강인한 직접 적응 퍼지 제어기

Robust Direct Adaptive Fuzzy Controller

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Abstract : In this paper is proposed a new direct adaptive fuzzy controller that can be applied for tracking control of a class of uncertain nonlinear SISO systems. It is shown that, in the presence of the perturbations such as fuzzy approximation error and external disturbance, boundedness of all the signals in the system is ensured, while under the assumption of no perturbations, the stability of the overall system is guaranteed. Also, the concept of persistent excitation in the adaptive fuzzy control systems is introduced to guarantee the convergence and the boundedness of adaptation parameter in the proposed controllers. Simulation example shows the effectiveness of the proposed method in the presence of fuzzy approximation error and external disturbance.

Keywords – Adaptive fuzzy control, robustness, stability, boundedness.

1. Introduction

The stability of adaptive fuzzy control systems has been studied by many researchers [1, 2, 3, 4]. Wang has proposed stable direct and indirect adaptive fuzzy controllers for nonlinear systems [1, 2]. There are proposed adaptive fuzzy controllers combined with sliding mode control regime [3, 4]. Chen *et al.* proposed indirect and direct adaptive fuzzy controllers in view of H^∞ control technique to achieve H^∞ tracking performance [5].

The approximation error in the adaptive fuzzy control systems is generally inevitable [1, 2]. The approximation error can be considered as a disturbance in the adaptive fuzzy control systems, which may cause the divergence of the adaptive process. Also, in the practical applications, system variables are continually contaminated by external disturbances and noise. Thus, it is important to investigate the behavior of the adaptive fuzzy control system in the presence of both external disturbances and approximation error. The adaptive fuzzy controller should be designed to be robust to cope with external disturbances and approximation error. That is, in the absence of approximation error and external disturbances, the objective is to design a stable adaptive fuzzy controller that would ensure asymptotic trajectory tracking; however, in the presence of approximation error and external disturbances, it is no longer possible to ensure asymptotic trajectory tracking, and the objective is to design a controller that ensure the boundedness of all the signals in the system and satisfy the specified performances. In this study, design of the adaptive fuzzy controller to achieve both objectives is discussed.

The stability of certain adaptive fuzzy control systems can be assumed as shown in [1, 2]. However, the approximation error of the fuzzy system and external disturbances may deteriorate the tracking performance due to the fact that the influence of approximation error and external disturbances on the tracking error cannot be efficiently eliminated [5, 6]. In these cases, the parameter error in the adaptation process may not be small even when the tracking error is small and, in addition, it can diverge as in the case of conventional adaptive control systems. In this regard, it could be a challenging realistic problem to design an adaptive fuzzy controller that guarantees the boundedness of both tracking error and parameter error in the presence of approximation error and external disturbances. Wang[1, 2] and Chen *et al.*[5] employed the projection technique in the update law to keep the parameters within an upper norm bound of parameter vector. This method requires *a priori* information regarding the bound on the parameter vector and the parameter may converge to its upper or lower bound under continual perturbations. Lu *et al.*[3] and Su *et al.*[4] used an additional switching controller in sliding mode control to attenuate the influence of approximation error and external disturbances. However, this latter method may lead to generation of higher frequency switching control signals and may excite high frequency modes of unmodeled dynamics as in sliding mode control. To resolve these problems, we propose a new adaptive algorithm which can improve the robustness of the adaptive fuzzy control system.

In this paper, a new direct adaptive fuzzy controllers is proposed so that not only the stability of adaptive fuzzy control system is guaranteed but also, in the presence of approximation error and external disturbances, boundedness of all the signals in the system is ensured. Also, the concept of *persistent excitation* in the adaptive fuzzy control systems is first introduced to guarantee the convergence and boundedness of adaptation parameter in the fuzzy system. Simulation results indicate the effectiveness of the proposed method in the presence of approximation error and external disturbances.

2. Robust Direct Adaptive Fuzzy Control

2.1 Fuzzy Logic systems

In this section, we consider a fuzzy logic system whose basic configuration is shown in Fig. 1.

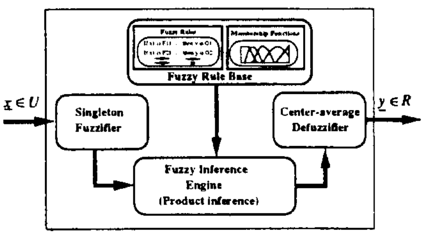


Fig. 1. Basic configuration of fuzzy logic system.

Specifically, consider the fuzzy logic system studied in [1, 2] as:

$$y(\underline{x}) = \frac{\sum_{k=1}^{M} \theta_k (\Pi_{i=1}^{n} \mu_{F_{k,i}}(x_i))}{\sum_{k=1}^{M} (\Pi_{i=1}^{n} \mu_{F_{k,i}}(x_i))}, \tag{1}$$

where M is the number of the fuzzy rules, θ_k is the point at which μ_{G_k} achieves its maximum value. Let us defene *fuzzy basis function* as $\xi_k(\underline{x}) = \Pi_{i=1}^{n} \mu_{F_{k,i}}(x_i) / \sum_{k=1}^{M} (\Pi_{i=1}^{n} \mu_{F_{k,i}}(x_i))$, $k = 1, 2, \cdots, M$. Then, the fuzzy logic system (1) can be rewritten as the fuzzy basis function expansion [2]:

$$y(\underline{x}) = \sum_{k=1}^{M} \xi_k(\underline{x}) \theta_k = \underline{\xi}^T(\underline{x}) \underline{\theta}, \tag{2}$$

where $\underline{\theta} = (\theta_1, \cdots, \theta_M)^T$ is an adjustable parameter vector and $\underline{\xi} = (\xi_1, \cdots, \xi_M)^T$ is a vector of fuzzy basis functions, which in this study is called a *fuzzy basis vector*. It is remarked that the fuzzy logic system (2) is capable of uniformly approximating any nonlinear function to any degree of accuracy [2]. In this study, we use the fuzzy logic systems (2) as basic building blocks of adaptive fuzzy controller for nonlinear systems.

2.2 Design of a direct adaptive fuzzy controller

Consider a class of single-input nonlinear systems described by

$$x^{(n)} = f(x, \dot{x}, \cdots, x^{(n-1)}) + u + d(t), \quad y = x \tag{3}$$

where f is unknown but bounded continuous function, d denotes the external disturbance which is unknown but bounded, and $u \in R$ and $y \in R$ are the input and output of the system, respectively. Let $\underline{x} = (x, \dot{x}, \cdots, x^{(n-1)})^T = (x_1, x_2, \cdots, x_n)^T \in R^n$ be the state vector of the system which is assumed to be available.

The control objective is to force y to follow a given bounded reference signal y_m. Let us denote the output tracking error $e = y_m - y$, the error state vector $\underline{e} = (e, \dot{e}, \cdots, e^{(n-1)})^T = (e_1, e_2, \cdots, e_n)^T \in R^n$ and the tracking error metric as

$$s = \underline{c}^T \underline{e}, \tag{4}$$

where $\underline{c} = (c_1, \cdots, c_{n-1}, 1)^T$ such that all roots of $h(p) = p^{n-1} + \cdots + c_2 p + c_1 = 0$ are in the left-half plane.

The direct adaptive fuzzy control(DAFC) in [1] has attempt to directly approximate the following control law

$$u^* = -f(\underline{x}) + y_m^{(n)} + \underline{k}^T \underline{e}. \tag{5}$$

Let us consider the following control law

$$u_{fz}(\underline{x}/\underline{\theta}) = \underline{\xi}^T(\underline{x}) \underline{\theta}. \tag{6}$$

From (3) and (6), the following tracking dynamic equation can be obtained.

$$e^{(n)} = -\underline{k}^T \underline{e} - (u_{fz}(\underline{x}/\underline{\theta}) - u^*) - d. \tag{7}$$

Define the optimal parameter vector $\underline{\theta}^*$ as follows:

$$\underline{\theta}^* \equiv argmin_{\underline{\theta}\in\Omega_\theta}[\sup_{\underline{x}\in\Omega_x} |u^* - u_{fz}(\underline{x}/\underline{\theta})|], \quad (8)$$

where Ω_θ and Ω_x are the sets of suitable bounds on $\underline{\theta}$ and $\underline{x}$, respectively. Also, the minimum approximation error is defined as

$$\omega_d \equiv u_{fz}(\underline{x}/\underline{\theta}^*) - u^*. \quad (9)$$

The error dynamic equation(7) can be rewritten as

$$e^{(n)} = -\underline{k}^T\underline{e} + \underline{\xi}^T(\underline{x})\underline{\phi} - (\omega_d + d), \quad (10)$$

where $\underline{\phi} = \underline{\theta}^* - \underline{\theta}$. Here, we can find that, when $\underline{\phi}, \omega_d$ and d are bounded, the tracking errors $e^{(i)}$, $i = 0, 1, \cdots, n-1$ will be bounded.

2.3 *Robustness of the proposed DAFC system*

From (6) and (10), the time derivative of the tracking error metric (4) can then be written as

$$\begin{aligned} \dot{s} &= -k_d s + (u_{fz}(\underline{x}/\underline{\theta}^*) - u_{fz}(\underline{x}/\underline{\theta})) - (\omega_d + d) \\ &= -k_d s + \underline{\xi}^T(\underline{x})\underline{\phi} - (\omega_d + d). \quad (11) \end{aligned}$$

Let us assume that $|d(t)| < D_d$, $\epsilon_d = D_d + |\omega_d|$ and $\lambda = k_d + \frac{1}{k_d}$. Then, we get the following theorem which shows the properties of the proposed direct adaptive fuzzy controller. For clarification of the remaining proof, the notations are simplified as follows: $u_{fz} \triangleq u_{fz}(\underline{x}/\underline{\theta})$ and $u^*_{fz} \triangleq u_{fz}(\underline{x}/\underline{\theta}^*)$.

Theorem 1 *Consider the nonlinear system (3) with the control law (6) and the adaptive law*

$$\dot{\underline{\theta}} = \eta \cdot (\dot{s} + \lambda_d s) \cdot \underline{\xi}(\underline{x}). \quad (12)$$

Then, the tracking error and the approximation error of fuzzy system are uniformly bounded. These bounds depend on the bound on approximation error ω_d and bound on external disturbance d. Moreover, If the fuzzy basis vector $\underline{\xi}(\underline{x})$ is persistently exciting, the parameter error vector $\underline{\phi}$ is bounded. Also, whenever ω_d and d tend to zero, tracking error and approximation error of fuzzy system will converge to zero. That is, if $\omega_d, d, \epsilon_d \in \mathcal{L}_2$, then $s, \underline{e}, u^*_{fz} - u_{fz} \in \mathcal{L}_2$,

Proof

Consider the Lyapunov function candidate

$$V(s, \underline{\phi}) = \frac{1}{2}(\frac{1}{k_d}s^2 + \frac{1}{\eta}\underline{\phi}^T\underline{\phi}). \quad (13)$$

Differentiating the Lyapunov function V with respect to time,

$$\begin{aligned} \dot{V} &= \frac{1}{k_d}s\dot{s} + \frac{1}{\eta}\dot{\underline{\phi}}^T\underline{\phi} \quad (14) \\ &\le -s^2 + \frac{1}{k_d}\epsilon_d|s| - (u^*_{fz} - u_{fz})^2 + \epsilon_d|u^*_{fz} - u_{fz}| \end{aligned}$$

From above inequality, $\dot{V} < 0$ outside a compact region Ω_d, where the set Ω_d is defined as

$$\Omega_d \triangleq \{(s, \underline{\phi}) \mid |s| \le (\frac{1}{2} - \frac{1}{k_d})\epsilon_d,\ |\underline{\xi}^T(\underline{x})\underline{\phi}| \le (1 + \frac{1}{2k_d})\epsilon_d\}. \quad (15)$$

Since $V(s, \underline{\phi})$ is a scalar function with continuous partial derivatives in Ω_d^c and satisfy 1) $V(s, \underline{\phi}) > 0\ \forall\ s, \underline{\phi} \in \Omega_d^c$, 2) $\dot{V}(s, \underline{\phi}) \le 0\ \forall\ s, \underline{\phi} \in \Omega_d^c$, and 3) $V(s, \underline{\phi})$ belong to class $\mathcal{K}_\infty$, s is uniformly bounded, i.e., $|s| \le \Delta_d$. Also, (4) can be rewritten as

$$e^{(n-1)} + c_{n-1}e^{(n-2)} + \cdots + c_2\dot{e} + c_1 e = s. \quad (16)$$

From the input-to-state stability theory[9] and the design assumption about the vector $\underline{c}$, if the input s of the system (16) is bounded, then the tracking errors $e^{(i)}$, $i = 0, 1, \cdots, n-1$ are bounded.

The adaptation law (12) can be rewritten as

$$\dot{\underline{\phi}} = -\eta\underline{\xi}(\underline{x})\underline{\xi}(\underline{x})^T\underline{\phi} + \eta((\lambda_d - k_d)s - \omega_d - d)\underline{\xi}(\underline{x}). \quad (17)$$

Since $\underline{\xi}(\underline{x})$ is a piecewise continuous and bounded, when $\underline{\xi}(\underline{x})$ is persistent exciting, i.e.,

$$\int_t^{t+T_0} \underline{\xi}(\underline{x}(\tau))\underline{\xi}^T(\underline{x}(\tau))\, d\tau \ge \alpha I \quad \forall\ t \ge t_0, \quad (18)$$

it can be shown [7] that the nominal system $\dot{\underline{\phi}} = -\eta\underline{\xi}(\underline{x})\underline{\xi}(\underline{x})^T\underline{\phi}$ has an exponentially stable equilibrium state $\underline{\phi} = 0$. Since $\underline{\xi}(\underline{x}), s, \omega_d$ and d are bounded, if s is assumed to be bounded, the perturbation term $\underline{\rho} = \eta((\lambda_d - k_d)s - \omega_d - d)\underline{\xi}(\underline{x})$ is also bounded. Since the system (17) has the exponentially stable nominal dynamics and the perturbation $\underline{\rho}$ is bounded, the parameter error vector $\underline{\phi}$ is bounded [9]. Therefore, the parameter vector $\underline{\theta}$ also is bounded.

From the equation (14),

$$\dot{V} \le -\frac{1}{2}s^2 - \frac{1}{2}(u^*_{fz} - u_{fz})^2 + \frac{1}{2}(1 + \frac{1}{k_d^2})\epsilon_d^2. \quad (19)$$

Integrating both side of (19), we obtain

$$\begin{aligned} \int_0^\infty [s^2 + (u^*_{fz} - u_{fz})^2]dt &\le 2V_0 - 2V_\infty \\ &+ (1 + \frac{1}{k_d^2})\int_0^\infty \epsilon_d^2 dt. \end{aligned}$$

Because V_0, V_∞ are bounded, if $\omega_d, d, \epsilon_d \in \mathcal{L}_2$, it follows that $s, \underline{e}, u^*_{fz} - u_{fz} \in \mathcal{L}_2$. Also, from (6), (10) and(11), we can find that $s, \underline{e}, u^*_{fz} - u_{fz} \in \mathcal{L}_\infty$. Therefore, from the lemma in [7](if $g \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{g} \in \mathcal{L}_\infty$, then $\lim_{t\to\infty}|g| = 0$), $s, \underline{e}$ and $u^*_{fz} - u_{fz}$ converge to zero as $t \to \infty$. This completes the proof. ■

Also, in the absence of approximation error and external disturbance, we obtain the following result.

Corollary 1 *Consider the system (3) with the control law (6) and the learning law (12). Then, the tracking error and the approximation error of fuzzy system asymptotically converge to zero, and the parameter error vector $\underline{\phi}$ remains bounded. Moreover, If the fuzzy basis vector $\underline{\xi}(\underline{x})$ is persistently exciting, the parameter error vector $\underline{\phi}$ asymptotically converges to zero.*

Proof Similar to Theorem 1.

Remark 1 If the condition of Theorem 1 are satisfied and a tracking error metric (4) is defined by $s(t) = (\frac{d}{dt} + \gamma)^{n-1} e(t)$, $\gamma > 0$, then the actual tracking errors can be shown [8] to be asymptotically bounded by

$$|e^{(i)}(t)| \leq 2^i \gamma^{i-n+1} (\frac{1}{2} - \frac{1}{k_d})\epsilon, \ i = 0, \cdots, n-1.$$

Remark 2 The equation (17) represents a stable time-varying filter. Thus, the parameter search goes along a "filtered" direction. Also, this indicate that parameter and tracking error convergence can be smoother and faster than the adaptive fuzzy controllers with the adaptive law of the form

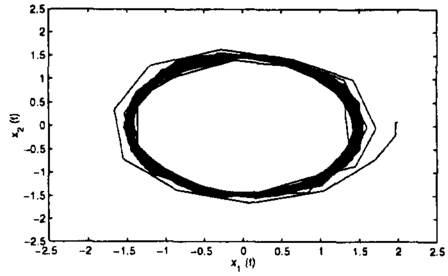
$$\dot{\underline{\theta}} = -\ \eta \cdot s \cdot \underline{\xi}(\underline{x}). \quad (20)$$

3. Simulation Example

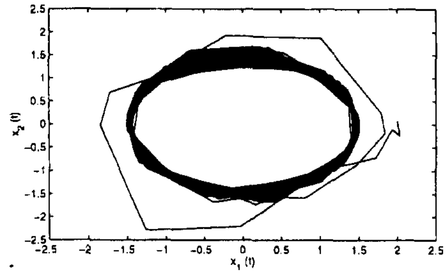
Consider the Duffing forced oscillation system [2]

$$\begin{aligned} \dot{x_1} &= x_2, \\ \dot{x_2} &= -0.1 x_2 - x_1^3 + 12\cos t + u + d. \end{aligned}$$

The system is chaotic if $u(t) = 0$ and $d(t) = 0$. The reference signal is assumed to be $y_m(t) = 1.5\sin(t)$ and the external disturbance $d(t) = \sin(6t)$ is present. Let the initial state $\underline{x}(0) = (2,0)^T$ and the parameter vectors $\underline{\theta}(0) = \underline{0}$. We simply choose $k_2 = 2$, $k_1 = 1$, $\eta = 2.5$, and $\lambda_d = 2$. Also, the membership functions for x_1 and x_2 are selected as follows: $\mu_{F_{1,i}}(x_i) = exp[-((x_i + 2.5)/0.4)^2]$, $\mu_{F_{2,i}}(x_i) = exp[-((x_i + 1.5)/0.4)^2]$, $\mu_{F_{3,i}}(x_i) = exp[-((x_i + 0.5)/0.4)^2]$, $\mu_{F_{4,i}}(x_i) = exp[-((x_i)/0.3)^2]$, $\mu_{F_{5,i}}(x_i) = exp[-((x_i + 0.5)/0.4)^2]$, $\mu_{F_{6,i}}(x_i) = exp[-((x_i + 1.5)/0.4)^2]$ and $\mu_{F_{7,i}}(x_i) = exp[-((x_i + 2.5)/0.4)^2]$, which cover the interval $[-2.5, 2.5]$. We directly integrate the differential equations of the closed-loop system and the adaptive law with step size 0.02. Fig. 2-3 show the simulation results by proposed method and the conventional method using the adaptive law (20). Also, we can see that, in the conventional method, even though the tracking error is bounded, the parameter vectors $\underline{\theta}$ diverges as $t \to \infty$. However, in the proposed method, we can find that the effects of both the approximation error and external disturbance are attenuated efficiently, and the all signals in the system are bounded.

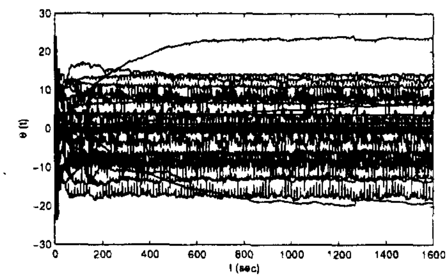


(a) Using proposed adaptive law(12).



(b) Using the adaptive law(20).

Fig. 2. State trajectory in the phase plane



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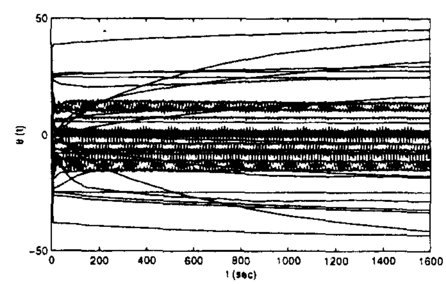

(b) Using the adaptive law(20).

Fig. 3. Trajectories of element of parameter vector $\underline{\theta}$

4. Conclusions

In the paper, a direct adaptive fuzzy controller is designed based on the new adaptation scheme which can guarantee the boundedness of all the signals in the system. In the proposed method, the concept of persistent excitation is first introduced to assure the convergence and the boundedness of adaptation parameters of fuzzy system. Simulation results shows that, in the presence of approximation error and external disturbance, the adaptation parameters of the method with adaptive law (20) can diverge. However, in the proposed method, we can find that the effects of both the approximation error and external disturbance are attenuated efficiently.

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