FUZZY r-PRECONTINUOUS MAPS

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ABSTRACT

We introduce new concepts of fuzzy r-preopen(r-preclosed) sets and fuzzy r-precontinuous(r-preopen, r-preclosed) maps as generalizations of the concepts of fuzzy preopen and fuzzy precontinuous of Shahna [7].

1. PRELIMINARIES

A *Chang's fuzzy topology* on X is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.
(2) If $\mu_1, \mu_2 \in T$ then $\mu_1 \wedge \mu_2 \in T$.
(3) If $\mu_i \in T$ for each i, then $\bigvee \mu_i \in T$.

The pair (X, T) is called a *Chang's fuzzy topological space.*

A *fuzzy topology* on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$,
(2) $\mathcal{T}(\mu_1 \wedge \mu_2) \geq \mathcal{T}(\mu_1) \wedge \mathcal{T}(\mu_2)$,
(3) $\mathcal{T}(\bigvee \mu_i) \geq \bigwedge \mathcal{T}(\mu_i)$.

The pair $(X, \mathcal{T})$ is called a *fuzzy topological space.*

For $r \in I_0$, we call μ a *fuzzy r-open set* of X if $\mathcal{T}(\mu) \geq r$ and μ a *fuzzy r-closed set* of X if $\mathcal{T}(\mu^c) \geq r$.

Let $(X, \mathcal{T})$ be a fuzzy topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the *fuzzy r-closure* is defined by

$$\mathrm{cl}(\mu, r) = \bigwedge\{\rho \in I^X : \mu \leq \rho, \mathcal{T}(\rho^c) \geq r\}$$

and the *fuzzy r-interior* is defined by

$$\mathrm{int}(\mu, r) = \bigvee\{\rho \in I^X : \mu \geq \rho, \mathcal{T}(\rho) \geq r\}.$$

DEFINITION 1.1 ([5]). Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map from a fuzzy topological space X to another fuzzy topological space Y and $r \in I_0$. Then f is called

(1) a *fuzzy r-continuous* map if $f^{-1}(\mu)$ is a fuzzy r-open set of X for each fuzzy r-open set μ of Y,

(2) a *fuzzy r-open* map if $f(\mu)$ is a fuzzy r-open set of Y for each fuzzy r-open set μ of X,

(3) a *fuzzy r-closed* map if $f(\mu)$ is a fuzzy r-closed set of Y for each fuzzy r-closed set μ of X.

2. Fuzzy r-preopen sets

Definition 2.1. Let μ be a fuzzy set of a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then μ is said to be

(1) *fuzzy r-preopen* if $\mu \leq \operatorname{int}(\operatorname{cl}(\mu, r))$,
(2) *fuzzy r-preclosed* if $\operatorname{cl}(\operatorname{int}(\mu, r)) \leq \mu$.

It is clear that a fuzzy set μ is fuzzy r-preopen if and only if μ^c is fuzzy r-preclosed.

Remark 2.2. It is obvious that every fuzzy r-open set (r-closed) is a fuzzy r-preopen (r-preclosed) set. But the converse need not be true. Also, the intersection (union) of any two fuzzy r-preopen (r-preclosed) sets need not be fuzzy r-preopen (r-preclosed).

Theorem 2.3. (1) *Any union of fuzzy r-preopen sets is fuzzy r-preopen.*
(2) *Any intersection of fuzzy r-preclosed sets is fuzzy r-preclosed.*

Definition 2.4. Let $(X, \mathcal{T})$ be a fuzzy topological space. For each $r \in I_0$ and for each $\mu \in I^X$, the *fuzzy r-preclosure* is defined by

$$\operatorname{pcl}(\mu, r) = \bigwedge\{\rho \in I^X : \mu \leq \rho,\ \rho \text{ is fuzzy } r\text{-preclosed}\}$$

and the *fuzzy r-preinterior* is defined by

$$\operatorname{pint}(\mu, r) = \bigvee\{\rho \in I^X : \mu \geq \rho,\ \rho \text{ is fuzzy } r\text{-preopen}\}.$$

Obviously $\operatorname{pcl}(\mu, r)$ is the smallest fuzzy r-preclosed set which contains μ and $\operatorname{pint}(\mu, r)$ is the greatest fuzzy r-preopen set which contained in μ. Also, $\operatorname{pcl}(\mu, r) = \mu$ for any fuzzy r-preclosed set μ and $\operatorname{pint}(\mu, r) = \mu$ for any fuzzy r-preopen set μ. Also we have

$$\operatorname{int}(\mu, r) \leq \operatorname{pint}(\mu, r) \leq \mu \leq \operatorname{pcl}(\mu, r) \leq \operatorname{cl}(\mu, r).$$

Moreover, we have the following results:

(1) $\operatorname{pint}(\tilde{0}, r) = \tilde{0}, \operatorname{pint}(\tilde{1}, r) = \tilde{1}$; $\operatorname{pcl}(\tilde{0}, r) = \tilde{0}, \operatorname{pcl}(\tilde{1}, r) = \tilde{1}$.
(2) $\operatorname{pint}(\mu, r) \leq \mu$; $\operatorname{pcl}(\mu, r) \geq \mu$.
(3) $\operatorname{pint}(\mu \wedge \rho, r) \leq \operatorname{pint}(\mu, r) \wedge \operatorname{pint}(\rho, r)$; $\operatorname{pcl}(\mu \vee \rho, r) \geq \operatorname{pcl}(\mu, r) \vee \operatorname{pcl}(\rho, r)$.
(4) $\operatorname{pint}(\operatorname{pint}(\mu, r), r) = \operatorname{pint}(\mu, r)$; $\operatorname{pcl}(\operatorname{pcl}(\mu, r), r) = \operatorname{pcl}(\mu, r)$.

Theorem 2.5. *For a fuzzy set μ of a fuzzy topological space X and $r \in I_0$,*

(1) $\operatorname{pint}(\mu, r)^c = \operatorname{pcl}(\mu^c, r)$.
(2) $\operatorname{pcl}(\mu, r)^c = \operatorname{pint}(\mu^c, r)$.

THEOREM 2.6. *For a fuzzy set μ of a fuzzy topological space X and $r \in I_0$,*

(1) $\mathrm{pint}(\mathrm{pcl}(\mathrm{pint}(\mathrm{pcl}(\mu, r), r), r), r) = \mathrm{pint}(\mathrm{pcl}(\mu, r), r)$.
(2) $\mathrm{pcl}(\mathrm{pint}(\mathrm{pcl}(\mathrm{pint}(\mu, r), r), r), r) = \mathrm{pcl}(\mathrm{pint}(\mu, r), r)$.

THEOREM 2.7. *Let μ be a fuzzy set of a fuzzy topological space $(X, \mathcal{T})$ and $r \in I_0$. Then μ is fuzzy r-preopen (r-preclosed) in $(X, \mathcal{T})$ if and only if μ is fuzzy preopen (preclosed) set in $(X, \mathcal{T}_r)$.*

THEOREM 2.8. *Let μ be a fuzzy set of a Chang's fuzzy topological space (X, T) and $r \in I_0$. Then μ is fuzzy preopen (preclosed) in (X, T) if and only if μ is fuzzy r-preopen (r-preclosed) in (X, T^r).*

3. FUZZY r-PRECONTINUOUS MAPS

DEFINITION 3.1. Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map from a fuzzy topological space X to another fuzzy topological space Y and $r \in I_0$. Then f is called

(1) a *fuzzy r-precontinuous* map if $f^{-1}(\mu)$ is a fuzzy r-preopen set of X for each fuzzy r-open set μ of Y, or equivalently, $f^{-1}(\mu)$ is a fuzzy r-preclosed set of X for each fuzzy r-closed set μ of Y,

(2) a *fuzzy r-preopen* map if $f(\rho)$ is a fuzzy r-preopen set of Y for each fuzzy r-open set ρ of X,

(3) a *fuzzy r-preclosed* map if $f(\rho)$ is a fuzzy r-preclosed set of Y for each fuzzy r-closed set ρ of X.

REMARK 3.2. It is obvious that every fuzzy r-continuous (r-open, r-closed) map is also a fuzzy r-precontinuous (r-preopen, r-preclosed) map for each $r \in I_0$. But the converse need not be true.

Now, we characterize fuzzy r-precontinuous by fuzzy r-closure and fuzzy r-interior.

THEOREM 3.3. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map and $r \in I_0$. Then the following statements are equivalent:*

(1) *f is a fuzzy r-precontinuous map.*
(2) $\mathrm{cl}(\mathrm{int}(f^{-1}(\mu), r), r) \leq f^{-1}(\mathrm{cl}(\mu, r))$ *for each fuzzy set μ of Y.*
(3) $f(\mathrm{cl}(\mathrm{int}(\rho, r), r)) \leq \mathrm{cl}(f(\rho), r)$ *for each fuzzy set ρ of X.*

Also, we characterize fuzzy r-precontinuous by fuzzy r-preclosure and fuzzy r-preinterior.

THEOREM 3.4. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map and $r \in I_0$. Then the following statements are equivalent:*

(1) *f is a fuzzy r-precontinuous map.*
(2) $f(\mathrm{pcl}(\rho, r)) \leq \mathrm{cl}(f(\rho), r)$ *for each fuzzy set ρ of X.*
(3) $\mathrm{pcl}(f^{-1}(\mu), r) \leq f^{-1}(\mathrm{cl}(\mu, r))$ *for each fuzzy set μ of Y.*
(4) $f^{-1}(\mathrm{int}(\mu, r)) \leq \mathrm{pint}(f^{-1}(\mu), r)$ *for each fuzzy set μ of Y.*

THEOREM 3.5. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a bijection and $r \in I_0$. Then f is a fuzzy r-precontinuous map if and only if* $\mathrm{int}(f(\rho), r) \leq f(\mathrm{pint}(\rho, r))$ *for each fuzzy set ρ of X.*

THEOREM 3.6. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map and $r \in I_0$. Then the following statements are equivalent:*

(1) *f is a fuzzy r-preopen map.*
(2) $f(\mathrm{int}(\rho, r) \leq \mathrm{pint}(f(\rho), r)$ *for each fuzzy set ρ of X.*
(3) $\mathrm{int}(f^{-1}(\mu), r) \leq f^{-1}(\mathrm{pint}(\mu, r))$ *for each fuzzy set μ of Y.*

THEOREM 3.7. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map and $r \in I_0$. Then the following statements are equivalent:*

(1) *f is a fuzzy r-preclosed map.*
(2) $\mathrm{pcl}(f(\rho), r) \leq f(\mathrm{cl}(\rho, r))$ *for each fuzzy set ρ of X.*

THEOREM 3.8. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a bijection and $r \in I_0$. Then f is a fuzzy r-preclosed map if and only if $f^{-1}(\mathrm{pcl}(\mu, r)) \leq \mathrm{cl}(f^{-1}(\mu), r)$ for each fuzzy set μ of Y.*

THEOREM 3.9. *Let $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ be a map from a fuzzy topological space X to another fuzzy topological space Y and $r \in I_0$. Then f is fuzzy r-precontinuous(r-preopen, r-preclosed) if and only if $f : (X, \mathcal{T}_r) \to (Y, \mathcal{U}_r)$ is fuzzy precontinuous(preopen, preclosed).*

THEOREM 3.10. *Let $f : (X, T) \to (Y, U)$ be a map from a Chang's fuzzy topological space X to another Chang's fuzzy topological space Y and $r \in I_0$. Then f is fuzzy precontinuous (preopen, preclosed) if and only if $f : (X, T^r) \to (Y, U^r)$ is fuzzy r-precontinuous (r-preopen, r-preclosed).*

References

[1] K. K. Azad, *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl. **82** (1981), 14–32.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.

[3] K. C. Chattopadhyay and S. K. Samanta, *Fuzzy topology : Fuzzy closure operator, fuzzy compactness and fuzzy connectedness*, Fuzzy Sets and Systems **54** (1993), 207–212.

[4] K. C. Chattopadhyay, R. N. Hazra and S. K. Samanta, *Gradation of openness : fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237–242.

[5] S. J. Lee and E. P. Lee, *Fuzzy r-semicontinuous sets and fuzzy r-semicontinuous maps*, Proc. of KFIS Spring Conference'97 **7** (1997), 29-32.

[6] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371–375.

[7] A. S. Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinuity*, Fuzzy Sets and Systems **44** (1991), 303–308.