SMOLDERING IGNITION OF FLAMMABLE SUBSTRATE

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ABSTRACT

A theoretical model for the interaction of the moving heat source and a solid substrate when they are in contact is described. For purposes of the model the substrate is assumed to act as a continuum and the Fourier equation for transient, three-dimensional conduction is solved using Laplace and Fourier transformations. Unlike most previous models, this model shows the explicit relations between the properties of heat source and that of the substrate. Since the size, shape and speed of heat source impact the ignition of substrate, considerable attention is devoted to evaluating these parameters. Results are presented which show the effects of the size, shape and speed of heat source on the substrate.

INTRODUCTION

When a substrate, such as a upholstered furniture, is in contact with a moving heat source such as a burning cigarette the continuous flux of heat to the substrate results in a temperature increase at any point in the substrate. If and when the temperature of substrate rise to a critical temperature for the initiation of smoldering, the ignition occurs.

Almost 50,000 fires initiated from the cigarette ignition in 1984. This caused an estimated 1,530 deaths and 3,950 injuries[1]. To minimize the casualties from the fires started from the cigarette, Public Laws, the "Cigarette Safety Act 1984 and 1990" were passed by the Congress.

NIST has conducted cigarette ignition propensity studies under the Acts. One of the tasks under the Acts was to model mathematically the behavior of upholstered furniture and of the cigarette, when a smoldering cigarette is dropped on it. As results, two prototype computer programs, TMPSUB[2] and TEMPSUB[3] were developed. Both computer programs calculate the temperature of the upholstered furniture as a function of time and position, when it is exposed to a prescribed heating flux. This heating flux can be taken to be a moving source.

In this study, we concentrated on modeling the temperature distribution in the substrate in three dimensions in terms of the net heat flux, Q_{net}. The effects of model parameters of the temperature distribution on a flexible PU foam covered by #12 cotton duck are performed.

FORMULATION OF THE MODEL

When the heat source moves, the problem must be treated in three dimensions. When the heated spot moves in a straight line along the surface, the problem simplifies slightly. To formulate the transient, three-dimensional model following assumptions can be made;

1) During natural smoldering, the solid and gas phases inside the coal* have the same temperature. Baker[4] has demonstrated that the temperature distribution of the solid and gas phases inside the burning cigarette are in near thermal equilibrium during the inter puff smolder period.

2) During natural smoldering the linear burn rate is a constant.

3) The net heat flux, Q, from a burning cigarette at the surface of a substrate is a function of the spatial variables, x and y.

The heat from the cigarette is transferred to the media through the window, W, at the top of the substrate, which is initially at room temperature, T_o. Figure 1 shows the schematic diagram of the heat transfer form the cigarette to the substrate through the window.

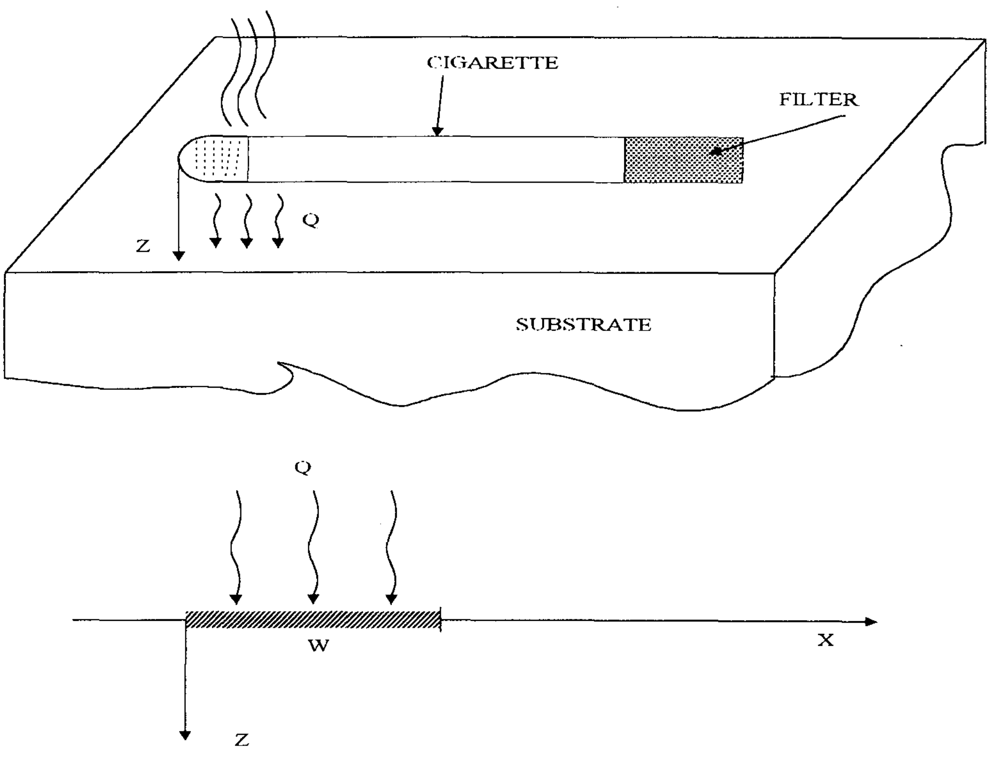

Figure 1. Schematic diagram of heat transfer from the smoldering cigarette to the substrate through the windows, W.

* The burning cone at the lighted end of a cigarette.

Utilizing the first law of thermodynamics, the general equation for heat conduction through a solid medium is[5]

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+S=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

This equation relates the time rate of change of temperature at a point, to the temperature gradients at that point. Here S is any internal heat source (or sink), and α is the thermal diffusivity of the substrate. To determine the temperature in the substrate, the solution of Eqn. (1) in the infinite region was found. The initial and boundary conditions of the Eqn. (1) are, at any point,

$$T(x,y,z,t=0)=T_O \tag{2}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0}=-\frac{Q_{net}(x,y)}{k} \qquad (x,y)\in W \tag{3}$$

$$\lim_{z\to\infty} T(x,y,z,t)=T_o \tag{4}$$

Defining $U=T-T_o$ and thermal diffusivity, a, includes the effect of heat of reaction through the effective heat capacity, we find that U satisfies the partial differential equation for the substrate

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=\frac{1}{\alpha}\frac{\partial U}{\partial t} \tag{5}$$

with the initial condition

$$U(x,y,z,t)=0 \tag{6}$$

and the boundary condition

$$\left.\frac{\partial U}{\partial z}\right|_{z=0}=-\frac{Q_{net}(x,y)}{k} \qquad (x,y)\in W \tag{7}$$

If the X coordinate, moving at the same speed, V, and in the same direction as the heat flux i.e., window W, is taken along the longitudinal axis of the cigarette, the temperature profile will be stationary with respect to X coordinate. Consequently, Eqn. (5) can be rewritten as Eqn. (9), using the relationships among the X coordinate, x coordinate, and the smoldering speed.

$$X=x-Vt;\quad Y=y;\quad Z=z,$$
$$\frac{\partial X}{\partial x}=1;\quad \frac{\partial X}{\partial t}=-V;\quad \frac{\partial \tau}{\partial t}=1 \tag{8}$$

$$\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}+\frac{\partial^2 U}{\partial Z^2}+\frac{V}{\alpha}\frac{\partial U}{\partial X}=\frac{1}{\alpha}\frac{\partial U}{\partial \tau} \tag{9}$$

Let

$$U=\phi e^{-\frac{VX}{2\alpha}} \tag{10}$$

then by substitution in Eqn. (9), a partial differential equation in ϕ and resulting boundary conditions are as follows:

$$\frac{\partial^2 \phi}{\partial X^2}+\frac{\partial^2 \phi}{\partial Y^2}+\frac{\partial^2 \phi}{\partial Z^2}-\frac{V^2}{4\alpha^2}\phi=\frac{1}{\alpha}\frac{\partial \phi}{\partial \tau} \tag{11}$$

$$\phi(X,Y,Z,\tau\le 0)=0 \tag{12}$$

$$\lim_{Z\to\infty}\phi(X,Y,Z,\tau)=0 \tag{13}$$

$$\left.\frac{\partial\phi}{\partial Z}\right|_{Z=0} = \frac{Q_{net}(X,Y)}{k}e^{\frac{VX}{2\alpha}} \qquad (X,Y)\in W \tag{14}$$

Taking the Laplace transform with respect to t and the Fourier transform with respect to the variables, X and Y to solve Eqn. (11) gives;

$$U(X,Y,Z,\tau) = e^{-\frac{VX}{2\alpha}}\Phi(X,Y,Z,\tau) \tag{15}$$

where

$$\Phi(X,Y,Z,\tau) = \frac{1}{4\pi^{\frac{3}{2}}k\alpha^{\frac{1}{2}}}\int_0^a\int_{-\eta(\xi)}^{\eta(\xi)} Q_{net}(\xi,\eta)e^{\frac{V\xi}{2\alpha}}\int_0^\tau \frac{e^{-\left[\left\{\frac{(\xi-X)^2+(\eta-Y)^2+Z^2}{4\alpha\tau}\right\}+\frac{V^2\tau}{4\alpha}\right]}}{\tau^{\frac{3}{2}}}d\tau d\eta d\xi \tag{16}$$

For the case that the window shape through which heat is transferred to the substrate is elliptic

$$\eta(\xi) = \pm b\left[1-\frac{\xi^2}{a^2}\right]^{\frac{1}{2}} \tag{17}$$

where a is a coal length and b is a cigarette radius.

Eqn. (16) describes the temperature of substrate relative to room temperature as a function of the spatial variables, x, y, and z, time, t, substrate properties, k, and α, the heat flux, Q_{net}, and the smoldering speed, V. The temperature profile at any fixed point in the substrate has a local maximum at the time corresponding to the solution of Eqn. (18)

$$\int_0^a\int_{-\eta(\xi)}^{\eta(\xi)} Q_{net}(\xi,\eta)e^{\frac{V\xi}{2\alpha}}\left[\frac{e^{-\gamma_1}}{\tau^{\frac{3}{2}}} + \int_0^\tau\left(V^2 - \frac{V(\xi-X)}{2\alpha\tau'}\right)\frac{e^{-\gamma}}{\tau'^{\frac{3}{2}}}d\tau'\right]d\xi d\eta = 0 \tag{18}$$

where

$$\gamma_1 = \frac{(\xi-X)^2+(\eta-Y)^2+Z^2}{4\alpha\tau} + \frac{V^2\tau}{4\alpha} \tag{19}$$

$$\gamma = \frac{(\xi-X)^2+(\eta-Y)^2+Z^2}{4\alpha\tau'} + \frac{V^2\tau'}{4\alpha} \tag{20}$$

Providing that the heat flux, Q_{net} and the smoldering speed, V are known, Eqn. (16) can be used to determine the temperature at any given time and position in the substrate.

THEORETICAL RESULTS AND DISCUSSIONS

Figures 2 and 3 illustrate the influence of smoldering speed of the heat source on the temperature profile of the substrate at different locations.

Both figures show the temperature increases to a maximum, then decreases to a level close to the room temperature as a heat source moves with a uniform speed. Each local maximum temperature depends on the smoldering speed of the cigarette (the lower the smoldering speed the higher the temperature). These maxima are enveloped by a curve that is independent of a smoldering speed and increases monotonically to a constant value.

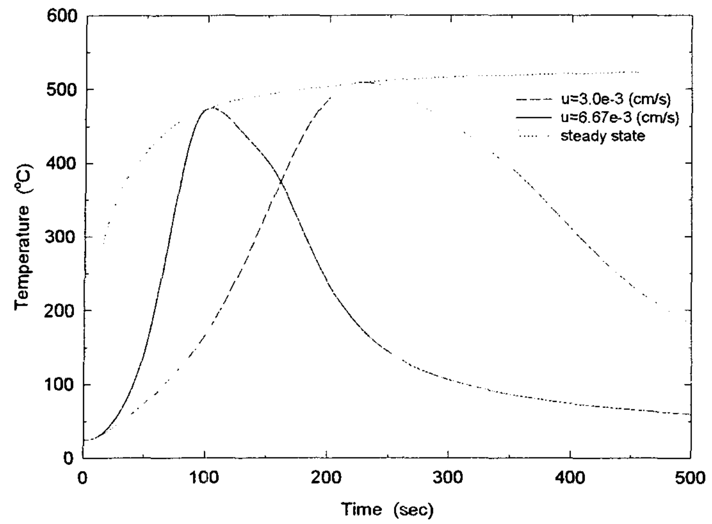

Figure 2. Time dependence of temperature at the fabric surface for different smoldering speed (x=1.5, y=0.0, z=0.0)

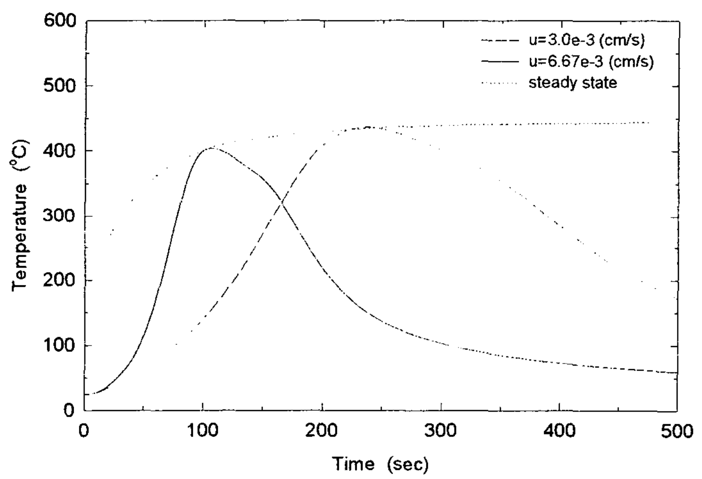

Figure 3. Time dependence of temperature at the fabric surface for different smoldering speed (x=1.5, y=0.0, z=0.0)

Figure 4 is comparison of theoretical temperature profiles at the surface of the fabric with various βs and a constant heat flux. As can be seen in figure, the theoretical analysis predicts that there is a significant difference in the temperature profiles among the variable vs. constant heat flux with respect to spatial positions on the substrate, x and y. The smaller shape factor, β, predicts the lower peak temperature since the higher heat flux. The constant heat flux shows the lower peak temperature as well as more flattened temperature profile.

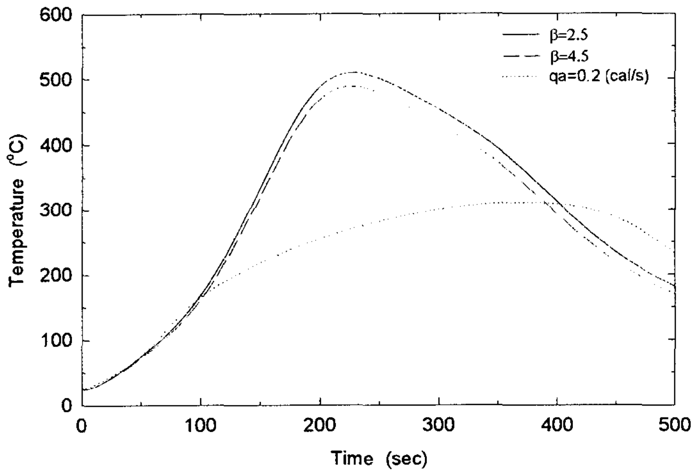

Figure 4. Influence of shape parameter and constant heat flux on theoretical temperature profile. (x=1.5, y=0.0, z=0.0)

SUMMARY AND CONCLUSIONS

This paper has outlined the development of a theoretical model for the ignition of the substrate. A transient, three-dimensional formulation of the Fourier conduction equation with subject to a moving boundary condition is solved via Laplace and Fourier transformations analytically. This model shows an explicit relations between the properties of a burning cigarette such as a smoldering speed, a coal diameter, and a coal radius and that of the substrate such as a thermal conductivity, a density, and a specific heat. The developed model will be applied to the experimental data to investigate how the cigarette properties will influence its ignition propensity.

ACKNOWLEDGEMENT

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SYMBOLS

Q	heat flux (W/m^2)
S	internal heat source(sink) (W/m^3)
T	temperature (K)
U	temperature difference, Eqn. (5)
V	smoldering speed (m/s)
X	moving coordinate

Y,Z coordinates; same as y and z
W window
a coal length (m)
b cigarette radius (m)
h convective heat loss coefficient (W/m^2K)
k thermal conductivity (W/mK)
t time (s)
x, y, z coordinates

Greek symbols

α thermal diffusivity of the substrate (m^2/s)
β shape parameter of the burning zone
γ parameter in equations (19) and (20)
ε emissivity of the substrate surface
Φ solution of equation (16)
η transformed coordinate of x
σ Stefan-Boltzmann constant(W/m^2K^4)
τ time (s)
ξ transformed coordinate of y

Subscript

c convective
cig cigarette
g gas
net net value
r radiative
s substrate
o initial

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