# ON FUZZY $p$-CONTINUOUS MAPPINGS

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## 1. INTRODUCTION AND PRELIMINARIES

In 1991, Bin Shahna [2] introduced the concept of fuzzy pre-continuous mappings by using the concept of fuzzy preopen sets [2,13]. Park and Park [9] defined fuzzy pre-irresolute mappings between fuzzy topological spaces, which is srtonger than fuzzy pre-continuity and independent of fuzzy continuity of Chang [3]. Recently, Park and Ha [10] introduced fuzzy weakly pre-irresolute mappings and fuzzy strongly pre-irresolute mappings, one of which is weaker than and the other is stronger than fuzzy pre-irresolute mappings, and pointed out that these concepts are still independent of fuzzy continuity.

In this paper, we first introduce three new classes of mappings between fuzzy topological spaces, under the terminologies fuzzy $p$-continuous, fuzzy weakly $p$-continuous and fuzzy strongly $p$-continuous mappings, and give several characterizing theorems for these types of mappings and study these mappings in relation to some other types of already known mappings. Finally, we investigate for the suitable conditions under which the implications concerning fuzzy $p$-continuous, fuzzy weakly $p$-continuous and fuzzy strongly $p$-continuous mappings.

Throughout this paper, by $(X, \tau)$ (or simply $X$) we mean a fuzzy topological space in Chang's [3] sense. A fuzzy point in $X$ with support $x \in X$ and value $\alpha$ $(0 < \alpha \leq 1)$ is denoted by $x_\alpha$. For a fuzzy set $A$ in $X$, $\mathrm{Cl}(A)$, $\mathrm{Int}(A)$ and $1 - A$ will respectively denote the closure, interior and complement of $A$, whereas the constant fuzzy sets taking on the values 0 and 1 on $X$ are denoted by $0_X$ and $1_X$, respectively. A fuzzy set $A$ in $X$ is said to be q-coincident with a fuzzy set $B$, denoted by $A\mathrm{q}B$, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [12]. It is known [12] that $A \leq B$ if and only if $A$ and $1 - B$ are not q-coincident, denoted by $A\bar{\mathrm{q}}(1 - B)$. For definitions and results not explained in this paper, the reader is referred to [1, 2, 12] in the assumption they are well known. The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd' and 'fts', respectively.

**Definition 1.1 [2, 13].** A fuzzy set $A$ in a fts $X$ is said to be fuzzy preopen (resp. fuzzy preclosed) if $A \leq \mathrm{Int}(\mathrm{Cl}(A))$ (resp. $\mathrm{Cl}(\mathrm{Int}(A)) \leq A$).

**Definition 1.2 [9].** A fuzzy set $A$ in a fts $X$ is said to be fuzzy pre-q-nbd (resp. fuzzy pre-nbd) of fuzzy point $x_\alpha$ if there exists a fuzzy preopen set $B$ such that $x_\alpha \mathrm{q} B \leq A$ (resp. $x_\alpha \in B \leq A$).

**Theorem 1.3 [9].** *A fuzzy set $A$ is a fuzzy preopen if and only if for each fuzzy point $x_\alpha qA$, $A$ is a fuzzy pre-q-nbd of $x_\alpha$.*

**Definition 1.4 [2,13].** Let $A$ be any fuzzy set of a fts $X$. Then fuzzy pre-closure (pCl) and pre-interior (pInt) of $A$ are defined as follows:

$$pClA = \bigwedge\{B \mid B \text{ is fuzzy preclosed and } A \leq B\},$$

$$pIntA = \bigvee\{B \mid B \text{ is fuzzy preopen and } B \leq A\}.$$

**Theorem 1.5 [9].** *Let $A$ be a fuzzy set in $X$ and $x_\alpha$ be a fuzzy point in $X$. Then $x_\alpha \in pCl(A)$ if and only if for each fuzzy preopen pre-q-nbd $U$ of $x_\alpha$, $UqA$.*

**Definition 1.6.** A fuzzy point $x_\alpha$ is said to be fuzzy pre-$\theta$-cluster point of a fuzzy set $A$ in $X$ if the fuzzy pre-closure of every fuzzy preopen pre-q-nbd of $x_\alpha$ is q-coincident with $A$. The union of all fuzzy pre-$\theta$-cluster points of $A$ is called the fuzzy pre-$\theta$-closure of $A$ and is denoted by $[A]_{p\text{-}\theta}$. A fuzzy set $A$ is called fuzzy pre-$\theta$-closed if $A = [A]_{p\text{-}\theta}$, and the complement of a fuzzy pre-$\theta$-closed set is fuzzy pre-$\theta$-open.

**Theorem 1.7.** *For a fuzzy preopen set $A$ in a fts $X$, $pCl(A) = [A]_{p\text{-}\theta}$.*

**Corollary 1.8.** *If $A$ is a fuzzy pre-clopen in $X$, then $A$ is fuzzy pre-$\theta$-closed as well as fuzzy pre-$\theta$-open.*

**Definition 1.9.** A fuzzy set $A$ is said to be a fuzzy pre-$\theta$-nbd of a fuzzy point $x_\alpha$ if there exists a fuzzy preopen pre-q-nbd $V$ of $x_\alpha$ such that $\mathrm{pCl}V\bar{q}(1 - A)$.

**Theorem 1.10.** *A fuzzy set $A$ in a fts $X$ is fuzzy pre-$\theta$-open if and only if for each fuzzy point $x_\alpha$ in $X$ with $x_\alpha qA$, $A$ is fuzzy pre-$\theta$-nbd of $x_\alpha$.*

## 2. Fuzzy $p$-continuous mappings

According to Park and Park [9], a mapping $f : X \to Y$ is said to be fuzzy pre-irresolute if $f^{-1}(B)$ is fuzzy preopen for each fuzzy preopen set $B$ in $Y$. It is proved in [9] that $f : X \to Y$ is fuzzy pre-irresolute if and only if for each fuzzy point $x_\alpha$ in $X$ and each fuzzy preopen pre-q-nbd $V$ of $f(x_\alpha)$, there exists a fuzzy preopen pre-q-nbd $U$ of $x_\alpha$ such that $f(U) \leq V$. Now, let us define as follows.

**Definition 2.1.** A mapping $f : X \to Y$ is said to be

(a) fuzzy $p$-continuous (briefly, f$p$c) if for each fuzzy point $x_\alpha$ in $X$ and each fuzzy preopen pre-q-nbd $V$ of $f(x_\alpha)$, there exists a fuzzy open q-nbd $U$ of $x_\alpha$ such that $f(U) \leq V$;

(b) fuzzy weakly $p$-continuous (briefly, fw$p$c) if for each fuzzy point $x_\alpha$ in $X$ and each fuzzy preopen pre-q-nbd $V$ of $f(x_\alpha)$, there exists a fuzzy open q-nbd $U$ $x_\alpha$ such that $f(U) \leq \mathrm{pCl}(V)$

(b) fuzzy strongly $p$-continuous (briefly, fs$p$c) if for each fuzzy point $x_\alpha$ in $X$ and each fuzzy preopen pre-q-nbd $V$ of $f(x_\alpha)$, there exists a fuzzy open q-nbd $U$ $x_\alpha$ such that $f(\mathrm{Cl}(U)) \leq V$.

Clearly, every fs$p$c mapping implies f$p$c and every f$p$c mapping implies fw$p$c. However, the converses need not be true in general.

**Theorem 2.2.** *A mapping $f : X \to Y$ is f$p$c (resp. fw$p$c, fs$p$c) if and only if for each fuzzy point $x_\alpha$ in $X$ and each fuzzy preopen set $V$ containing $f(x_\alpha)$, there exists a fuzzy open set $U$ containing $x_\alpha$ such that $f(U) \leq V$ (resp. $f(U) \leq pCl(V)$, $f(Cl(U)) \leq V$).*

**Theorem 2.3.** *For a mapping $f : X \to Y$, then the following are equivalent:*
(a) *$f$ is fpc.*
(b) *For each fuzzy set $B$ in $Y$, $Cl(f^{-1}(B)) \leq f^{-1}(pCl(B))$.*
(c) *For each fuzzy set $A$ in $X$, $f(Cl((A)) \leq pCl(f(A))$.*
(d) *For each fuzzy preopen set $B$ in $Y$, $f^{-1}(B)$ is fuzzy open in $X$.*
(e) *For each fuzzy preclosed set $B$ in $Y$, $f^{-1}(B)$ is fuzzy closed in $X$.*

**Theorem 2.4.** *For a mapping $f : X \to Y$, then the following are equivalent:*
(a) *$f$ is fwpc.*
(b) *For each fuzzy set $B$ in $Y$, $Cl(f^{-1}(B)) \leq f^{-1}([B]_{p\text{-}\theta})$.*
(c) *For each fuzzy set $A$ in $X$, $f(Cl((A)) \leq [f(A)]_{p\text{-}\theta}$.*
(d) *For each fuzzy pre-$\theta$-open set $B$ in $Y$, $f^{-1}(B)$ is fuzzy open in $X$.*
(e) *For each fuzzy pre-$\theta$-closed set $B$ in $Y$, $f^{-1}(B)$ is fuzzy closed in $X$.*
(f) *For each fuzzy point $x_\alpha$ in $X$ and each fuzzy pre-$\theta$-nbd $N$ of $f(x_\alpha)$, $f^{-1}(N)$ is a fuzzy open q-nbd of $x_\alpha$.*

**Lemma 2.5 [10].** *If $A$ is any fuzzy set and $B$ is a fuzzy preopen set in a fts $X$ such that $A\bar{q}B$, then $pCl(A)\bar{q}B$.*

**Theorem 2.6.** *For a mapping $f : X \to Y$, then the following are equivalent:*
(a) *$f$ ia fwpc.*
(b) *For each fuzzy preopen set $V$ in $Y$, $f^{-1}(V) \leq Int(f^{-1}(pCl(V)))$.*
(c) *For each fuzzy preopen set $V$ in $Y$, $Cl(f^{-1}(V)) \leq f^{-1}(pCl(V))$.*

**Theorem 2.7.** *For a mapping $f : X \to Y$, then the following are equivalent:*
(a) *$f$ is fspc.*
(b) *For each fuzzy set $B$ in $Y$, $[f^{-1}(B)]_\theta \leq f^{-1}(pCl(B))$.*
(c) *For each fuzzy set $A$ in $X$, $f([A]_\theta) \leq pCl(f(A))$.*
(d) *For each fuzzy pre-open set $B$ in $Y$, $f^{-1}(B)$ is fuzzy $\theta$-open in $X$.*
(e) *For each fuzzy pre-closed set $B$ in $Y$, $f^{-1}(B)$ is fuzzy $\theta$-closed in $X$.*
(f) *For each fuzzy point $x_\alpha$ in $X$ and each fuzzy pre-nbd $N$ of $f(x_\alpha)$, $f^{-1}(N)$ is a fuzzy $\theta$-nbd of $x_\alpha$.*

**Theorem 2.8.** *Let $X$ and $Y$ be fts's such that $X$ is product related to $Y$ and let $f : X \to Y$ be a mapping. Then if the graph $g : X \to X \times Y$ of $f$ is fpc (resp. fwpc, fspc), then $f$ is also fpc (resp. fwpc, fspc).*

**Theorem 2.9.** *Let $f : X \to Y$ be a fpc injection. If $Y$ is fuzzy pre-$T_i$ space, then $X$ is fuzzy $T_i$, for $i = 0, 1, 2$.*

We now investigate for the suitable conditions under which the implications concerning fspc, fpc and fwpc mappings can be reversed.

**Definition 2.10.** A fts $X$ is said to be fuzzy regular [7] (resp. fuzzy p-regular [10]) if for each fuzzy point $x_\alpha$ in $X$ and each fuzzy open q-nbd (resp. fuzzy preopen pre-q-nbd) $V$ of $x_\alpha$, there exists a fuzzy open q-nbd (resp. fuzzy preopen pre-q-nbd) $U$ of $x_\alpha$ such that $\mathrm{Cl}(U) \leq V$ (resp. $\mathrm{pCl}(U) \leq V$).

**Lemma 2.11[7,10].** *Let $X$ be a fts.*
(a) *$X$ is fuzzy regular if and only if $Cl(A) = [A]_\theta$ for each fuzzy set $A$ in $X$.*
(b) *$X$ is fuzzy p-regular if and only if $pCl(A) = [A]_{p\text{-}\theta}$ for each fuzzy set $A$ in $X$.*

**Theorem 2.12.** *Let $f : X \to Y$ be a mapping.*
(a) *If $f$ is fwpc and $Y$ is fuzzy p-regular, then $f$ is fpc.*
(b) *If $f$ is fpc and $X$ is fuzzy regular, then $f$ is fspc.*

**Definition 2.13 [5,8,11].** Two non-null fuzzy sets $A$ and $B$ in a fts $X$ are sait to be fuzzy separated (resp. fuzzy $\theta$-separated, fuzzy pre-separated) if $\mathrm{Cl}(A)\mathrm{q}B$ and $\mathrm{Cl}(B)\mathrm{q}A$ (resp. $[A]_\theta \mathrm{q}B$ and $[B]_\theta \mathrm{q}A$, $\mathrm{pCl}(A)\mathrm{q}B$ and $\mathrm{pCl}(B)\mathrm{q}A$).

**Definition 2.14 [5,8,11].** A fuzzy set $A$ in a fts $X$ is said to be fuzzy connected (resp. fuzzy $\theta$-connected, fuzzy pre-connected) if it cannot be expressed as the union of two fuzzy separated (resp. fuzzy $\theta$-separated, fuzzy pre-separated) sets.

**Theorem 2.15.** *Let $f : X \to Y$ be a surjection. Then we have*
*(a) If $f$ is fspc and $A$ is fuzzy $\theta$-connected subset in $X$, then $f(A)$ is fuzzy pre-connected in $Y$.*
*(b) If $f$ is fpc and $A$ is fuzzy connected subset in $X$, then $f(A)$ is fuzzy pre-connected in $Y$.*

**Corollary 2.16 [11].** *If $f : X \to Y$ be a fuzzy completely pre-irresolute surjection and $A$ is fuzzy connected subset in $X$, then $f(A)$ is fuzzy pre-connected in $Y$.*

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