Wave Propagation in Unidirectionally Reinforced Composites

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Abstract

Wave propagation was studied for an unidirectionally reinforced composite materials. The velocities, the particle directions and the amplitudes of reflected and transmitted waves were obtained. This analysis involves an immersion C-scan procedure. Snell's law was modified to get the velocities of waves. This analysis could be applied to the detection of flaws in a transversely isotropic composite motor case.

1. Introduction

Some motor cases were made by composite materials using filament winding technique. Due to the complexity of manufacturing, flaws were found. Those flaws could be bigger and finally become critical cites to failure. Ultrasonic nondestructive testing works well for detecting gross composites defects. Porosity, local variation in fiber orientation, segregation of reinforcing fibers are difficult to identify with conventional data analysis procedures. Since defects such as these will principally affect the local moduli, ultrasonic velocity measurements are quite useful in analyzing these types of problems[1]. The ultrasonic tests are required to examine directional dependence of the properties by measuring all pertinent elastic moduli. The ultrasonic tests were used to completely characterize all five elastic moduli for transversely isotropic materials and all nine elastic moduli of orthotropic materials[2-5].

Previous investigators used contact transducers at normal incidence. These approaches are not

suitable for the scanning of large parts due to the difficulties of maintaining shear coupling as the transducer is scanned. Immersion transducer with mode conversion to generate the required waves is another alternative approach. Unfortunately, the mode conversion approach to the generation of waves in an anisotropic media is significantly more complicated than the isotropic case. Since the generated waves do not have their normal along symmetry axes, they will not usually be pure mode waves but rather quasilongitudinal or quasitransverse waves. Furthermore, as has been observed many times in anisotropic media, the energy flux associated with wave propagation will often deviate from the wave normal complicating the problem considerably. A further problem stems from the variation in wave speed with propagation direction in the composite and the associated problem with determining the angle of refraction via Snell's law.

The principal objective of this work was to develop a simplified method for analyzing reflection-refraction phenomena in transversely isotropic materials for arbitrary angles of incidence. Ideally, a method was sought which was rapid, accurate and sufficiently compact to be implemented on a laboratory microcomputer so that it would be useful for the detection of flaws of transversely isotropic composite motor case.

2. Governing Equation

First, it is useful to review the equation of motion and the constitutive equation. The equation of motion is

$$\sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i \qquad (1)$$

where, σ = stress u = displacement
ρ = density b = body force.

The constitutive equation is

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \qquad (2)$$

where, c = the stiffness matrix
ε = strain.

and the strain and displacement relationship is

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (3).$$

Substituting equations (2) and (3) into equation (1) without body force gives the following equation

$$\rho \ddot{u}_i = c_{ijkl}u_{k,lj} \qquad (4)$$

The displacement may be represented as

$$u_i = A_0\alpha_i e^{i(kl_jx_j - \omega t)} \qquad (5)$$

where, k = wave number
l = wave normal vector
ω = frequency
A_0 = amplitude
α = polarization vector

Substituting equation (5) into equation (4) gives

$$-\rho\omega^2 u_i = c_{ijkl}(-k^2 l_l l_j)u_k \qquad (6)$$

Rearranging equation (6) gives the governing equation as

$$(\underset{\approx}{\lambda}-\rho v^2 \underset{\approx}{I})\underset{\approx}{\alpha}=0 \qquad (7)$$

where, I = identity matrix

v = velocity

$\lambda_{ik} = c_{ijkl} l_j l_l$.

3. Application to orthotropic medium

The wave normal for the refracted waves in the orthotropic media is

$$\tilde{l} = \begin{pmatrix} \cos\theta' \\ 0 \\ \sin\theta' \end{pmatrix}$$

Once the wave normal vector is found, $\underset{\approx}{\lambda}$ can be evaluated as

$$\underset{\approx}{\lambda} = \begin{bmatrix} C_{11}\cos^2\theta' + C_{55}\sin^2\theta' & 0 & C_{13}\sin\theta'\cos\theta' + C_{55}\sin\theta'\cos\theta' \\ 0 & C_{66}\cos^2\theta' + C_{44}\sin^2\theta' & 0 \\ C_{13}\sin\theta'\cos\theta' + C_{55}\sin\theta'\cos\theta' & 0 & C_{55}\cos^2\theta' + C_{33}\sin^2\theta' \end{bmatrix} \qquad (8)$$

In solving the eigenvalue problem, the most difficult problem is that the directional cosines of the refracted wave can not be determined from Snell's law because of the directional dependence of the wave velocities.

Defining the slowness vector as

$$\tilde{m} = \frac{1}{\omega}\tilde{k} = \frac{1}{v}\tilde{l}$$

where, $\tilde{k}$ = wave vector = $|\tilde{k}|\tilde{l}$

k = $|\tilde{k}|$ = wave number

$\tilde{l}$ = wave normal.

The slowness surface represents the locus of the endpoints of the slowness vectors. For anisotropic media, there are three distinct sheets of arbitrary shape. The shape of slowness surface is an important factor in determining the nature of reflected and refracted waves in anisotropic media.

The problem under consideration consists of a plane longitudinal wave in water incident upon the boundary of a unidirectional composite panel.

For the cases to be studied, the wave vector for the incident wave lies in a plane either parallel or perpendicular to the fiber reinforcement.

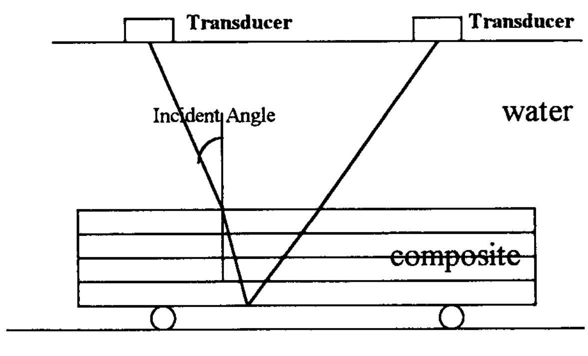

Fig. 1 Experimental Arrangement

With this geometry, the incident wave may be represented as

$$\tilde{U}_{in} = \tilde{A} e^{i\omega^{in}(m_k^{in}x_k - t)}$$

Similarly, the reflected longitudinal wave may be represented as

$$\tilde{U}_{re} = \tilde{A}_{re} e^{i\omega^{re}(m_k^{re}x_k - t)}$$

For the transmitted waves, we have

$$\tilde{U}^{i} = \tilde{A}_{i} e^{i\omega^{i}(m_k^{i}x_k - t)}$$

where, the superscript i is used to differentiate between the transmitted work. In order to satisfy the boundary conditions at the interface, the frequencies of all waves must be equal, i.e.

$$\omega = \omega^{in} = \omega^{re} = \omega^{i}$$

and

$$m_k^{in}x_k = m_k^{re}x_k = m_k^{i}x_k \qquad (9)$$

which is equivalent to

$$a_i = \varepsilon_{ijk} m_j^{in} v_k = \varepsilon_{ijk} m_j^{re} v_k = \varepsilon_{ijk} m_j^{i} v_k$$

where,

$\tilde{v}$ = normal to interface

$\tilde{a}$ = constant vector quantity.

In the case that

$$\tilde{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and,

$$\tilde{m}^{in} = \frac{1}{V_w}\begin{pmatrix} \cos\theta^{in} \\ 0 \\ \sin\theta^{in} \end{pmatrix}$$

$$\tilde{m}^{re} = \frac{1}{V_w}\begin{pmatrix} -\cos\theta^{re} \\ 0 \\ \sin\theta^{re} \end{pmatrix}$$

$$\tilde{m}^{i} = \frac{1}{V_w}\begin{pmatrix} \cos\theta^{i} \\ 0 \\ \sin\theta^{i} \end{pmatrix}$$

where, the negative sign in the slowness vector of the reflected wave is included to indicate that it is propagating away from the interface, then we have the Snell's law as

$$\frac{\sin\theta^{in}}{V_w} = \frac{\sin\theta^{re}}{V_w} = \frac{\sin\theta^{i}}{V_i} \qquad (10)$$

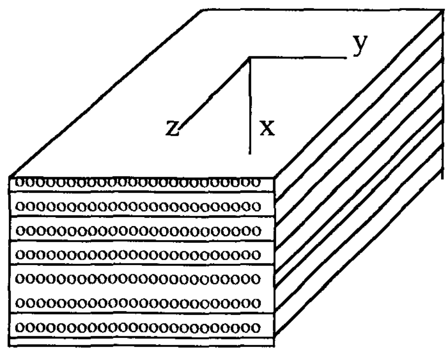

Fig.2 Coordinate System

4. Eigenvalue Problem

We can rewrite the Snell's law as

$$\sin\theta^{i} = \frac{V_i}{V_w}\sin\theta^{in} = Kv_i$$

for each mode, then equation (8) becomes as

$$\begin{bmatrix} c_{11}\left[1-(Kv_i)^2\right]+c_{55}(Kv_i)^2-\rho v_i^2 & 0 & (c_{13}+c_{55})[1-(Kv_i)^2]^{\frac{1}{2}}Kv_i \\ 0 & c_{66}\left[1-(Kv_i)^2\right]+c_{44}(Kv_i)^2-\rho v_i^2 & 0 \\ (c_{13}+c_{55})[1-(Kv_i)^2]^{\frac{1}{2}}Kv_i & 0 & c_{55}\left[1-(Kv_i)^2\right]+c_{33}(Kv_i)^2-\rho v_i^2 \end{bmatrix} = 0$$

Since the pure mode shear wave could not be excited in the experimental arrangement, we restrict attention to the characteristic equation of remaining two waves as

$$\left\langle \{c_{11}\left[1-(Kv_i)^2\right]+c_{55}(Kv_i)^2-\rho v_i^2\}\{c_{55}\left[1-(Kv_i)^2\right]+c_{33}(Kv_i)^2-\rho v_i^2\}\right\rangle$$
$$-(c_{13}+c_{55})^2(Kv_i)^2\left[1-(Kv_i)^2\right]=0$$

This is a simple equation for v_i^2 which may be solved numerically for the two real roots.

5. Energy Flux Deviation

Once the velocities associated with quasilongitudinal(QL) and quasitransverse (QT) wave propagation have been determined, it is possible to calculate the energy flux vector for each mode of propagation. Unlike the propagation along a symmetry axis, the energy flux vector for wave propagation in an arbitrary direction does not coincide with the wave normal.

The energy flux vector is given by

$$E_j = -\sigma_{ij}\dot{u}_i \qquad (11)$$

Hence, calculation of the eigenvectors for each mode is required. Then the particle displacements become as

$$u_i = \alpha_i A e^{-i[\omega t - k(\cos\theta * x + \sin\theta * z)]} \qquad (12)$$

The stresses are then determined from the linearly elastic constitutive equation for the composite. For an orthotropic medium, the directional cosines for the energy flux vector then become

$$\frac{E}{|E|} = -\begin{pmatrix} \frac{E_1}{|E|} \\ 0 \\ \frac{E_3}{|E|} \end{pmatrix} \quad (13)$$

Using equations (2) and (12), we can evaluate equation (13) as

$$E_1 = c_{11}\alpha_1^2\cos\theta + (c_{13} + c_{55})\alpha_1\alpha_3\sin\theta + c_{55}\alpha_3^2\cos\theta$$

$$E_3 = c_{33}\alpha_3^2\sin\theta + c_{13}\alpha_1\alpha_3\cos\theta + c_{55}\alpha_1^2\sin\theta + c_{55}\alpha_1\alpha_3\cos\theta$$

and $|E| = \sqrt{E_1^2 + E_3^2}$.

6. Amplitude considerations

The particle displacements for the incident wave are represented as

$$\tilde{U}_{in} = A_0\begin{pmatrix} \cos\theta^{in} \\ 0 \\ \sin\theta^{in} \end{pmatrix} e^{ik'[\cos\theta^{in}x+\sin\theta^{in}z-\omega t]}$$

Similarly, the particle displacements for the reflected wave are

$$\tilde{U}_{re} = A_r\begin{pmatrix} -\cos\theta^{re} \\ 0 \\ \sin\theta^{re} \end{pmatrix} e^{ik'[-\cos\theta^{re}x+\sin\theta^{re}z-\omega t]}$$

For wave propagation in the composite, similar expressions for the generated quasilongitudinal and quasitransverse waves are

$$\tilde{U}_{QL} = A_{QL}\begin{pmatrix} \alpha_1^{QL} \\ 0 \\ \alpha_3^{QL} \end{pmatrix} e^{ik_{QL}[\cos\theta^{QL}x+\sin\theta^{QL}z-\omega t]}$$

$$\tilde{U}_{QT} = A_{QT}\begin{pmatrix} \alpha_1^{QT} \\ 0 \\ \alpha_3^{QT} \end{pmatrix} e^{iK_{QT}[\cos\theta^{QTL}x+\sin\theta^{QT}z-\omega t]}$$

Here, the eigenvectors $\tilde{\alpha}^{QL}$ and $\tilde{\alpha}^{QT}$ are perpendicular to one another, but in general $\tilde{\alpha}^{QL} \star \tilde{l}^{QL} \neq 1$ and $\tilde{\alpha}^{QT} \star \tilde{l}^{QT} \neq 0$. Therefore they are called as quasilongitudinal and quasitransverse waves. Three boundary conditions at the fluid solid interface are required to calculate the reflection and transmission coefficients at the interface.

(1) Continuity of Normal Displacement

$$u_{water}\big|_{x=0} = u_{composite}\big|_{x=0}$$

which leads to an expression of Snell's law for the composite as before as well as the relationship

$$A_0\cos\theta^{in} - A_r\cos\theta^{re} = A_{QL}\alpha_1^{QL} + A_{QT}\alpha_1^{QT}$$

(2) Continuity of Normal Stress

$$\sigma_{11(water)}\Big|_{x=0} = \sigma_{11(composite)}\Big|_{x=0}$$

which becomes as

$$\lambda k_l\left[\left(\cos^2\theta^{in}+\sin^2\theta^{in}\right)A_0+\left(\cos^2\theta^{e}+\sin^2\theta^{e}\right)A_r\right]$$
$$=c_{11}\left[A_{QL}\alpha_1^{QL}k_{QL}\cos\theta^{QL}+A_{QT}\alpha_1^{QT}k_{QT}\cos\theta^{QT}\right]$$
$$+c_{13}\left[A_{QL}\alpha_3^{QL}k_{QL}\sin\theta^{QL}+A_{QT}\alpha_3^{QT}k_{QT}\sin\theta^{QT}\right]$$

(3) Zero Transverse Stress

Since the fluid can not support a shear stress,

$$\sigma_{13(water)}\Big|_{x=0} = 0 = \sigma_{13(composite)}\Big|_{x=0}$$

which becomes as

$$A_{QL}k_{QL}\left(\alpha_1^{QL}\sin\theta^{QL}+\alpha_3^{QL}\cos\theta^{QL}\right)+$$
$$A_{QT}k_{QT}\left(\alpha_1^{QT}\sin\theta^{QT}+\alpha_3^{QT}\cos\theta^{QT}\right)=0$$

Those yields three equations in three unknowns which can be solved for the reflection and transmission coefficients.

Rearranging the above three expressions gives in matrix form as

$$\begin{bmatrix} \cos\theta^{r} & \alpha_1^{QL} & \alpha_1^{QT} \\ 0 & c_{22} & v_{QL}\left(\alpha_3^{QT}\cos\theta_{QT}+\alpha_1^{QT}\sin\theta_{QT}\right) \\ -\lambda v_{QL}v_{QT} & c_{32} & c_W v_{QL}\left(c_{11}\alpha_1^{QT}\cos\theta_{QT}+c_{13}\alpha_3^{QT}\sin\theta_{QT}\right) \end{bmatrix} \begin{Bmatrix} A_{re}/A_{in} \\ A_{QL}/A_{in} \\ A_{QT}/A_{in} \end{Bmatrix} = \begin{Bmatrix} \cos\theta_{in} \\ 0 \\ \lambda v_{QL}v_{QT} \end{Bmatrix}$$

$c22=v_{QT}\left(\alpha_3^{QL}\cos\theta_{QL}+\alpha_1^{QL}\sin\theta_{QL}\right)$

c32=

$c_W v_{QT}\left(c_{11}\alpha_1^{QL}\cos\theta_{QL}+c_{13}\alpha_3^{QL}\sin\theta_{QL}\right)$

7. Results and Consideration

T300 Carbon Fiber / 5208 Epoxy composite was studied for wave propagation at oblique angles incidence. The stiffness coefficients are as

$$\tilde{C} = \begin{bmatrix} 11.2 & 5 & 7.1 & 0 & 0 & 0 \\ 5 & 11.2 & 7.1 & 0 & 0 & 0 \\ 7.1 & 7.1 & 153 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.1 \end{bmatrix} x10^9\frac{kg}{m^2}$$

and the density $\rho = 1.8\frac{g}{cm^3}$.

The oblique incident beam was projected to a unidirectionally reinforced composite in the reinforcement plane parallel and perpendicular to the fiber axis.

A) x-y plane (isotropic plane)

(1) longitudinal wave slowness radius:

$$R_l = \frac{1}{V_1} = \sqrt{\frac{\rho}{C_{11}}} = 0.4mm$$

(2) SV slowness radius:

$$R_{SV} = \frac{1}{V_2} = \sqrt{\frac{\rho}{C_{44}}} = 0.5mm$$

(3) SH slowness radius:

$$R_{SH} = \frac{1}{V_3} = \sqrt{\frac{\rho}{c_{66}}} = 0.76mm$$

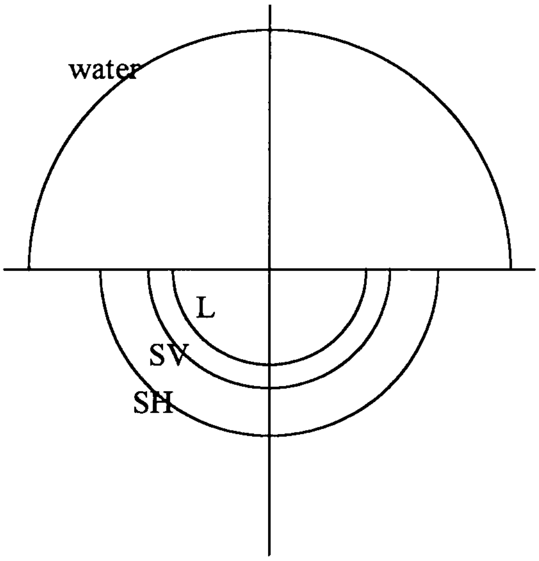

Fig.3 Slowness surface (normal to fiber)

B) x-z plane

(1) S-H wave

$$R_{SH}(\theta) = \frac{1}{V_{SH}} = \sqrt{\frac{\rho}{c_{44}\sin^2\theta + c_{66}\cos^2\theta}}$$

$$= \sqrt{\frac{1.8}{7.1\sin^2\theta + 3.1\cos^2\theta}} x10^{-3}mm$$

(2) quasilongitudinal velocity(v_{QL}) and quasitransverse velocity(v_{QT})

$$R_{QL}(\theta) = \frac{1}{V_{QL}} = \sqrt{\frac{2\rho}{M + \sqrt{M^2 - 4N}}}$$

$$R_{QT}(\theta) = \frac{1}{V_{QT}} = \sqrt{\frac{2\rho}{M - \sqrt{M^2 - 4N}}}$$

where,

$$M = c_{11}\cos^2\theta + c_{33}\sin^2\theta + c_{44}$$

$$N = (c_{11}\cos^2\theta + c_{44}\sin^2\theta)(c_{11}\cos^2\theta + c_{44}\sin^2\theta) - (c_{13} + c_{44})^2\sin^2\theta\cos^2\theta$$

So, according to incident angle (θ), the slowness surfaces are obtained.

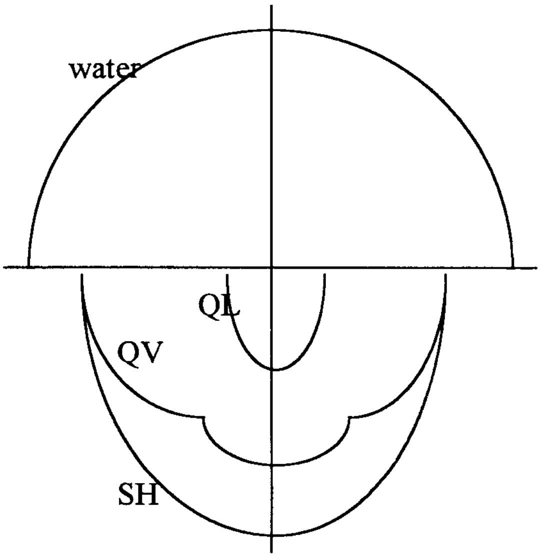

Fig. 4 Slowness surface (parallel to fiber)

8. Conclusions

Wave propagation features in transversely isotropic materials were evaluated for a microcomputer based technique. Using oblique angles of incidence, important information about laminate properties could be obtained. This approach may be used in a scanning mode to detect local flaws of a big composite motor case if the shell effect does not play an important role.

9. References

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