0129
2	7.6095×10^{-4}	0.0311
3	6.7000×10^{-4}	0.1340
4	1.3100×10^{-3}	0.3310
5	5.9860×10^{-4}	1.2600
6	1.7640×10^{-4}	3.2100

Table 3 Deviations between approximate values and the exact ones.

K_β ($1+\alpha$)	Exact time to reach peak power (ms)	Approximately calculated time to reach peak power (ms)	Error (%)	Exact value of peak power (MW)	Approximately calculated peak power (MW)	Error (%)
0.8	63.551	63.396	-0.24	5.1154×10^5	5.0408×10^5	-1.46
0.9	70.568	70.493	-0.11	4.6592×10^5	4.6395×10^5	-0.42
1.0	77.590	Reference	NA	4.2386×10^5	Reference	NA
1.1	84.614	84.687	+0.08	3.8497×10^5	3.8378×10^5	-0.31
1.2	91.637	91.784	+0.16	3.4898×10^5	3.4364×10^5	-1.53
1.3	98.655	98.881	+0.23	3.1555×10^5	3.0353×10^5	-3.81
1.4	105.667	105.979	+0.30	2.8480×10^5	2.6343×10^5	-7.50

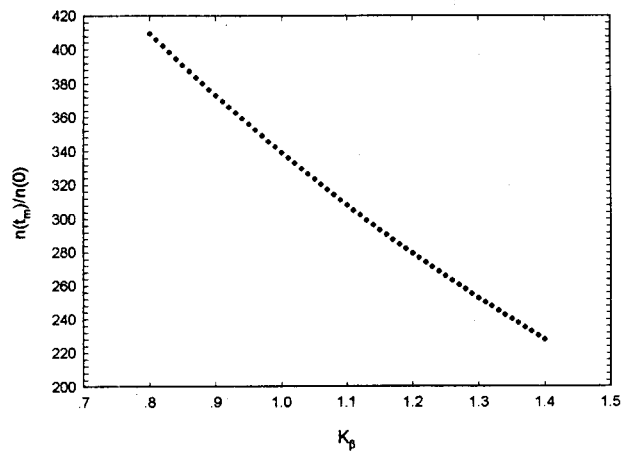


Figure 1 Exact variations of peak powers vs. sensitivity parameters

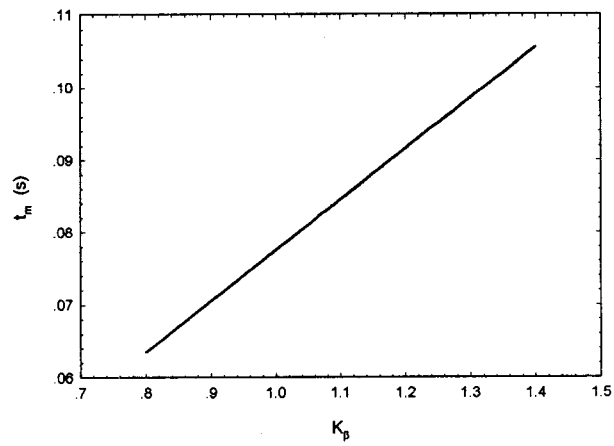


Figure 2 Exact variations in time to reach peak power vs. sensitivity parameters