CHARACTERIZATIONS OF COMPACTNESS IN FUZZY TOPOLOGICAL SPACES

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Abstract

The concept of fuzzy sets was introduced by Zadeh in his highly influential paper [5]. Using this concept, Chang [1] introduced a notion of fuzzy topological spaces which formally is the same one as for ordinary topological spaces. Observing that with Chang's definition constant maps between fuzzy topological spaces are not necessarily continuous, Lowen [2] gave an alternative and more natural definition for a fuzzy topological spaces and characterized the fuzzy compact spaces by means of prefilters in [4].

In this paper we give new characterizations of fuzzy compact spaces introduced in [2]. These results explain more clearly fuzzy compactness in fuzzy topological spaces.

1. Characterizations of compactness in fuzzy topological spaces

Definition 1.1 [2]. Let $\alpha \in I^X$ such that $0 < inf_{x \in X}\alpha(x)$. Then α is said to be fuzzy compact in a fuzzy topological space (X, δ) if for any $\{\beta_j\}_{j \in J} \subset \delta$ such that $\alpha \leq sup_{j \in J}\beta_j$ and each ε such that $0 < \varepsilon < inf_{x \in E}\alpha(x)$, there is a finite subset F of J such that $\alpha - \varepsilon \leq sup_{j \in F}\beta_j$.

A fuzzy topological space (X, δ) is said to be fuzzy compact if every nonzero constant fuzzy set is fuzzy compact.

This is a part of a forthcoming paper with the same title.

Definition 1.2 [3]. Let $a \in [0, 1)$. A fuzzy topological space (X, δ) is a-compact if and only if for any $\{\beta_j\}_{j\in J} \subset \delta$ such that $a < sup_{j\in J}\beta_j$, there is a finite subset F of J such that $a < sup_{j\in F}\beta_j$.

Definition 1.3. Let $\alpha, \beta \in I^X$. Then we say that α is a perfect subset of β if $inf_{x\in X}(\beta(x) - \alpha(x)) > 0$. In this case we write $\alpha < \beta$.

Definition 1.4. Let $\alpha \in I^X$ such that $sup_{x\in X}\alpha(x) < 1$. Then a fuzzy topological space (X, δ) is said to be α-fuzzy compact if for any $\{\beta_j\}_{j\in J} \subset \delta$ such that $\alpha < sup_{j\in J}\beta_j$, there is a finite subset F of J such that $\alpha \leq sup_{j\in F}\beta_j$.

Proposition 1.5. *Let $a \in [0, 1)$. If a fuzzy topological space (X, δ) is a-compact, then it is a-fuzzy compact.*

Theorem 1.6. *For a fuzzy topological space (X, δ), the following are equivalent.*

(1) *(X, δ) is fuzzy compact.*

(2) *For every $a \in [0, 1)$, (X, δ) is a-fuzzy compact.*

(3) *For every $a \in D-\{1\}$, (X, δ) is a-fuzzy compact, where D is a dense subset of I for the usual topology.*

Using the above theorem and Proposition 1.6 we obtain R. Lowen's result as corollary.

Corollary 1.7 [3]. Let (X, δ) be a fuzzy topological space and D a dense subset of I for the usual topology. If for *every* $a \in D-\{1\}$ *the space* (X, δ) is a-compact then it is fuzzy compact.

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