A Novel Nonlinear Synapse Neuron Model
--Wavelet Neuron--

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1. Introduction

The ordinary neural networks are established only by learning and thus exhibit the following three demerits; (1) Poor designability. Designers cannot estimate how many layers are necessary for the network structure and how many neural elements are necessary for each layer to model the nonlinear system. Furthermore, designers cannot estimate how many data are needed to complete the learning. (2) Long learning term. It takes a long term to achieve the learning, the period of which is depending upon the complexity of the system to be modeled. (3) Local minima. After learning, a neural network can easily fall into local minima and does not guarantee the optimum design.

To cope with these problems peculiar to neural networks, the author developed a new neuron model, the synaptic characteristics of which is nonlinear and represented by a set of wavelets and the weight corresponding to each wavelet is determined by learning rather than by analytical procedure. The new neuron model was named a *wavelet neuron* [1, 2].

2. Wavelet Neuron

Furrier expansion is very significant to represent a variety of periodic functions by a constant value and sinusoidal functions of a basic frequency and its harmonics. however it is not so suitable for representing a non-periodic function or a function defined within a limited interval. The wavelet expansion [3, 4] is very suitable for representing these types of functions and the coefficients are analytically obtained by integrating the original function and the wavelet from -infinity to +infinity under the admissibility condition. Thus it takes a long time to calculate all the coefficients corresponding to a numerous wavelets constructing the original function. In the technological sense, the approximate representation of an original function by employing as small number of wavelets as possible within a permissible error is preferable, because it will realize a rapid generalization of any functions. This is just the concept of Soft Computing.

The author presented the new neuron model having nonlinear synaptic characteristics and guaranteeing a global minimum in contrast with ordinary neural nets and RBF networks. It is a *wavelet neuron,* the synaptic weights of which can be obtained by learning rather than analytical way. The wavelet function in the wavelet neuron is a non-orthogonal, convex and compactly supported function. Practically we adopted a cosine-like function as the wavelet. In the usual wavelet expansion, dyadic wavelets (the number of wavelets increases such as 1, 2, 4, 8, 16, ... in accordance with the scaling parameter) are employed. However, the wavelet neuron employs over-complete number of wavelets (the number of wavelets increases such as 1, 2, 3, 4, 5, 6 ......in accordance with the scaling parameter). The completeness of the wavelets has been proved in the reference [2].

3. Generalization Characteristics

The attempt was made to generalize a one-dimensional complicated function constructed with a piecewise linear part and a smooth part by the wavelet neuron having only one *wavelet synapse*. The initial values of all the weights in the wavelet neuron are assigned to be zero, while those in ordinary neuron models are assigned to be random numbers. The examination was achieved for a scaling parameter of 15 which means 136 weights in the wavelet neuron. The ordinary neural network of three layers having the same number of parameters (weights and thresholds) was also examined for comparison. 100 times of training in the wavelet neuron presents about 1% of R.M.S. error of generalization, while even 10,000 times of training in the ordinary neural network cannot reach the same accuracy and it easily falls into a local minimum.

The chaotic behavior of a nonlinear dynamical system was expected by the wavelet neuron having three synapses with 100 times of learning. A 4-layered ordinary neural network (3 neurons - 5 neurons - 5 neurons - 1 neuron) with 100,000 times of learning was also examined for comparison. The wavelet neuron could expect seven steps ahead without calibration, while the ordinary neural network cannot do one step ahead at all.

4. Application to Solving Differential Equations

The attempt was also made to solve differential equations by using the integral form of the wavelets, i.e. sine function, and the initial values. All the weights in the wavelet neuron having cosine functions as basis functions are assigned in the training mode and those wavelets are replaced with the integral form to get the solution in the solving mode. The results are significantly reasonable for technical use.

5. Conclusions

The new neuron model with nonlinear synapses was presented, which was constructed with a definite number of and over-complete number of weights and compuctly supported and convex wavelets. All the weights of the wavelet neuron are assigned by learning. The wavelet neuron guarantees a global minimum and much higher speed with much less error than ordinary multilayer neural networks.

References

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