퍼지관계방정식의 해의유계

THE BOUNDED OF SOLUTIONS OF FUZZY RELATION EQUATION

김현미

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0.ABSTRACT. In a fuzzy relation equation $R \circ U = T$, we find the bounded by U_* and U^* of the equation, where U_* is lower bound and U^* is upper bound.

1. INTRODUCTION

Sanchez introduced a fuzzy relation equations [1].
This paper study the bounded of fuzzy relation equation.
Some operations are defined.
Let us consider the lattice L = ([0 , 1], $\vee, \wedge, \rightarrow, \ll$), where

$$a \vee b = \max\ (a, b),$$
$$a \wedge b = \min\ (a, b),$$
$$a \rightarrow b = \begin{cases} 1 & \text{if } a \leq b, \\ b & \text{if } a > b. \end{cases}$$
$$a \ll b = \begin{cases} 1 & \text{if } a \geq b, \\ a & \text{if } a < b. \end{cases}$$

We need the following definitions and properties.
Let X be non-empty finite set.

DEFINITION1.1

A fuzzy binary relation on X and Y is a fuzzy subset R on X × Y.
We are only interested in the case in which X = Y.

DEFINITION1.2

Suppose R and U are two fuzzy relation on X.

$$(R \cdot U)(x, z) = \bigvee(R(x, y) \wedge S(y, z)) \qquad \text{for } x, z \in X,$$

where $\cdot$ operation is called a sup-inf composition.

PROPOSITION1.3

Let R, U, S and T be fuzzy relations on X.

[1] $(R \cdot S) \cdot T = R \cdot (S \cdot T)$.

[2] If $R \leq T$, then $R \cdot S \leq T \cdot S$.

DEFINITION1.4

We say that I is called an identity relation on X if $R \cdot I = I \cdot R$,

where $I(x, y) = 1$ if $x = y$,

0 if $x \neq y$.

2. PRELIMINARIES

The existence of solution of the relation equation

$$R \cdot U = T \qquad \text{------------------------------} (1)$$

(with unknown relation U and given relation R, T) was characterized by Sanchez [1],[3].

THEOREM2.1[3]

Eq. (1) has solutions if $R \cdot U^* = T$, where

$$U^*(x, z) = \bigwedge(R(y, x) \rightarrow T(y, z)) \text{ for } x, z \in X.$$

If Eq.(1) has solutions, then the above formula gives the greatest one. In general, we always have $R \cdot U^* \leq T$.

DEFINITION2.2 [2]

[1] A fuzzy relation R is said to reflxive if $I \leq R$.

[2] If $R \cdot R \leq R$, then R is called transitive.

3. RESULT

THEOREM.3.1

Let R be reflexive relation on X.

[1] If $R \leq T$, *then* $U_* \leq U^*$.

[2] For any U such that $U_* \leq U \leq U^*$, then $R \circ U \leq T$.

where $U_*(x, z) = \bigwedge[R(x, y) \ll T(y, z)]$.

Proof.

Let $U_*(x, z) = \bigwedge[R(x, y) \ll T(y, z)]$ --------------------------(2)

The right-hand member of (2) contains terms

$$R(x, x) \ll T(x, z)\ ,\ R(x, z) \ll T(z, z)\ ,\ R(x, y) \ll T(y, z).$$

Let $U^*(x, z) = \bigwedge[R(y, x) \rightarrow T(y, z)]$ --------------------------(3)

The right -hand member of (3) contains terms

$$R(x, x) \rightarrow T(x, z)\ ,\ R(z, x) \rightarrow T(z, z)\ ,\ R(y, x) \rightarrow T(y, z).$$

1] Suppose that $R(x, x) \geq T(x, z)$. Since $R \geq I$,we have $R(x, x) \ll T(x, z) = 1 = U_*$.
If $R(x, x) < T(x, z)$, then $R(x, x) \ll T(x, z) = 1 = U_*$.

2] Suppose that $R(x, z) \geq T(z, z)$. Since $R \leq T$ and $R \geq I$, $R(x, z) \ll T(z, z) = 1 = U_*$.
If $R(x, z) < T(z, z)$, then $R(x, z) \ll T(z, z) = R(x, z) = U_*$.

3] Suppose that $R(x, y) \geq T(y, z)$. Then $R(x, y) \ll T(y, z) = 1 = U_*$.
If $R(x, y) < T(y, z)$, then $R(x, y) \ll T(y, z) = R(x, y) = U_*$.

So $\bigwedge[1, 1, 1, R(x, z), 1, R(x, y)] = U_*$ ----------------------------(4)

4] Suppose that $R(x, x) > T(x, z)$. Then $R(x, x) \rightarrow T(x, z) = T(x, z) = U^*$.
If $R(x, x) \leq T(x, z)$, then $R(x, x) \rightarrow T(x, z) = 1 = U^*$.

5] Suppose that $R(z, x) > T(z, z)$. Since $R(z, z) \leq T(z, z)$, $R(z, x) \rightarrow T(z, z) = 1 = U^*$.
If $R(z, x) \leq T(z, z)$, then $R(z, x) \rightarrow T(z, z) = 1 = U^*$.

6] Suppose that $R(y, x) > T(y, z)$. Then $R(y, x) \rightarrow T(y, z) = T(y, z) = U^*$.
If $R(y, x) \leq T(y, z)$, then $R(y, x) \rightarrow T(y, z) = 1 = U^*$.

So $\bigwedge[T(x, z), 1, 1, 1, T(y, z), 1] = U^*$ -----------------------(5)

Suppose $R(x,z) \leq R(x,y)$ in (4). Since $R(x,z) < T(z,z)$ and $R(x,y) < T(y,z)$ in 2] and 3], $U_* = R(x,z)$ and $U^* = T(x,z)$ or $T(y,z)$. Thus $U_* \leq U^*$.
If $R(x,y) < R(x,z)$, then $U_* = R(x,y)$ and $U^* = T(x,z)$ or $T(y,z)$. $U_* \leq U^*$.

[2] Since $\circ$ operration is isotony in Proposition1.3 [2], it is trivial. ///

EXAMPLE.3.2

$$R = \begin{pmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.1 \\ 0.6 & 0.2 & 1 \end{pmatrix} \leq T = \begin{pmatrix} 1 & 0.2 & 0.5 \\ 0.4 & 1 & 0.3 \\ 0.6 & 0.3 & 1 \end{pmatrix}.$$

$$U^*(x,z) = \bigwedge[R(y,x) \rightarrow T(y,z)]$$

$$= \begin{pmatrix} (1\rightarrow 1 \wedge 0.4 \rightarrow 0.4 \wedge 0.6 \rightarrow 0.6) & (1 \rightarrow 0.2 \wedge 0.4 \rightarrow 1 \wedge 0.6 \rightarrow 0.3) & (1 \rightarrow 0.5 \wedge 0.4 \rightarrow 0.3 \wedge 0.6 \rightarrow 1) \\ (0.2 \rightarrow 1 \wedge 1 \rightarrow 0.4 \wedge 0.2 \rightarrow 0.6) & (0.2 \rightarrow 0.2 \wedge 1 \rightarrow 1 \wedge 0.2 \rightarrow 0.3) & (0.2 \rightarrow 0.5 \wedge 1 \rightarrow 0.3 \wedge 0.2 \rightarrow 1) \\ (0.3 \rightarrow 1 \wedge 0.1 \rightarrow 0.4 \wedge 1 \rightarrow 0.6) & (0.3 \rightarrow 0.2 \wedge 0.1 \rightarrow 1 \wedge 1 \rightarrow 0.3) & (0.3 \rightarrow 0.5 \wedge 0.1 \rightarrow 0.3 \wedge 1 \rightarrow 1) \end{pmatrix}$$

$$= \begin{pmatrix} (1 \wedge 1 \wedge 1) & (0.2 \wedge 1 \wedge 0.3) & (0.5 \wedge 0.3 \wedge 1) \\ (1 \wedge 0.4 \wedge 1) & (1 \wedge 1 \wedge 1) & (1 \wedge 0.3 \wedge 1) \\ (1 \wedge 1 \wedge 0.6) & (0.2 \wedge 1 \wedge 0.3) & (1 \wedge 1 \wedge 1) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.3 \\ 0.6 & 0.2 & 1 \end{pmatrix}$$

$$U_*(x,z) = \bigwedge[R(x,y) \ll T(y,z)]$$

$$= \begin{pmatrix} (1 \ll 1 \wedge 0.2 \ll 0.4 \wedge 0.3 \ll 0.6) & (1 \ll 0.2 \wedge 0.2 \ll 1 \wedge 0.3 \ll 0.3) & (1 \ll 0.5 \wedge 0.2 \ll 0.3 \wedge 0.3 \ll 1) \\ (0.4 \ll 1 \wedge 1 \ll 0.4 \wedge 0.1 \ll 0.6) & (0.4 \ll 0.2 \wedge 1 \ll 1 \wedge 0.1 \ll 0.3) & (0.4 \ll 0.5 \wedge 1 \ll 0.3 \wedge 0.1 \ll 1) \\ (0.6 \ll 1 \wedge 0.2 \ll 0.4 \wedge 1 \ll 0.6) & (0.6 \ll 0.2 \wedge 0.2 \ll 1 \wedge 1 \ll 0.3) & (0.6 \ll 0.5 \wedge 0.2 \ll 0.3 \wedge 1 \ll 1) \end{pmatrix}$$

$$= \begin{pmatrix} (1 \wedge 0.2 \wedge 0.3) & (1 \wedge 0.2 \wedge 1) & (1 \wedge 0.2 \wedge 0.3) \\ (0.4 \wedge 1 \wedge 0.1) & (1 \wedge 1 \wedge 0.1) & (0.4 \wedge 1 \wedge 0.1) \\ (0.6 \wedge 0.2 \wedge 1) & (1 \wedge 0.2 \wedge 1) & (1 \wedge 0.2 \wedge 1) \end{pmatrix}$$

$$= \begin{pmatrix} 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \end{pmatrix}$$

So $U_* \leq U^*$.

For any U such that $U_* \leq U \leq U^*$, let $U = \begin{pmatrix} 0.3 & 0.2 & 0.2 \\ 0.3 & 0.9 & 0.2 \\ 0.4 & 0.2 & 0.5 \end{pmatrix}$.

$$\text{Then } R \circ U = \begin{pmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.1 \\ 0.6 & 0.2 & 1 \end{pmatrix} \circ \begin{pmatrix} 0.3 & 0.2 & 0.2 \\ 0.3 & 0.9 & 0.2 \\ 0.4 & 0.2 & 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3 & 0.2 & 0.3 \\ 0.3 & 0.9 & 0.2 \\ 0.4 & 0.2 & 0.5 \end{pmatrix} \leq T.$$

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