FORMULATION OF THE TIDAL PREDICTION SYSTEM AND IT'S APPLICATION

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ABSTRACT

With the combination of existing tidal prediction model and numerical tidal model, the efficient tidal prediction system was formulated and applied to the neighboring area of Pusan port. Beacuse all tidal constituents for prediction (normally 69 constituents are used) can't be considered due to difficulties on computing efforts, some errors between the observed and predicted values were inevitably occurred. But it was confirmed that the practical results with about 10% of relative errors were obtained if four major tidal constituents(M_2, S_2, K_1, O_1) are used at least.

Thus, if other constituents than four major tidal constituents are additionaly used, more accurate results will be obtained. Futhermore, if the databases of harmonic constants in coastal waters is made in advance using the numerical tidal model, prompt tidal prediction could be achived whenever required.

INTRODUCTION

Tides, one of the important natural phenomena, occurred at sea are closely concerned with our life. Information about tides can be used as important data for safe navigation, efficient fishing and many kinds of oceanographic researchs. Applicable fields are very broad such as oil spill modelling, position estimation for drifting object, life saving at sea and contaminants dispersion modelling, etc. Tides are caused by the relative motion of the sun, moon and earth. Those seem to be a peridical motion with the time, but actually not a peridical one.

Those are complicated motion which is composed of many number of peridical motions, so called tidal constituents. Thus, tidal prediction can be acomplished by combining the harmonic constants of tidal constituents which can be determined by harmonic analysis based on observed tidal data for the long period of time. Like this, tidal prediction is usually undretaken on a certain point where long period of tidal observation data exist, but we generally need tidal prediction for a certain region, not a certain point.

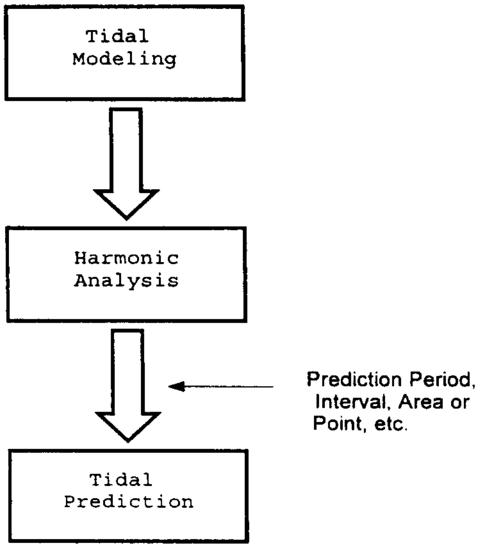

Fig. 1. Block Diagram of Tidal Prediction System.

Therefore, this paper aims, with the combination of existing tidal prediction model and numerical tidal model, to formulate the efficient tidal prediction system which can predicts tides rapidly for a certain point or region whenever required. This work can be accomplished by conventional prediction prediction method using the computing results of numerical tidal model. Say again, that is to combine the spatial prediction capability of numerical tidal model and the time prediction capability of tidal prediction model. Hereinafter, details will be presented about the outline of tidal prediction system and application example to neighboring area of Pusan port.

OUTLINE OF THE TIDAL PREDICTION SYSTEM

The numerical tidal model is used for computing the tidal elevation and currents in an area where open boundary condition is given. In this case, open boundary condition is given as water level changes of specific tidal constituent with the time and the computing results show the periodic changes of specific tidal constituent which is different from actual tides. if the computing results of all tidal constituents is combined by the tidal prediction model, actual tides can be predicted. However, it is very difficult to make tidal computation for all tidal constituents (69 constituents except for shallow water constituents) because long computing time and open boundary conditions for all constituents are required. Considerd only the effective major constituents for the concerend area, this problem can be resolved to a certain extent even though it results some error. Fig. 1 represents the block diagram of tidal prediction system.

Numerical Tidal Model

The numerical tidal model used in this study is two-dimensional depth integrated model which is widely used for computing the flow field of shallow coastal waters. The governing equations used in this model, which are derived from three-dimensional continuity equation and momentum equation by integrating them for vertical plane($-h \leq z \leq \zeta$), considering the effects of rotating earth, bottom friction and gravity as external driving forces and ignoring wind effects on water surface, are as follows.

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}[(\zeta+h)U]+\frac{\partial}{\partial y}[(\zeta+h)V]=0 \tag{1}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}-fV+g\frac{\partial \zeta}{\partial x}+\frac{gU\sqrt{U^2+V^2}}{C^2(\zeta+h)}=0 \tag{2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+fU+g\frac{\partial \zeta}{\partial y}+\frac{gV\sqrt{U^2+V^2}}{C^2(\zeta+h)}=0 \tag{3}$$

where U, V = depth averaged velocity in x, y direction, ζ = water surface elevation above Datum, h = water depth below Datum, g = gravitational acceleration, f = Coriolis parameter, and C = Chezy friction coefficient, respectively.

This model adopts ADI scheme which has been widely used for tidal computation since Leendertse(1967) for computing the above governing equations. Details about ADI scheme are omitted because it is already introduced in several publications (See Leedertse[1] and Lardner et al[2]). The open boundary condition, given as water level changes of each tidal constituent, is as follow.

$$\zeta(t)=A\cos(\omega t-\phi) \tag{4}$$

where $\zeta(t)$ = water level changes, A = amplitute of constituent, ω = angular speed of constituent, and ϕ = phase angle of constituent, respectively.

Harmonic Analysis

Through the harmonic analysis on computing results of numerical tidal model, harmonic constants of tidal elevation and currents can be obtained. The process of harmonic analysis is as follow. In first, the changes of tidal elevation and currents with the time are assumed as sinusoidal curve like Eqn. (5).

$$y(t)=acos(\omega t-\phi) \tag{5}$$

where angular velocity is given and amplitute, a and phase, ϕ are determined by least-squares approximation. The least-squares approximation is method to seek a and ϕ on the condition of minimizing the differences between the computed values and function values based on Eqn. (5).

$$E=\sum_{t=1}^{n}[y(t)-acos(\omega t-\phi)]^2 \tag{6}$$

From the Eqn. (6), the equations to seek a and ϕ on the condition of minimizing E are as follows.

$$\frac{\partial E}{\partial a} = -2\sum_{t=1}^{n}\cos(\omega t-\phi)[y(t)-a\cos(\omega t-\phi)]=0 \qquad (7)$$

$$\frac{\partial E}{\partial \phi} = 2\sum_{t=1}^{n}a\sin(\omega t-\phi)[y(t)-a\cos(\omega t-\phi)]=0 \qquad (8)$$

Because the above equations are not linear simultaneous equation, they have to be transformed as follows, that is, being started from transformation of Eqn. (1).

$$\begin{aligned} y(t) &= a\cos(\omega t-\phi) \\ &= a\cos\omega t\cos\phi + a\sin\omega t\sin\phi \\ &= \alpha\cos\omega t + \beta\sin\omega t \qquad (9) \end{aligned}$$

where $\alpha = a\cos\phi$ and $\beta = a\sin\phi$. Because α and β are regardless of time, a and ϕ can be calculated from following equations (10) when α and β are determined by least-squares approximation.

$$a=\sqrt{\alpha^2+\beta^2}$$

$$\phi=\tan^{-1}\left(\frac{\beta}{\alpha}\right) \quad (10)$$

Equations for determining α and β by least-squares approximation are as follows.

$$E=\sum_{t=1}^{n}[y(t)-(\alpha\cos\omega t+\beta\sin\omega t)]^2$$

$$\frac{\partial E}{\partial \alpha} = -2\sum_{t=1}^{n}[y(t)-(\alpha\cos\omega t+\beta\sin\omega t)]\cos\omega t=0$$

$$\frac{\partial E}{\partial \beta} = -2\sum_{t=1}^{n}[y(t)-(\alpha\cos\omega t+\beta\sin\omega t)]\sin\omega t=0$$

$$\alpha\sum_{t=1}^{n}\cos^2\omega t+\beta\sum_{t=1}^{n}\sin\omega t\cos\omega t=\sum_{t=1}^{n}y(t)\cos\omega t$$

$$\alpha\sum_{t=1}^{n}\sin\omega t\cos\omega t+\beta\sum_{t=1}^{n}\sin^2\omega t=\sum_{t=1}^{n}y(t)\sin\omega t$$

Tidal Prediction Model

Real-time tidal elevation and currents are predicted from following Eqs. (9) to (11) using harmonic constants for major tidal constituents obtained through the predescribed harmonic analysis on computing results of numerical tidal model (See Foreman[3,4]).

$$U(t) = \sum_{i=1}^{N} f_i(t) A_{ui} \cos 2\pi [\omega_i(t) - \phi_{ui} + V_i(t) + u_i(t)] \qquad (9)$$

$$V(t) = \sum_{i=1}^{N} f_i(t) A_{vi} \cos 2\pi [\omega_i(t) - \phi_{vi} + V_i(t) + u_i(t)] \qquad (10)$$

$$\zeta(t) = \sum_{i=1}^{N} f_i(t) A_{\zeta}i \cos 2\pi [\omega_i(t) - \phi_{\zeta}i + V_i(t) + u_i(t)] \qquad (11)$$

where $U(t)$, $V(t)$ = real time velocity in x, y direction, $\zeta(t)$ = real-time tidal elevation, ω_i = angular speed of constituent i, A_i = amplitute of constituent i and ϕ_i = local phase angle of constituent i, respectively. Astronomical variables, $f_i(t)$, $u_i(t)$ are nodal factor of amplitute and phase, respectively and $V_i(t)$ is phase of the equilibrium tide at given time

The variables, f_i, u_i and V_i are computed from equations concerned with orbital components of the sun and moon. This model adopts the equations expressed by function of time elapsed from Jan. 1, 1900 suggested by Doodson(1921) (See Doodson[5]).

APPLICATION EXAMPLE OF THE SYSTE

The tidal prediction system derived from this study was applied to the neighboring area of Pusan port.

Numerical Tidal Model

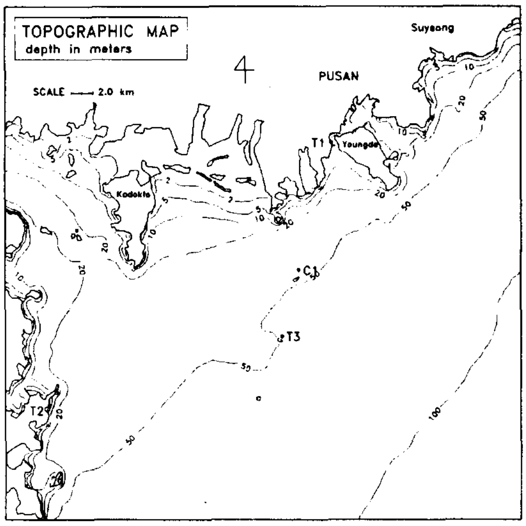

Fig. 2. Map of the Study Area.

Fig. 2 shows the study area including eastern part of Geoje island on west, Jang -seungpo harbor on south and Daebyun harbor on north, respectively. In this figure, legend T and C denote the observation point of tidal elevation and currents, respectively. The computatio -nal grid map, composed with a mesh of 150×145 constant grid points, for adopting model to the study area is shown in Fig. 3, For a reason of computational efficiency, comp -utings for four major tidal constituents were only made. Considering the stability of numerical solution, computing for each tidal constituent was repeated for six tidal period, then computing results of last tidal period were stored in

48 equal spaced time interval. Then, using these computing data the databases of harmonic constants for each constituent were constructed through the harmonic analysis, and these databases were used for plotting the co-tidal chart and tidal current ellipses for each constituent as presented in Appendix. For the verification of computing results, comparison between the observed and calculated harmonic constants for three stations (T_1, T_2, T_3) was made as shown in Table 1.

In results of the above comparison, it is known that there exist almost no errors for semi-diurnal tides, but exist 10% or more of relative errors for diurnal tides. Considering the study area is located near the ephidermic point in the case of diurnal tides, these are considered to be a desirable results.

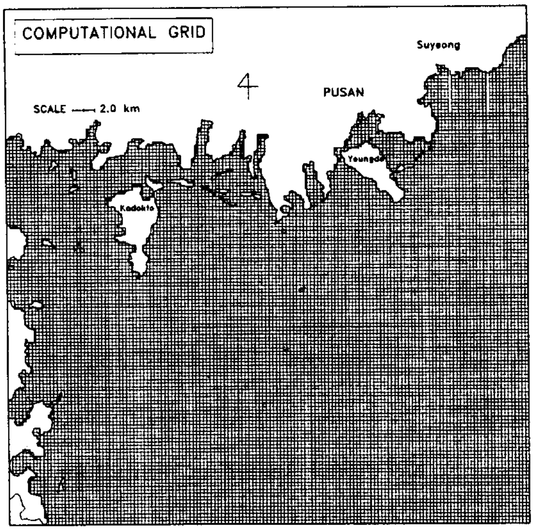

Fig. 3. Computational Grid Map.

Table 1. Comparison between observed and calculated constants

Station	Constituent	Observed		Calculated		Diff.		Rel. Error
		Amp.	Phs.	Amp.	Phs.	Amp.	Phs.	Amp.
T_1	M_2	39.6	234.9	39.2	234.4	0.4	0.5	1.0
	S_2	19.4	271.1	19.1	271.0	0.3	0.1	1.5
	K_1	4.8	136.5	4.4	135.6	0.4	0.9	8.3
	O_1	1.7	107.2	1.8	108.0	0.1	0.8	5.9
T_2	M_2	49.9	242.8	49.0	241.3	0.9	1.5	1.8
	S_2	24.7	277.9	24.7	277.2	0.0	0.7	0.0
	K_1	7.9	159.1	6.9	158.2	1.0	0.9	12.6
	O_1	4.4	132.8	4.0	130.5	0.4	1.7	9.1
T_3	M_2	46.3	241.7	44.8	241.6	1.5	0.1	3.2
	S_2	23.3	277.1	22.6	277.0	0.7	0.1	3.0
	K_1	6.5	160.2	5.7	165.7	0.8	5.5	12.3
	O_1	3.3	137.3	2.8	130.0	0.5	7.3	15.2

unit: amplitude(cm), phase(°), rel. error(%)

Tidal Prediction Model

Tidal Elevation

For the verification of tidal elevation computed by tidal prediction system, the observed, predicted and calculated values were compared, respectively. Herein, the observed values mean computed ones by tidal prediction model using the harmonic constants of 69 constituents which is obtaind through the harmonic analysis of observation data series, the predicted values mean computed ones using harmonic constants of 4 major constituents(M_2, S_2, K_1, O_1) chosen in 69 constituents, and the calculated values mean computed

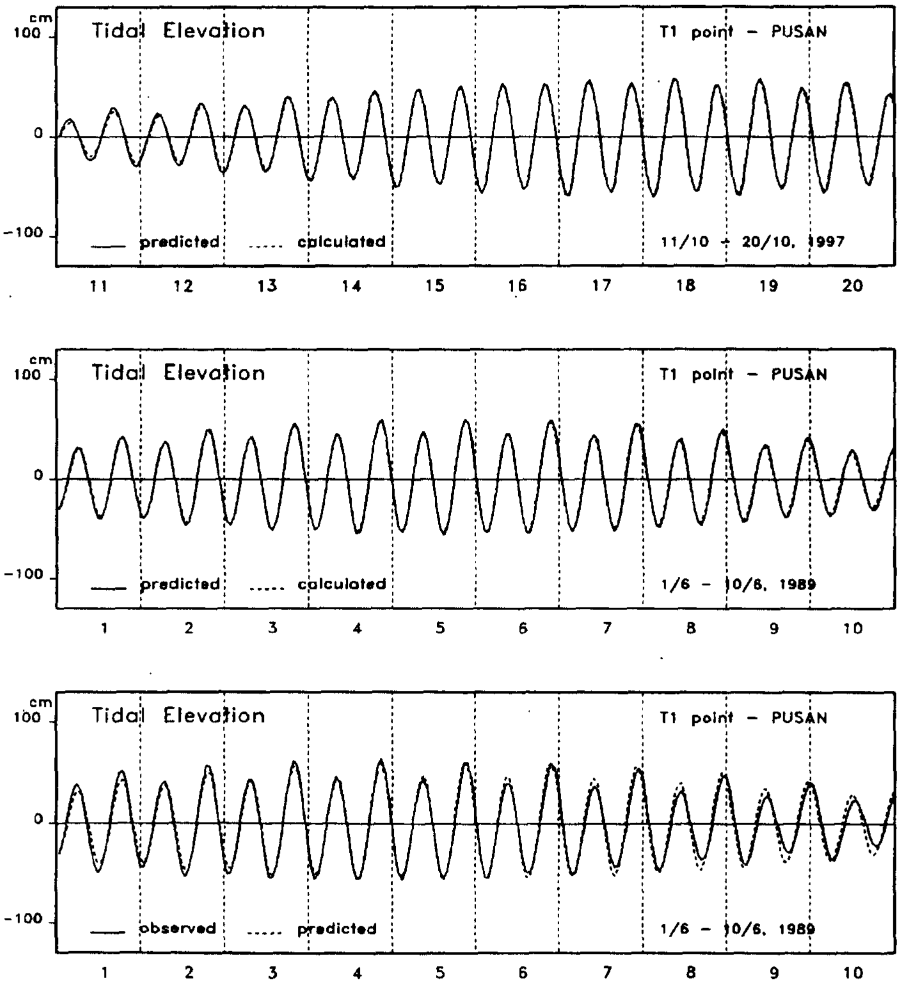

Fig. 4. Changes in tidal elevation at T_1 station.

ones using harmonic constants of 4 major constituents computed by numerical tidal model. Fig. 4 shows the changes of observed, predicted and calculated values with the time for station T_1 and Fig. 5 for station T_3, respectively. Firstly, Fig. 4 exhibits maximum 12 cm errors (about 10% of relative errors) for amplitute between the observed and predicted values, but almost no errors between the predicted and calculated ones. The former is resulted from tidal prediction using only four major constituents and the latter means the computing results of numerical tidal model are almost correct, respectively. Thus, the errors between the observed and predicted values will be diminished if more constituents are used for prediction.

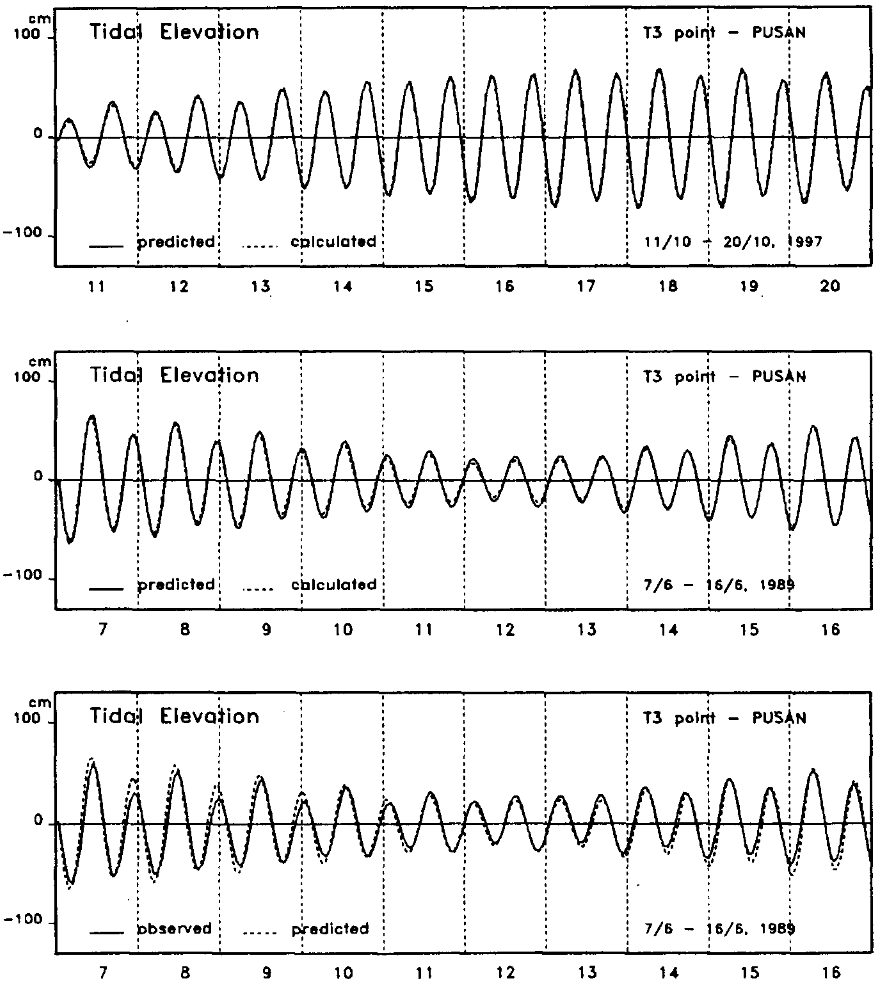

Fig. 5. Changes in tidal elevation at T_3 station.

Secondly, Fig. 5 exhibits maximum 13 cm errors (about 10% of relative errors) for amplitute between the observed and predicted values, but almost no errors between the predicted and calculated ones like station T_1.

Tidal Currents

For the verification of tidal currents computed by tidal prediction system, the observed, predicted and calculated values were compared like the tidal elevation. Fig. 6 and 7 shows the observed, predicted

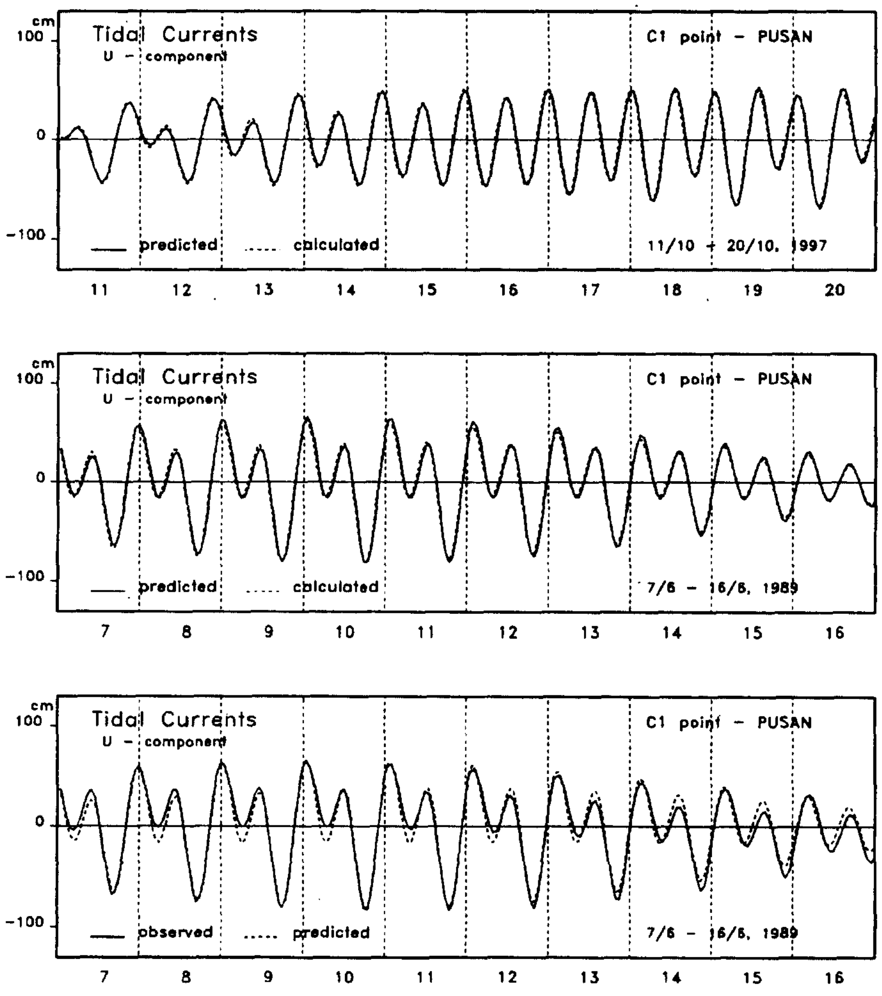

Fig. 6. Changes in U-component of tidal currents at C_1 station.

and calculated values of U, V-component of velocity with the time for station C_1, respectively. The meaning of observed, predicted and calculated values is same to those of tidal elevation. Firstly, Fig. 6 showing the changes of U-component of velocity exhibits maximum 18 cm/sec errors (about 13% of relative errors) for amplitute between the observed and predicted values, but almost no errors between the predicted and calculated ones. Like the case of tidal elevation, the former is resulted from tidal prediction using only four major constituents and the latter means the computing results of numerical tidal model are almost correct, respectively. Thus, the errors between the observed and predicted values will be diminished if more constituents are used for prediction.

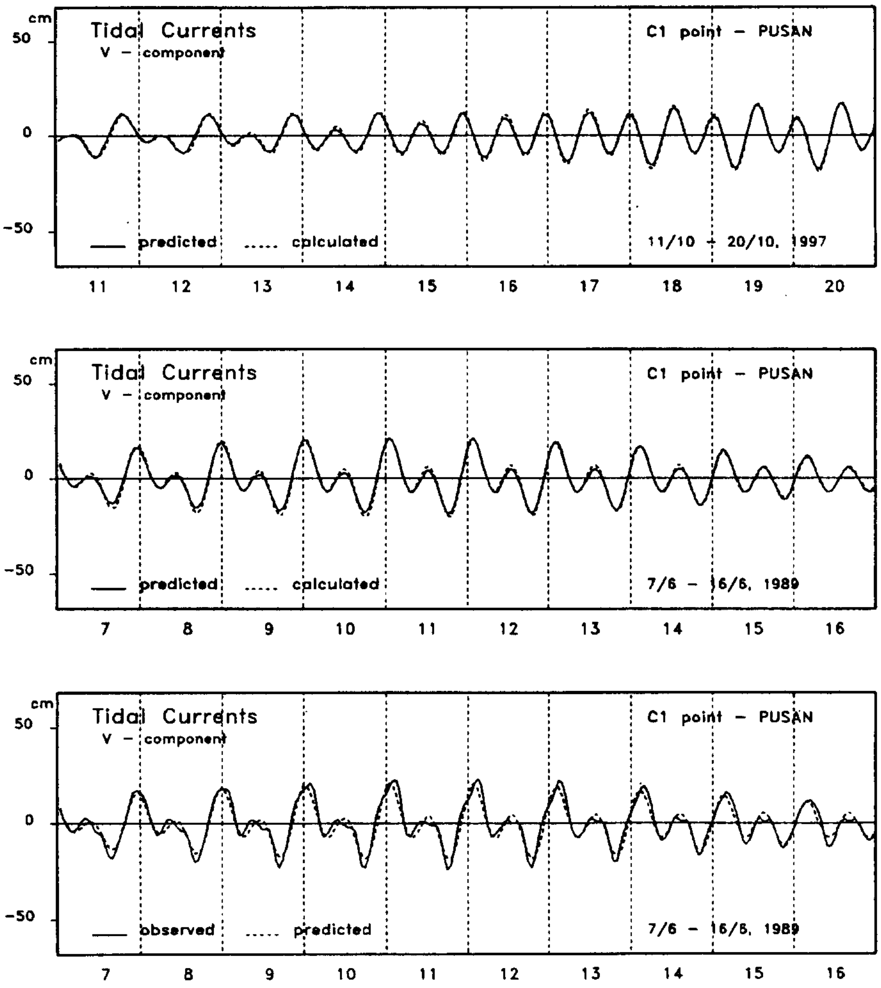

Fig. 7. Changes in V-component of tidal currents at C_1 station.

Secondly, Fig. 7 showing the changes of V-component of velocity exhibits maximum 5 cm/sec errors (about 10% of relative errors) for amplitute between the observed and predicted values, but almost no errors between the predicted and calculated values like the U-component of velocity.

Fig. 8 and 9 are velocity vector diagram which exhibits the distribution of tidal currents at the time of maximum flood on 7 AM and at the time of maximum ebb on 2 PM, October 19, 1997, respectively. The magnitute of velocities are maximum 80 cm/sec at flood tide and maximum 65 cm/sec at ebb tide.

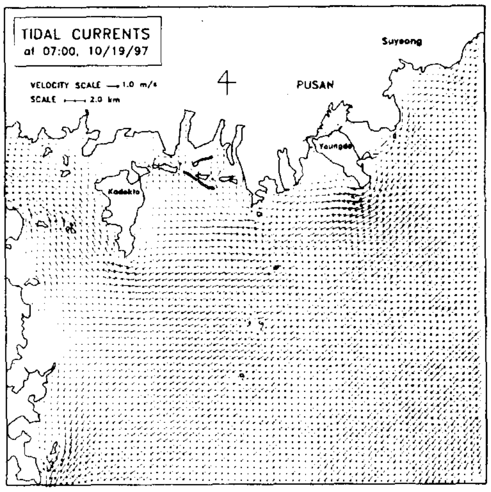

Fig. 8. Tidal Currents at 7 AM on 10/19/1997.

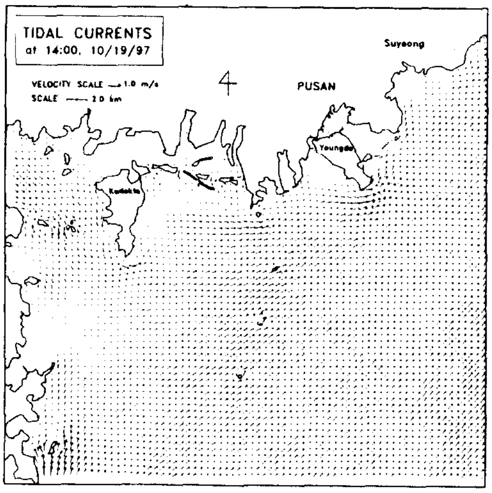

Fig. 9. Tidal Currents at 2 PM on 10/19/1997.

CONCLUSIONS

With the combination of existing tidal prediction model and numerical tidal model, the efficient tidal prediction system was formulated. This system is expected to give us real-time tidal data at a certain point or region whenever required. Thus, this system will make a contribution to improve several fields of oceanographic researchs as well as the safety of navigation. The unique point of weakness for this system is that system has a physical limit such as all tidal constituents can't be considered for prediction, and it causes some error between the prediction and actual values.

It was confirmed that the practical results with about 10% of relative errors could be obtained at least if four major tidal constituents(M_2, S_2, K_1, O_1) are used. Thus, if other constituents than four major tidal constituents are additionaly used, more accurate results will be made. Futhermore, if the databases of harmonic constants in coastal waters is made in advance using the numerical tidal model, prompt tidal prediction will be obtained whenever required.

REFERENCES

[1] Leendertse J. J., "Aspects of a computational model for long-period water-wave propagation." Rand Corporation Report RM-5294-PR, 1967, pp. 52~65.

[2] Lardner R. W., Belen M. S., and Cekirge H. M., "Finite Difference Model for Tidal Flows in The Arabian Gulf", Comp. & Maths. with Appls. Vol. 8. No. 6, 1982, pp. 425~444.

[3] Foreman, M. G. G., Manual for tidal heights analysis and prediction. Pacific Marine Science Report 77-10, Institute of Ocean Sciences, Sidney, B. C., Canada, 1977, 101 pp.

[4] Foreman, M. G. G. and Henry R. F., Tidal analysis based on high and low water observations. Pacific Marine Science Report 79-15, Institute of Ocean Sciences, Sidney, B. C., Canada, 1979, 39 pp.

APPENDIX

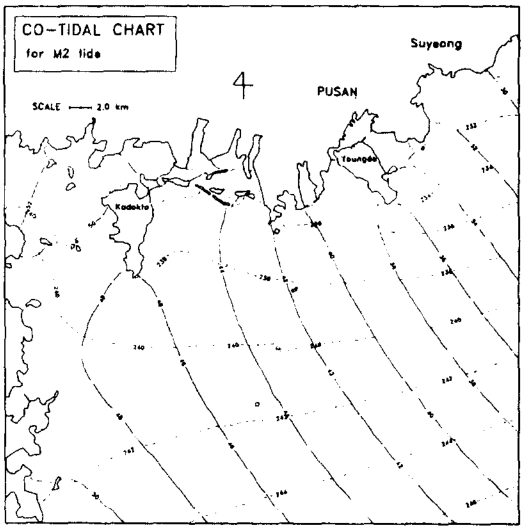

Fig. A1. Co-tidal Chart for M_2 tide.

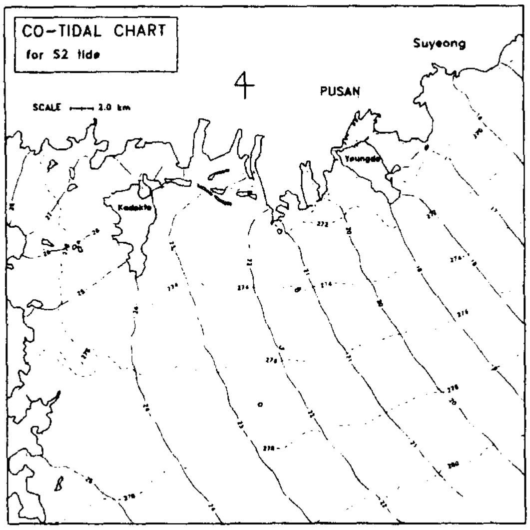

Fig. A2. Co-tidal Chart for S_2 tide.

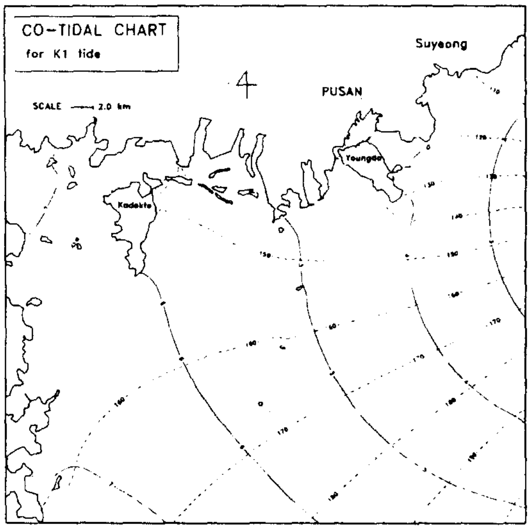

Fig. A3. Co-tidal Chart for K_1 tide.

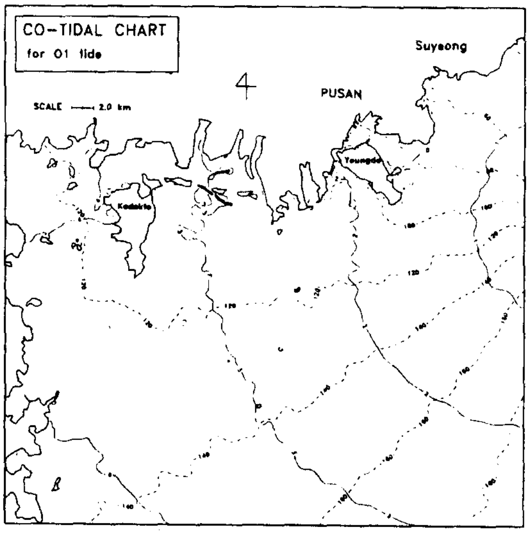

Fig. A4. Co-tidal Chart for O_1 tide.

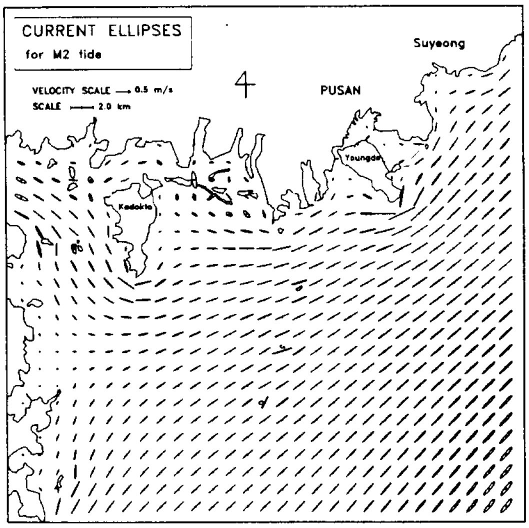

Fig. A5. Tidal Current Ellipses for M_2 tide.

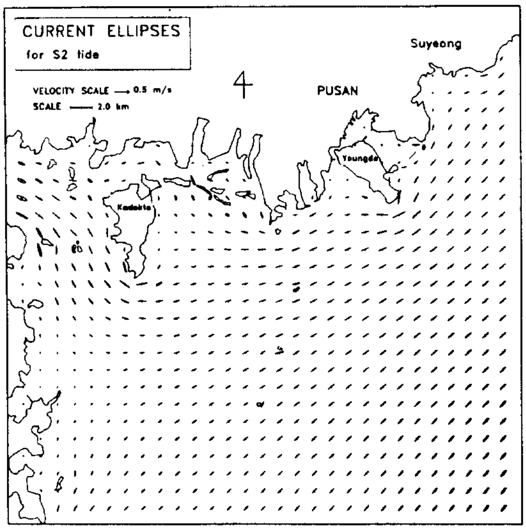

Fig. A6. Tidal Current Ellipses for S_2 Tide.

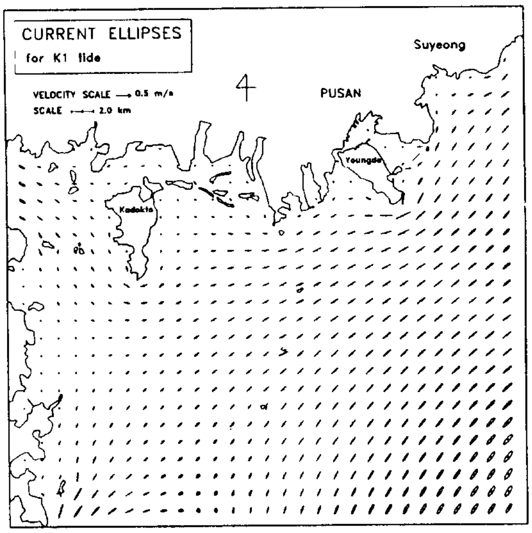

Fig. A7. Tidal Current Ellipses for K_1 Tide.

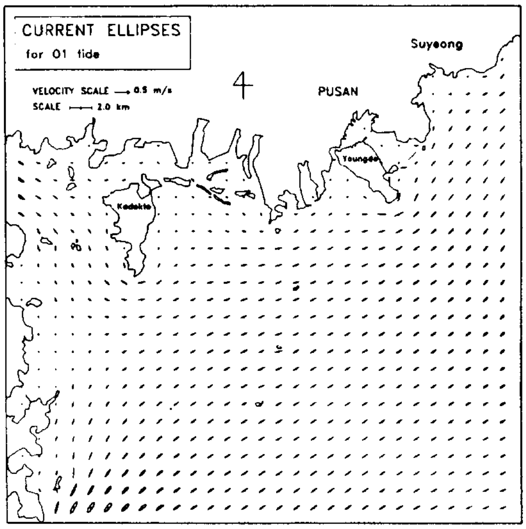

Fig. A8. Tidal Current Ellipses for O_1 Tide.