퍼지미분방정식의 해의 존재성과 유일성

EXISTENCE AND UNIQUENESS OF SOLUTIONS FOR THE FUZZY DIFFERENTIAL EQUATION

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1. Introduction

This paper is concerned with fuzzy-set-valued mapping of real variable whose values are normal, convex, upper semicontinuous and compactly supported fuzzy set in E^n.

We study differential and integral properties of such functions and give an existence and uniqueness theorem for a solution to a following fuzzy differential equation with fuzzy initial value.

$$(1.1)\qquad \begin{cases} \dfrac{dx(t)}{dt} = f(t, x, \displaystyle\int_0^t k(t,s,x)ds) \\ x(0) = x_0 \end{cases}$$

A differential and integral calculus for fuzzy set valued, shortly fuzzy-valued, mappings was developed in papers of Dubois and Prade [2, 3, 4] and Puri Ralescu [9,10].

As preliminaries we give in Chapter 2 some basic properties of E^n space and their metric.

Finally, we prove the existence and uniqueness of solutions to a fuzzy differential equation with fuzzy initial value when the mapping f on the right hand side of the differential equation (1.1) satisfies a global Lipschitz condition.

2. Preliminaries

In this chapter, we introduce definition and some properties of E^n space and their metric .

Define E^n space satisfies (i) - (iv) below ;

(i) u is normal i.e there exists an $x_0 \in R^n$ such that $u(x_0) = 1$.
(ii) u is fuzzy convex.
(iii) u is upper semicontinuous.
(iv) $[u]^0 = \overline{\{x \in R^n | u(x) > 0\}}$ is compact.

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Let x be a point in R^n and A a nonempty subset of R^n.

We define the distance $d(x,\ A)$ from x to A by

$$d(x, A) = inf\{\|x - a\| | a \in A\}$$

Thus $d(x, A) = d(x, \overline{A}) \geq 0$ and $d(x, A) = 0$ if and only if $x \in \overline{A}$, the closure of A in R^n. Now let A and B be nonempty subsets of R^n. We define the Hausdorff separation of B from A by

$$d_H^*(B, A) = \sup\{d(b, A) | b \in B\}$$

Thus we have $d_H^*(B, A) \geq 0$ with $d_H^*(B, A) = 0$ if and only if $B \subseteq \overline{A}$ In addition, the triangle inequality

$$d_H^*(B, A) \leq d_H^*(B, C) + d_H^*(C, A)$$

holds for all nonempty A, B and C of R^n. In general,however

$$d_H^*(A, B) \neq d_H^*(B, A)$$

This is schematically shown in Figure 2.1

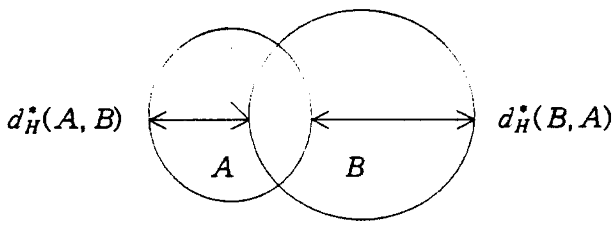

Figure 2.1

We define the Hausdorff distance between nonempty subsets A and B of R^n by

$$d_H(A, B) = \max\{d_H^*(A, B), d_H^*(B, A)\} \tag{2.1}$$

This is now symmetric in A and B. Consequently

$$\begin{aligned} &(i)\ \ d_H(A, B) \geq 0 \ \text{ with } \ d_H(A, B) = 0 \ \text{ if and only if } \ \bar{A} = \bar{B}, \\ &(ii)\ \ d_H(A, B) = d_H(B, A), \\ &(iii)\ d_H(A, B) \geq d_H(A, C) + d_H(C, B) \end{aligned} \tag{2.2}$$

for any nonempty subsets A,B and C of R^n.

We shall meanly consider the following three spaces of nonempty subsets of R^n:

(i) C^n consisting of all nonempty closed subsets of R^n

(ii) K^n consisting of all nonempty compact (i.e closed and bounded) subsets of R^n

(iii) K_c^n consisting of all nonempty compact convex subsets of R^n.

Thus we have the strict inclusions

$$K_c^n \subset K^n \subset C^n$$

Restricting attention to the nonempty closed subsets of R^n. We see the Hausdorff distance (2.1) is a metric the Hausdorff metric. Thus (C^n, d_H) is a metric space.

Proposition 2.1. *(C^n, d_H) is a complete serarable metric space in which K^n and K_c^n are closed subset. Hence, (K^n, d_H) and (K_c^n, d_H) are also complete separable metric spaces. The support $[u]^0$ is also convex, which follows from the fact that*

$$d_H([u^\alpha],\ [u]^0) \to 0 \text{ as } \alpha \to 0^+$$

Combinning above proposition, we have the following

Proposition 2.2. *Let $u \in E^n$ and write $C_\alpha = [u]^\alpha$ for $\alpha \in I$. Then*

(i) *C_α is a nonempty compact comvex subset of R^n for each $\alpha \in I$,*
(ii) *$C_\beta \subseteq C_\alpha$ for $0 \leq \alpha \leq \beta \leq 1$,*
(iii) *$C_\alpha = \bigcap_{i=1}^{\infty} C_{\alpha_i}$ for any nondecresing $\alpha_i \to \alpha$ in I.*

Or equivalently,

$$d_H(C_{\alpha_i}, C_\alpha) \to 0 \quad \text{as} \quad \alpha_i \to \alpha.$$

The converse of Proposition 2.2 also holds.

Proposition 2.3. *Let $C = \{C_\alpha : \alpha \in I\}$ be a familly of subsets of R^n satisfying (i), (ii) and (iii) of Proposition 2.5, and define $u : R^n \to I$ by*

$$u(x) = \begin{cases} 0 & \text{if } x \notin C_0, \\ \sup\{\alpha \in I : x \in C_\alpha\} & \text{if } x \in C_0 \end{cases}$$

Then $u \in E^n$ with $[u]^\alpha = C_\alpha$ for $\alpha \in (0,1]$ and

$$[u]^0 = \overline{\bigcup_{\alpha \in (0,1]} C_\alpha} \subseteq C_0$$

3. Initial value problems of Fuzzy differential Equation.

The classical existence and uniqueness theorem for initial value problems assumes that the mapping nohomogeneous term on the right hand.side of the differential equation satisfies a Lipschitz condition.
In section, we concider the existence and uniqueness of solutions of the following fuzzy differential equation.

$$\text{(3.1)} \qquad \frac{dx(t)}{dt} = f(t, x, \int_0^t k(t,s,x)ds)$$
$$x(0) = x_0$$

On the complete metric space (E^n, d_∞) satisfies a global Lipschitz condition if there exist finite constants $K \geq 0$ and $M \geq 0$ such that

$$d_\infty(f(t,\phi_1,\psi_1), f(t,\phi_2,\psi_2)) \leq K\,(d_\infty(\phi_1,\phi_2) + d_\infty(\psi_1,\psi_2))$$
$$d_\infty(k(s,\tau,\xi_1(\tau)), k(s,\tau,\xi_2(\tau)) \leq M\,(d_\infty(\xi_1(\tau),\xi_2(\tau)))\,.$$

In this case we interpret the right hand side of (3.1) as mappings f and k defined the following that

$$f : [0,T] \times E^n \times E^n \to E^n$$

$$k : [0,T] \times [0,T] \times E^n \to E^n$$

we use the space $C([t_1,t_2],E^n)$ of continuous function $x : [t_1,t_2] \to E^n$ with respect to d_∞ metric on E^n ,on which we define the metric

$$H(x,y) = \max_{t_1 \le t \le t_2} d_\infty(x(t),y(t)). \tag{3.3}$$

Theorem3.1. *For any finite $T > 0$ and $x_0 \in E^n$, a fuzzy differential equation (3.1) on (E^n,d_∞) which satisfies a global Lipschitz condition (3.2) has a unique solution $x : [0,T] \to E^n$ corresponding to the initial data $x(0) = x_0$.*

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