# Numerical Method for Computing the Resonant Frequencies and Q-factor in Microwave Dielectric Resonator

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#### **Abstract**

The dielectric resonators(DRs) with dielectric properties are widely used in microwave integrated circuit(MICs) and monolithic microwave integrated circuits(MMICs). The variational method as numerical simulation scheme would be applied to calculate the resonant frequencies( $f_r$ ) and Q-factors of microwave dielectric resonators. The dielectric resonator with a cylindrical "puck" structure of high dielectric material is modeled in this simulation. The parameters, such as the diameter, the height, and the dielectric constant of dielectric resonator, would determine the resonant frequency and the Q-factor. The relationship between these parameters would effect each other to evaluate the approximate resonant frequency. This simulation method by the variational formula is very effective to calculate  $f_r$  and Q-factor in high frequency microwave dielectric resonator. The error rate of the simulation results and the measured results would be considered to design the microwave dielectric resonators.

#### I. Introduction

Dielectric resonator, which was known as DR, was a kind of passive component to fix certain frequency in the microwave systems. It was introduced by Richtmyer in 1939. In 1960's, DR was more focused and developed by the research of Okaya[1] and Barash. In spite of very high quality-factor(Q-factor) above 10000, it was not widely used in actual applications because of poor temperature stability. However, new materials and the advanced material techniques make it possible to improve the temperature stability very much. DR is now used very popularly in coupling with microstrip line in TEM mode because of some advantages such as small size, light weight, and low cost. Especially, because the size of DR is very smaller than that of the cavity resonator operated at the same frequency, DR is very useful in microwave integrated circuits(MICs) and monolithic microwave integrated circuits(MMICs).

In applying DR for microwave application circuits, DR should be modeled and simulated with microstrip lines at first. Hence in order to simulate DR, the metal shielding is used around DR and various microstrip lines. For finding an accurate resonant frequency and Q-factor of the DR, many parameters should be considered. In this paper, the several physical parameters — the height, the circumference, the dielectric constant, and the distance of conductor over the top and under the bottom — are discussed.

There are many numerical schemes to calculate the resonant frequency and the Q-factor of DR. The variational method by Kajfez and the 2-dimensional search by the model of Itoh and Rudokas are described and compared. The first method is known to very accurate in 1% of practical resonant frequency and Q-factor than the second scheme.

#### II. Standard Approach

When the dielectric constant of DR is over 30, the reflection coefficient  $\Gamma$  of the interface between DR body and the air goes to unity and most energy is reflected in the inside of DR<sup>[2]</sup>.

$$\Gamma = \frac{n_0 - n}{n_0 + n} = \frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} \approx 1, \quad \varepsilon_r \gg 1$$

where  $n_0$ =characteristic impedance of the air and n=characteristic impedance of the dielectric material.

When some reflections are in DR, the standing waves are formed and electromagnetic resonance is occurred. By these things, the resonant frequency of DR is depended on the resonance mode, the size of DR, and the dielectric constant. In this paper, the mode  $TE_{011}$ , the basic mode of  $TE_{01\delta}$ , is used in all parts.

In case of shape, cylindrical "puck" structure of DR is measured. There are several characteristics of microwave DR; one is the circumference of DR which is getting smaller by proportional to  $1/\sqrt{\varepsilon_i}$ . Therefore, in order to reduce diameter and height of DR, it is necessary to have high dielectric constant( $\varepsilon_{*}$ ). the other is the frequency-change-constant which depends on temperature. The change of resonant frequency results in the change of diameter and dielectric by temperature. The unloaded Quality factor Qu, known as frequency selectivity in an application, has inverse proportion of loss tangent(tano) of the material. For high frequency application, more higher Qu is needed<sup>[3]</sup>.

## III. Numerical Results for the Cylindrical DR

The analysis of cylindrical DR in1. the conductor housing is starting from th2.e electromagnetic wave which propagates to z direction in cylindrical coordinates shown in Figure 1<sup>[4]</sup>.

In order to simulate cylindrical dielectric

resonator of Figure 1, the root of the Maxwell's equation is given by Bessel function. The dielectric constant and size of all regions were considered as shown in Figure 2. For more simpled modeling, the former regions, 3~5. would be ignored. In order to propagate electromagnetic waves from the center of dielectric to the outside, the finite values have to be given. Therefore, the Bessel function of the first kind is satisfied with these conditions as the root of the inside of dielectric resonator, vice versa. In the outside of DR, by increasing the direction \( \rho \) (diameter of DR), electromagnetic field must be reduced rapidly. Consequently the modified Bessel function of the second kind is the root of the Maxwell's equation. If it is assumed that the root of TEo11 has no changes of electromagnetic field in the direction  $\phi$ , electric and magnetic field are able to be assumed  $E(\rho)e^{j\omega t-rz}$  and  $H(\rho)e^{j\omega t-rz}$  which are only the function  $\rho$  and z. Those are put into Maxwell's equation and the element of  $\rho, \phi, z$ directions can be found in the inside and the out side. Then the tangential of electric field equals to that of magnetic field at the interface. Finally, the characteristic equation of TE<sub>011</sub> mode is evoked<sup>[5]</sup>.

The main parameters of DR for resonant frequency and Q-factor are the diameter and the height. Additionally, the dielectric constant and the distance of two parts, which are the conductances over the top and below bottom, influence the resonant frequency and Q-factor, respectably. The several results simulated by the 2-dimensional search and the variational method are shown from Figure 3 to Figure 6. The 2-dimensional search by Itoh and Rudokas model has some errors over 5% between experimental results. The variational method corrects the interrupted part of the 2-dimensional model and takes the other way to calculate with accuracy. Therefore, the expected resonant frequency and Q-factor by the variational method are very accurate within 1%.

## IV. Conclusion

The several parameters of DR is so important to calculate the resonant frequency and Q-factor. The expectable characteristics for the parameters can it possible to expect the new structures of DRs before the actual fabrication. The numerical scheme such as variational method shows the accuracy results. For the more, the analysis of performance of DR can be applied to MICs and MMICs.

### Reference

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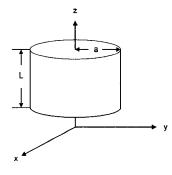


Fig.1 Cylindrical structure of DR

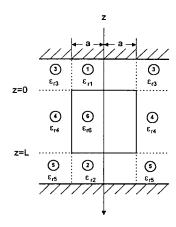


Fig.2 Shielded DR

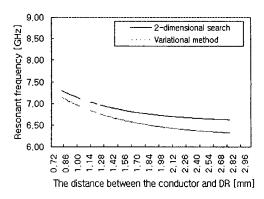


Fig.3 The distance between conductor and DR vs. resonant frequency

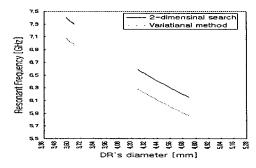


Fig.4 DR's diameter vs. resonant frequency

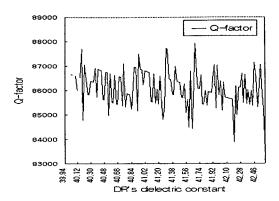


Fig.5 DR's dielectric constant vs. Q-factor

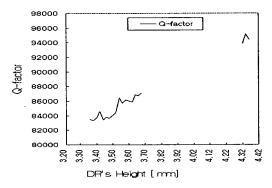


Fig.6 DR's height vs. Q-factor