FUZZY ALMOST C-CONTINUOUS MAPPINGS

KUL HUR, JU RAN MOON AND JANG HYUN RYOU

Department of Mathematics, Wonkwang University 344-2, Shinyong Iksan Chonbuk, Korea 570-749 TEL:(0653)850-6190, FAX:(0653)52-5139 E-Mail:kulhur@wonnms.wonkwang.ac.kr

1. Preliminaries.

Definition 1.1[1]. *A mapping $f : X \to Y$ is said to be* **fuzzy almost continuous** *(or simply,* **f-a-continuous***) at a fuzzy point x_λ in X if for each fuzzy open nbd V of $f(x_\lambda)$ in Y, exist a fuzzy open nbd U of $f(x_\lambda)$ in X such that $f(U) \subset int(clV)$. And f is said to be* **fuzzy almost continuous** *(or simply,* **f-a-continuous***)(on X), if it is f-a-continuous at each fuzzy point in X.*

Definition 1.2 [2]. *A mapping $f : X \to Y$ is said to be* **fuzzy c-continuous** *(or simply,* **f-c-continuous***) at a fuzzy point x_λ in X if for each fuzzy open nbd V of $f(x_\lambda)$ in Y such that V^c is fuzzy compact in Y, exist a fuzzy open nbd U of x_λ in X such that $f(U) \subset V$. And f is said to be* **fuzzy c-continuous** *(or simply,* **f-c-continuous***)(on X), if it is f-c-continuous at each fuzzy point in X.*

The following Lemmas are useful characterization of f-c-continuous, and f-a-continuous mappings.

Lemma 1.3 [1]. *Let $f : X \longrightarrow Y$ be a mapping. Then the followings are equivalent;*

(1) f is f-a-continuous.

(2) The inverse image of every fuzzy regularly open set in Y is fuzzy open in X.

(3) The inverse image of every fuzzy regularly closed set in Y is fuzzy closed in X.

Lemma 1.4 [2]. *Let $f : X \to Y$ be a mapping. Then the followings are equivalent;*

(1) f is f-c-continuous.

(2) The inverse image of every fuzzy open set in Y having fuzzy compact compliment is fuzzy open in X.

(3) The inverse image of every fuzzy closed compact set in Y is fuzzy closed in X.

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2. Basic properties of fuzzy almost c-continuous mappings.

From Definitions 1.1 and 1.2 , we introduce concept of fuzzy almost c-continuity.

Definition 2.1. *A mapping $f : X \longrightarrow Y$ is said to be* **fuzzy almost c-continuous** *(or simply,* **f-a-c-continuous***) at a fuzzy point x_λ in X if for each fuzzy open nbd V of $f(x_\lambda)$ in Y having fuzzy compact complement, exist a fuzzy open nbd U of x_λ in X such that $f(U) \subset int(cl\ V)$. And the mapping f is said to be* **fuzzy almost c-continuous** *(or simply,* **f-a-c-continuous***) (on X), if it is f-a-c-continuous at each fuzzy point in X.*

It is clear that all f-a-continuous mappings are f-a-c-continuous.

Theorem 2.2. *The following implications hold:*

f-continuous $\Longrightarrow$ f-c-continuous $\Longrightarrow$ f-a-continuous $\Longrightarrow$ f-a-c-continuous.

Proof. *Omitted.*

Theorem 2.3. *For a mapping $f : X \rightarrow Y$, the followings are equivalent.*

(1) f is f-a-c-continuous.

(2) The inverse image of every fuzzy regularly open set in Y having fuzzy compact complement is fuzzy open in X.

(3) The inverse image of every fuzzy reqularly closed compact set in Y is fuzzy closed in X.

(4) For each $x_\lambda \in Fp(X)$, and each fuzzy regularly open set in Y containg $f(x_\lambda)$ having fuzzy compact complement, exist a fuzzy open nbd U of x_λ in X such that $f(U) \subset V$.

(5) For each $x_\lambda \in Fp(X)$, and each fuzzy open nbd V of $f(x_\lambda)$ in Y having fuzzy compact complement, f^{-1} $(int(cl\ V)) \in FO(X)$.

Proof. *Omitted.*

Theorem 2.4. *Any restriction of a f-a-c-continuous mapping is also f-a-c-continuous.*

Proof. *Omitted.*

Theorem 2.5. *If $f : X \to Y$ is f-continuous and $g : Y \to Z$ is f-a-c-continuous, then $g \circ f : X \to Z$ is f-a-c-continuous.*

Proof. *Omitted.*

Theorem 2.6. *Let f:$X \to Y$ be surjective and f-open. If $g \circ f : X \to Z$ is f-a-c-continuous, then $f : Y \to Z$ is f-a-c-continuous.*

Proof. *Omitted.*

Lemma 2.7. *Let $f : X \to Y$ be a mapping and let $x_\lambda \in Fp(X)$. If there exists a fuzzy open nbd U of x_λ in X such that U=S(U) and $f|_U$ is f-a-c-continuous at x_λ, then f is f-a-c-continuous at x_λ.*

Proof. *Omitted.*

Theorem 2.8. *Let $\{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy open cover of X such that $U_\alpha = S(U_\alpha)$ for each $\alpha \in \Lambda$. If $f|_{U_\alpha}$ is f-a-c-continuous for each $\alpha \in \Lambda$, then f is f-a-c-continuous.*

Proof. *Omitted.*

Theorem 2.9. *Let $f : X \to Y$ be a mapping and $X = A \cup B$, where $A, B \in FC(X)$ such that A=S(A) and B=S(B). If $f|_A$ and $f|_B$ are f-a-c-continuous, then f is f-a-c-continuous.*

Proof. *Omitted.*

Theorem 2.10. *Let $f : X \to Y$ be a mapping and $X = A \cup B$, where A=S(A) and B=S(B). If $f|_A$ and $f|_B$ are f-a-c-continuous at $x_\lambda \in Fp(A \cap B)$, then f is f-a-c-continuous at x_λ.*

Proof. *Omitted.*

Theorem 2.11. *Let $f : X \to Y$ be a mapping and X a fuzzy compact space. If the graph mapping $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ for each $x \in X$ is f-a-c-continuous, then f is f-a-c-continuous.*

Proof. *Omitted.*

3. Further Results.

Lemma 3.1. *For any fts $(Y,\mathcal{T})$, let $\mathfrak{B}$ be the family of all the fuzzy regularly open sets in Y having fuzzy compact complement. Then $\mathfrak{B}$ is a base for a fuzzy topology $\mathcal{T}_*$ on Y. In this case, $\mathcal{T}_*$ is called* **fuzzy regular compact complement topology** *on Y by $\mathcal{T}$.*

Lemma 3.2. *Let $(Y,\mathcal{T})$ be a fts and let $\mathcal{T}_*$ the fuzzy regular compact complement topology on Y by $\mathcal{T}$. Let $f : X \to (Y, \mathcal{T})$ be a mapping at $f_* : X \to (Y, \mathcal{T}_*)$ a mapping defined by $f_*(x) = f(x)$ for each $x \in X$. Then;*

(1) $\mathcal{T}_* \subset \mathcal{T}$.

(2) f is f-a-c-continuous iff f_* is f-continuous.

(3) $id : (Y, \mathcal{T}) \to (Y, \mathcal{T}_*)$ is f-continuous and $id^{-1} : (Y, \mathcal{T}_*) \to (Y, \mathcal{T})$ is f-a-c-continuous.

Proof. *Omitted.*

Theorem 3.3. *For any $(Y,\mathcal{T})$, the space $(Y ,\mathcal{T}_*)$ is fuzzy compact.*

Proof. *Omitted.*

References

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