MIMO Robust Adaptive Fuzzy Controller[1]

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Abstract

A novel fuzzy basis function vector-based adaptive control approach for Multi-input and Multi-output (MIMO) system is presented in this paper,in which the nonlinear plants is first linearised,the fuzzy basis function vector is then introduced to adaptively learn the upper bound of the system uncertainty vector,and its output is used as the parameters of the compensator in the sense that both the robustness and the asymptotic error convergence can be obtained for the closed loop nonlinear control system.

Keywords: Stability, Robust control, Fuzzy basic function vector

1. Introduction

In recent years,fuzzy logic control technique has had an increasing impact in control engineering community. However,some important theoretical and practical problems have not be solved. For example,error convergence,stability and robustness have not fully proved for fuzzy logic control schemes where Mamdani-type linguistic models and Sugeno fuzzy models are used to deal with systems with uncertain dynamics.The recent developments in [1][2] using fuzzy basis function networks for model-reference adaptive control have made a great progress in solving the above problems.

In [1],a fuzzy basis function network is used to approximate an unknown system parameter vector and the weights of the fuzzy basis function network are then adaptively adjusted. But it lacks of the proof of the stability.In [2],a stable adaptive control approach using fuzzy systems and neural networks is presented,its control is comprised of a bounding-control term,a sliding-mode-control term,a certainty-control term.The hybrid control approach is designed in a very complicated way that it is hardly to be used in practice.Morever the emphasis in the above techniques is placed on the control of single-input single-output (SISO) plants.

In this paper, a fuzzy basis function vector based adaptive control is proposed for MIMO square nonlinear systems.It is shown that the nonlinear system is first linearised,the linearised nonlinear system is then treated as a partially known system. The known dynamics are used to design a nominal feedback controller to stabilise the nominal system,and a fuzzy basis function vector-based compensator is then designed to compensate the effects of system uncertainties. By the intensive design of Lyapnove function,we prove the stability of the closed loop nonlinear control system and obtain both the robustness with respect to unknown dynamics and the asymptotic error convergence for the system in the meantime.

2. Problem formulation

Consider the following MIMO square nonlinear system (i.e., a system with as many inputs as outputs)

[1] This work was supported by the national natural science foundation of China and KOSEF of Korea.

given by:

$$\dot{X} = f(X) + \sum_{j=1}^{m} g_j(X)u_j(t)$$
$$y_1(t) = h_1(X)$$
$$\cdots \qquad (1)$$
$$y_m(t) = h_m(X)$$

where $X \in R^n$ is the plant state vector, $U = [u_1, u_2 \cdots, u_m] \in R^m$ is the control input vector, $Y = [y_1, y_2 \cdots, y_m] \in R^m$ is the output vector, $f(.), g_i(.)$: $R^n \to R^n$, i=1,2,...,m are smooth functions vector, $h_i(.)$: $R^n \to R$, i=1,2,...,m are smooth functions. For convenience, the above equation will be rewritten in a condensed form:

$$\dot{X} = f(X) + G(X)U \qquad (2)$$
$$Y = h(X) \qquad (3)$$

where

$$U = col(u_1, u_2, \ldots u_m)$$
$$Y = col(y_1, y_2, \cdots y_m)$$
$$G(X) = [g_1(X), g_2(X), \ldots g_m(X)]$$
$$h(X) = col(h_1(X), h_2(X), \ldots h_m(X))$$

We recall the definition of the relative degree of a nonlinear system to characterize the system. it is necessary to define some notations. The derivative of a scalar function ϕ alone a vector $f=[f_1,\ldots,f_n]^T$ is defined as

$$L_f\phi(X) = \frac{\partial\phi}{\partial X} f(X) = [\frac{\partial\phi}{\partial x_1}, \ldots, \frac{\partial\phi}{\partial x_n}]\begin{bmatrix} f_1(X) \\ \cdots \\ f_n(X) \end{bmatrix}$$
$$= \sum_{i=1}^{n} \frac{\partial\phi}{\partial x_i} f_i(X) \qquad (4)$$

where $X=[x_1,\ldots,x_n]^T$, and the derivative of ϕ taken first along f and then along a vector g is

$$L_g L_f \phi(X) = \frac{\partial(L_f\phi)}{\partial X} g(X) \qquad (5)$$

If ϕ is being differentiated j times along f,the notation $L_f^j\phi$ is used with $L_f^0\phi(X) = \phi(X)$.

Differentiating the output y_i with respect to time t in eqn(1),we have

$$\dot{y}_i = L_f h_i(X) + \sum_{j=1}^{m} L_{g_j} h_i(X)u_j(t) \qquad (6)$$

where, $L_f h_i(X)$ and $L_{g_j} h_i(X)$ are the Lie derivatives of $h_i(X)$ with respect to f(X) and $g_j(X)$,respectively.

If $L_{g_j} h_i(X) = 0$, then $\dot{y}_i = L_f h_i(X)$.

Continuing in this process,we get:

$$y_i^{(r_i)} = L_f^{r_i} h_i(X) + \sum_{j=1}^{m} L_{g_j} L_f^{r_i-1} h_i(X)u_j(t) \qquad (7)$$

for $L_{g_j} L_f^l h_i(X) = 0, l<r_i-1$ and $L_{g_j} L_f^{r_i-1} h_i(X) \neq 0$.

In this way,we may rewrite the plant's input-output equation as:

$$\underbrace{\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix}}_{Y(t)} = \underbrace{\begin{bmatrix} L_f^{r_1}h_1 \\ \vdots \\ L_f^{r_m}h_m \end{bmatrix}}_{B(X,t)} + \underbrace{\begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1(X) \cdots L_{g_m}L_f^{r_1-1}h_1(X) \\ \vdots \quad \vdots \\ L_{g_1}L_f^{r_m-1}h_m(X) \cdots L_{g_m}L_f^{r_m-1}h_m(X) \end{bmatrix}}_{J(X,t)} \underbrace{\begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}}_{U(t)} \qquad (8)$$

An ideal Static-state feedback linearizing control law can be obtained by

$$U^* = J^{-1}(-B+V) \qquad (9)$$

We will define the term V below.In order for U^* to be defined,some assumptions about the plant have to be met. In particular,we need the following:

A.1 Plant Assumption

(i) The matrix J as defined above is nonsinqular.

(ii) The plant has a general vector relative degree $[r_1,\ldots,r_p]^T$, and its zero dynamics are exponentially attractive.

3. Fuzzy system formation

Definition 2

$|A|$ is modulus of matrix A,i.e.,matrix with modulus elements of A.

$\|A\|$ is a l_p norm of matrix A,which is with the performance of compatibility.

$|A|<|B|$ means $|a_{ij}|<|b_{ij}|$, $\forall a_{ij} \in A$ and $\forall b_{ij} \in B$, where $A=\{a_{ij}\}$,$B=\{b_{ij}\}$.

$|A|_m$ is a matrix,in which all the elements are equal to a_{max},and $a_{max} = \max_{i,j}|a_{ij}|$ where $A = \{a_{ij}\}$.

sign(A) is a sign matrix,which meets the condition of Asign(A)=|A|,where A is a vector.

tr(A) is the trace of the matrix A.

From eqn(8) we can obtain:

$$E(X)Y=F(X)+U \qquad (10)$$

where $F(X)=J^{-1}(X) \in R^{m\times m}$ (11)

$F(X)=J^{-1}(X)B(X) \in R^m$ (12)

and $E(X)^{-1}$ and F(X) are assumed to be bounded by the following unknown positive function $P_1(X)$ and vector $Q_1(X)$:

$$0 \le \|E(X)^{-1}\| < P_1(X) \quad (13)$$

$$0 \le |F(X)|_m < Q_1(X) \quad (14)$$

In practical situation,E(X) and F(X) may not be exactly known, E(X) and F(X) may then be expressed as

$$E(X) = E_0(X) + \Delta E(X) \quad (15)$$

$$F(X) = F_0(X) + \Delta F(X) \quad (16)$$

where $E_0(X)$ (nonsinqular) and $F_0(X)$ are known parts, and Δ E(X) and Δ F(X) are unknown parts.

Remark 1: According to the bounded properties of E(X) and F(X) in eqs(11)(12),uncertain dynamics Δ E(X) and Δ F(X) are also bounded by

$$\|\Delta E(X)^{-1}\| < P_2(X) \quad (17)$$

$$|\Delta F(X)| < Q_2(X) \quad (18)$$

where $P_2(X)$ and $Q_2(X)$ are unknown positive function and function vector of X.

Based on above analysis,eqn(10) can be written as the following form:

$$E_0(X)Y = F_0(X) + U + \rho(t) \quad (19)$$

where $\rho(t) = \Delta F(X) - \Delta E(X)Y$ (20)

is defined as the system uncertainty vector.

The following system without uncertainty is defined as a nominal system:

$$E_0(X)Y = F_0(X) + U \quad (21)$$

For the nominal system in eqn(21), let

$$U = E_0(X)V - F_0(X) \quad (22)$$

where $V=[v_1,v_2,\ldots,v_m]^T$ is chosen to provide stable tracking.Namely,let:

$$v_i = y_{d_i}^{(r_i)} - a_{r_i-2}^i \varepsilon_i^{(r_i-1)} -,\ldots,- a_0^i \varepsilon_i \quad (23)$$

the output tracking error $\varepsilon_i \underline{\Delta} y_i - y_{d_i}$ (24)

where y_{d_i} is the desired reference trajectories of y_i ,and the parameters $a_{r_i-2}^i,\ldots,a_0^i$ in eqn(23) are chosen such that the following polynomial

$$\Delta_i(s) = s^{r_i-1} + a_{r_i-2}^i s^{r_i-2} + \ldots + a_1^i s + a_0^i \quad (25)$$

is Hurwita, then the error dynamics of the nominal system has the following form:

$$\varepsilon_i^{(r_i)} + a_{r_i-2}^i \varepsilon_i^{(r_i-1)} +,\ldots,+ a_0^i \varepsilon_i = 0 \quad (26)$$

Eqn(26) means that the output tracking error ε_i will asymptotically converge to zero.

Consider the system in eqn(19) with uncertain dynamics,the control input of the closed loop system can be set in the following form:

$$U=U_0+U_1 \quad (27)$$

where U_0 is given in eqn(22) to stable control the nominal system(21) and U_1 is used to deal with the effects of the uncertainty.

From eqn(19),we have

$$E_0(X)Y = F_0(X) + U_0 + U_1 + \rho(t) \quad (28)$$

From eqn(23),we have

$$V = Y_d^{(n)} - A_{n-2}\varepsilon^{(n-1)} -,\ldots,- A_0\varepsilon \quad (29)$$

where

$$n = \max_i \{r_i\}$$

$$A_i = diag[a_i^1, a_i^2, \ldots, a_i^m] \quad (i = 0,1\ldots,n-2)$$

$$\varepsilon^{(i)} = [\varepsilon_1^i, \varepsilon_2^i, \ldots, \varepsilon_m^i]^T \quad (i = 1,2,\ldots,n-1)$$

$$Y_d^{(n)} = [y_{d_1}^{(r_1)}, y_{d_2}^{(r_2)}, \ldots, y_{d_m}^{(r_m)}]^T$$

Substitute eqn(22) into (28),we have

$$E_0(X)(Y - V) = U_1 + \rho(t) \quad (30)$$

or $Y - V = Y - Y_d^{(n)} + A_{n-2}\varepsilon^{(n-1)} +,\ldots,+ A_0\varepsilon$

(31)

Then the error dynamics of the closed loop system with uncertainty become

$$\varepsilon^{(n)} + A_{n-2}\varepsilon^{(n-1)} +,\ldots,+ A_0\varepsilon = E_0(X)^{-1}\rho(t) + E_0(X)^{-1}U_1 \quad (32)$$

For the bounded property of system uncertainty vector $\rho(t)$ in eqn (20) we have the following lemma:

Lemma: Consider the system uncertainty vector $\rho(t)$ in eqn(20).If the control input U is designed in the sense that the modulus vector of the control signal is upper bounded by a positive function vector $U_{max}(X)$,

$$|U(t)|_m < U_{max}(X) \quad (33)$$

then the modulus vector of the system uncertainty vector $\rho(t)$ is upper bounded by a positive function vector $\bar{\rho}(X)$

$$|\rho(t)| = |\Delta F(X) - \Delta E(X)Y| < \bar{\rho}(X) \quad (34)$$

Proof:

Eqn(10) can be written as the following form:

$$Y = E(X)^{-1}F(X) + E(X)^{-1}U(t) \quad (35)$$

Using eqn(35) in eqn(20),we have

$$\rho(t) = \Delta F(X) - \Delta E(X)(E(X)^{-1}F(x) + E(X)^{-1}U(t))$$
$$= \Delta F(X) - \Delta E(X)E(X)^{-1}F(x) - \Delta E(X)E(X)^{-1}U(t)$$

(36)

then

$$|\rho(t)| \le |\Delta F(X)| + |\Delta E(X)E(X)^{-1}F(x)| + |\Delta E(X)E(X)^{-1}U(t)|$$
$$\le |\Delta F(X)| + \|\Delta E(X)\| \|E(X)^{-1}\| \|F(x)\|_m + \|\Delta E(X)\| \|E(X)^{-1}\| \|U(t)\|_m$$
$$< Q_2(X) + P_2(X)P_1(X)Q_1(X) + P_2(X)P_1(X)U_{max}(X) = \bar{\rho}(X) \quad (37)$$

Remark 2: It is seen from above lemma that the bounded property of the system uncertainty $\rho(t)$ depends on the system structure and the form of the controller. It will be seen in the next section that the control signal satisfies the condition in eqn(33),and therefore,the bounded condition of the system uncertainty in eqn(34) is always held.

In this paper,we will use the following fuzzy basis function network to learn $\bar{\rho}(X) = [\bar{\rho}_1(X), \bar{\rho}_2(X), \ldots, \bar{\rho}_m(X)]^T$, the upper bound of the system uncertainty vector:

$$\hat{\bar{\rho}}_i(X, \theta_i) = \hat{\theta}_i^T \phi(X) \quad \text{(i=1,2,...,m)} \quad (38)$$

where $\hat{\theta}_i \in R^M$ is the weight vector of the fuzzy basis expansion vector $\phi(X)$, $\phi(X) = [\phi_1(X), \phi_2(X), \ldots, \phi_M(X)]^T$ in which the j-th fuzzy basis expansion defined as

$$\phi_j(X) = \frac{\prod_{i=1}^{n} \mu_{A_i^j}(x_i)}{\sum_{j=1}^{M}\prod_{i=1}^{n} \mu_{A_i^j}(x_i)} \quad (39)$$

with $\mu_{A_i^j}(x_i) = \exp\left[-\frac{(x_i - c_{Aij})^2}{\sigma_{Aij}^2}\right]$ (40)

where $\mu_{A_i^j}(x_i)$ is the membership function of x_i in fuzzy set A_i^j, c_{Aij} is the centre of $\mu_{A_i^j}(x_i)$, σ_{Aij} is the width of $\mu_{A_i^j}(x_i)$, and M is the number of fuzzy rules.

For the further analysis,the following assumptions are made:

A.2: Given an arbitrary small positive constant vector $\tilde{\varepsilon}^*$ and a continuous function vector $\bar{\rho}(X)$ defined in eqn(37) in a compact set Σ, there exists an optimal weight matrix θ^* such that the output vector of the optimal fuzzy network satisfies

$$\max_{X\in\Sigma}|\tilde{\varepsilon}(X)| = \max_{X\in\Sigma}|\theta^{*T}\phi(X) - \bar{\rho}(X)| < \tilde{\varepsilon}^* \quad (41)$$

with $\tilde{\varepsilon}(X) = \theta^{*T}\phi(X) - \bar{\rho}(X)$ (42)

where $\theta^{*T} = [\theta_1^*, \theta_2^*, \ldots, \theta_m^*]^T \in R^{m\times M}$

A.3 The modulus vector of the system uncertainty and its upper bound satisfy the following relationship in the compact set Σ:

$$\bar{\rho}(X) - |\rho(t)| > \tilde{\varepsilon} > \tilde{\varepsilon}^* \quad (43)$$

From (43) and (42) we know:

$$\theta^{*T}\phi(X) - \bar{\rho}(X) > O \quad (44)$$

where O is a zero vector.

The objective of this paper is to design a robust adaptive compensator U_1 using the fuzzy basis function network in eqn(39) so that the closed loop system has strong robustness and the output tracking error is guaranteed to asymptotically converge to zero.

4. Compensator design

Let $E = [\varepsilon, \dot{\varepsilon}, \ldots, \varepsilon^{n-1}]^T$, eqn(32) can then be written as the following state equation form:

$$\dot{E} = \Lambda E + \Psi E_0(X)^{-1}\rho(t) + \Psi E_0(X)^{-1}U_1 \quad (45)$$

where
$$\Lambda = \begin{bmatrix} O & I & O \ldots & O \\ O & O & I \ldots & O \\ \ldots & & & \\ -A_0 & -A_1 & -A_2 \ldots & -A_{n-2} \end{bmatrix} \quad (46)$$

$$\Psi = [O \ O \ O \ldots \ I]^T$$

$E \in R^{m^2\times 1}, \Lambda \in R^{m^2\times m^2}, \Psi \in R^{m^2\times m}$.

For the design of the adaptive compensator using the fuzzy basis function vector and analysis of error convergence of the closed loop system,we have the following theorem:

Theorem: Consider the error dynamics in eqn(45):

If (a) the compensator U_1 is designed as follows

$$U_1 = E_0(X)(C\Psi)^{-1}[-C\Lambda E - sign(S^T)\|C\Psi E_0(X)^{-1}\|\hat{\theta}^T\Phi(X) \quad (47)$$

where the vector $S = CE = [s_1, \ldots, s_m]^T$, $S \in R^m, C \in R^{m\times m^2}$;

(b) the matrix C should be chosen such that the polynomial s_i is Hurwitz about ε_i (i=1,2,...,m);

(c) the matrix $\hat{\theta}$ in eqn(47) is updated by the following adaptive mechanism:

$$\dot{\hat{\theta}} = \eta\phi(X)|E^T C^T|\|C\Psi E_0^{-1}(X)\| \quad (48)$$

with $\eta > 0$ and arbitrary positive initial values vector $\hat{\theta}$, *then* the output tracking errors vector ε asymptotically converges to zero vector.

Proof: Considering following Lyapunov function

$$v = \frac{1}{2}S^T S + \frac{1}{2}\eta^{-1} tr[\tilde{\theta}^T \tilde{\theta}] \quad (49)$$

where $\tilde{\theta} = \theta^* - \hat{\theta}$ (50)

$$\dot{\tilde{\theta}} = -\dot{\hat{\theta}} \quad (51)$$

Then

$$\begin{aligned} \dot{v} &= S^T \dot{S} + \eta^{-1} tr[\tilde{\theta}^T \dot{\tilde{\theta}}] = E^T C^T C\Psi E_0(X)^{-1} \rho(t) \\ &\quad - tr[\theta^{*T} \phi(X) |E^T C^T| \|C\Psi E_0^{-1}(X)\|] \\ &= E^T C^T C\Psi E_0(X)^{-1} \rho(t) - |E^T C^T| \|C\Psi E_0^{-1}(X)\| \theta^{*T} \phi(X) \\ &\le |E^T C^T| \|C\Psi E_0^{-1}(X)\| (|\rho(t)| - \theta^{*T} \phi(X)) \\ &< 0 \end{aligned} \quad (52)$$

5. Conclusion

An adaptive controller using fuzzy basis function vector is proposed in this paper.Our analysis demonstrates that the the weights of the fuzzy network converge to their optimal values,and the values of the weights are adaptively adjusted untill the variable vector S converges to zero.Then the weights will become constants to guarantee that the output tracking error asymptotically converges to zero after S=CE=0.

References

[1] Man Zhihong,X.H.Yu,Q.P.Ha:Adaptive control using fuzzy basis function expansion for SISO linearizable nonlinear systems.Proceedings of the 2nd ASCC,pp.695-698,1997.

[2] Jeffrey T.Spooner and Kevin M.Passino:Stable adaptive using fuzzy systems and neural networks.IEEE Trans on Fuzzy Systems,4(3),pp.339-359,1996.

[3]Chen Wen,Zhang Huaguang: Input/output linearization for nonlinear systems with uncertainties and disturbances using TDC.Int. J: Cybernetics and Systems(to appear).

[4] Dai Changguo: Linear algebra in control systems. Southeastern university publisher,1993(in Chinese).

[5] Isidori,A.:Nonlinear control systems: An introduction. Springer,Berlin,1989.

[6] S.Shankar Sastry and Alberto Isidori: Adaptive control of linearizable systems. IEEE Trans on AC,34(11),pp.1123-1131,1989.

[7] Y.S.Lu and J.S.Chen: A self-organizing fuzzy sliding-mode controller design for a class of nonlinear serve systems.IEEE Trans on Industrial Electronics, vol.41,no.5,pp.492-502,1994.

[8] C.Y.Su and Y.Stepanenko: Adaptive controller of a class of nonlinear systems with fuzzy logic. IEEE Trans. on Fuzzy Syst.,vol.2,no.4,pp.285-294,1994.