A Note On Fuzzy C-continuous Mappings

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1. Preliminaries.

Throughout this paper, we will use the concept of a fuzzy topological space (or simply, fts) in the sence of Chang[3] and denote the collection of all the fuzzy open(resp. closed)sets in a fts X as FO(X)(resp. FC(X)).

Definition 1.1 [4]. *Let $(X, \mathcal{T})$ be a fts. Then X is said to be* $\mathbf{T}_2$ *(or* **Hausdorff***) if and only if for two distinct point x_λ and y_μ in X;*

(i) If $x \neq y$, then there exists open sets A and B in X such that $x_\lambda \in A$, $y_\mu \in B$ and $A\bar{q}B$.

(ii) If $x = y$ and $\lambda < \mu$, then there exists open nbd A of x_λ and there exists open q-nbd B of y_μ such that $A\bar{q}B$.

Definition 1.2 [6]. Let $\mathfrak{B}$ be a collection of fuzy sets in a fts X. Then $\mathfrak{B}$ is called a **filter base** if for any finite subset $\{ U_i : i = 1, \dots n\}$ of $\mathfrak{B}$, $\bigcap_{i=1}^{n} U_i \neq \emptyset$.

Definition 1.3 [6]. A subset A of a fts $(X,\mathcal{T})$ is said to be **compact** if for each filter base $\mathfrak{B}$ such that every finite intersection of members of $\mathfrak{B}$ is q-coincident with A,

$$(\bigcap_{B\in \mathrm{B}} \mathrm{cl}\, B) \cap A \neq \emptyset.$$

Theorem 1.4 [6]. *A compact subset of a $\mathcal{T}_2$-space is closed.*

Theorem 1.5 [6]. *Let $(X, \mathcal{T})$ be a fts and $\{F_1, \cdots , F_k \}$ a family of closed subsets of X. In order that $F = \bigcup_{i=1}^{k} F_i$ be compact it is suffcient that the space F_i be compact for $i = 1,2, \cdots ,k$.*

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Definition 1.6 [8]. Let (X, $\mathcal{T}$) be a fts and let A be subset of X. Then the family $\mathcal{T}_A = \{U|_A : U \in \mathcal{T}\}$ is called the **relative fuzzy topology** of $\mathcal{T}$ to A. Such a fuzzy topological space (A,$\mathcal{T}_A$) is called a **subspace** of (X, $\mathcal{T}$). A **$\mathcal{T}_A$-open**(resp. **$\mathcal{T}_A$-closed**) fuzzy set is also called a **relative open**(resp. **closed**) fuzzy set in A.

It is clear that $\mathcal{T}_A$ is a fuzzy topology on A.

2. Basic properties of fuzzy c-continuous mapping.

Definition 2.1. Let X and Y be fuzzy topological spaces, let $f : X \longrightarrow Y$ be a mapping and let $x_\lambda \in F_p(X)$. Then f is said to be **fuzzy c-continuous** (or simple **f-c-continuous**) at x_λ if for each open nbd V of $f(x_\lambda)$ and V^c is fuzzy compact in Y, there exists an open nbd U of x_λ such that $f(U) \subset V$. The mapping f is said to be **fuzzy c-continuous** (or simple, **f-c-continuous**)(on X) if f is f-c-continuous at each fuzzy point in X.

It is clear that if f is f-continuous, then f is f-c-continuous.

Theorem 2.2. *Let X and Y be fuzzy topological spaces and let $f : X \longrightarrow Y$ be a mapping. Then the following statements are equivalent.*

(1) f is f-c-continuous.

(2) If V is a fuzzy open set in Y with compact complement, then $f^{-1}(V) \in FO(X)$.

These statements are implied by

(3) If C is a fuzzy compact set in Y, then $f^{-1}(C) \in FC(X)$. And moreover, if Y is Hausdorff, then all the statements are equivalent.

Proof. Omitted.

Theorem 2.3. *If $f : X \longrightarrow Y$ is f-c-continuous and A is a subset of X, then $f|_A : A \longrightarrow Y$ is f-c-continuous.*

Proof. Omitted.

Theroem 2.4. *If f: $X \longrightarrow Y$ is f-continuous and $g : Y \longrightarrow Z$ is f-c-continuous. Then $g \circ f : X \longrightarrow Z$ is f-c-continuous.*

Proof. Omitted.

Theorem 2.5. *Let X and Y be fuzzy topological space and let $X = A \bigcup B$, where A,B $\in$ FO(X) (resp. A, B $\in$ FC(X)) such that $A = S(A)$ and $B = S(B)$. Suppose $f : X \longrightarrow Y$ is a mapping such that $f|_A$ and $f|_B$ are f-c-continuous. Then f is f-c-continuous.*

Proof. First assume A,B $\in$ FO(X). Let V $\in$ FO(Y) such that V^c is fuzzy compact in Y. Since $f|_A$ and $f|_B$ are f-c-continuous. $(f|_A)^{-1}(V) \in FO(A)$ and $(f|_B)^{-1}(V) \in FO(B)$. Since A,B $\in$ FO(X), $(f|_A)^{-1}(V)$ and $(f|_B)^{-1}(V) \in FO(X)$. On the other hand, $f^{-1}(V) = (f|_A)^{-1}(V) \cup (f|_B)^{-1}(V)$. So $f^{-1}(V) \in FO(X)$. Hence f is f-c-continuous.

Now assume that A,B $\in$ FC(X). Let $X_\lambda \in F_p(X)$ and let V $\in$ FO(Y) such that $f(x_\lambda) \in V$ and V^c is fuzzy compact in Y. Then either x_λ in $A \cap B$, $x_\lambda \in A$, $x_\lambda \bar{q} B$ or $x_\lambda \bar{q} A$, $x_\lambda \in B$.

Case 1 : Suppose $x_\lambda \in A \cap B$. Since $f|_A$ is f-c-continuous, $f|_A$ is f-c-continuous at x_λ. Thus there exist $U \in FO(A)$ such that $x_\lambda \in U$ and $(f|_A)(U) \subset V$. Since $U \in FO(A)$, there exist $U' \in FO(X)$ such that $U = U' \cap A = U'|A$. Since $f|_B$ is f-c-continuous, $f|_B$ is f-c-continuous at x_λ. Thus there exists W $\in$ FO(B) such that $x_\lambda \in W$ and $(f|_B)(W) \subset V$. Since $W \in FO(B)$, there exist $W' \in FO(X)$ such that $E = W' \cap B = W|_B$. Let $O = U' \cap W'$. Then $O \in FO(X), x_\lambda \in O$, and $f(O) = f(U' \cap W') \subset f(U \cup W) = f(U) \cup f(W) \subset V$. Thus f is f-c-continuous at each x_λ in X. Hence f is continuous.

Case 2 : Suppose $x_\lambda \in A$ and $x_\lambda \bar{q} B$. Since $f|_A$ is f-c-continuous at x_λ, there exists $U \in FO(A)$ such that $x_\lambda \in U$ and $(f|_A)(U) \subset V$. Since $U \in FO(A)$, there exists $U' \in FO(X)$ such that $U = U' \cap A$. Let $W = U' \cap B^c$, Then clearly, $W \in FO(X)$. Since $x_\lambda \bar{q} B$, by theorem , $x_\lambda \in B^c$. Thus $x_\lambda \in U' \cap B^c = W$. Furthermore $f(W) \subset V$. So f is f-c-continuous at each x_i in X. Hence f is f-c-continuous. Case3 : Suppose $x_\lambda \in B$ and $x_\lambda \bar{q} A$. This case follows exactly like case2.

Theorem 2.6. *Let X be a fts and Y be a fuzzy Hausdorff space. If $f : X \longrightarrow Y$ is bijection and f-continuous, then $f^{-1} : Y \longrightarrow X$ is f-c-continuous.*

Proof. Omitted.

3. Further Results.

Definition 3.1[7]. Let $(X, \mathcal{T})$ be a fts and let $\mathfrak{B} \subset$ T. Then $\mathfrak{B}$ is called a base for T if for each U $\in$ T either U = $\emptyset$ or there exists a subfamily $\mathfrak{B}'$ of $\mathfrak{B}$ such that $U = \bigcup_{B \in \mathfrak{B}'} B$.

Lemma 3.2. *Let $\mathfrak{B}$ be a family of fuzzy sets in a set X. Then $\mathfrak{B}$ is a base for some fuzzy topology on X if and only if*

(1) $X = \bigcup_{B \in \mathfrak{B}'} B.$

(2) if $B_1, B_2 \in \mathfrak{B}$ and $x_\lambda \in B_1 \cap B_2$, then there exists $B \in \mathfrak{B}$ such that $x_\lambda \in B \subset B_1 \cap B_2$.

Proof. Omitted.

Thenorem 3.3. *Let $(X,\mathcal{T})$ be a fts and let $\mathfrak{B}$ be the family of members of $\mathcal{T}$ having compact complement. Then $\mathfrak{B}$ is a base for a fuzzy topology $\mathcal{T}^*$ on X. Moreover $\mathcal{T}^* \subset \mathcal{T}$ and $(X, \mathcal{T}^*)$ is a fuzzy compact space.*

In this case, $\mathcal{T}^*$ is called a **fuzzy compact complement topology** *on X by $\mathcal{T}$.*

Proof. Omitted.

Theorem 3.4. *Let X and $(Y,\mathcal{T})$ be any fuzzy topological space and let $\mathcal{T}^*$ be the fuzzy compact complement topology on Y by $\mathcal{T}$. Let $f : X \longrightarrow (X, \mathcal{T})$ be any mapping $id : (Y,\mathcal{T}) \longrightarrow (Y, \mathcal{T}^*)$ the identity mapping and $f^* : X \longrightarrow (Y,\mathcal{T}^*)$ the mapping defined by $f^*(x) = f(x)$ for all $x \in X$. Then;*

(1) $id \circ f = f^*$

(2) f is f-c-continuous iff f^* is f-continuous.

(3) id is f-continuous and id^{-1} is f-c-continuous.

Proof. Omitted.

Theorem 3.5. *Let $f : X \longrightarrow (Y, \mathcal{T})$ be f-c-continuous. If f^* is f-closed(resp. f-open). Then f is f-closed(resp. f-open).*

Proof. Omitted.

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