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THE CONSTRAINED ITERATIVE IMAGE RESTORATION ALGORITHM USING NEW REGULARIZATION OPERATORS

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ABSTRACT

This paper proposes the regularized constrained iterative image restoration algorithms which apply new space-adaptive methods to degraded image signals, and analyzes the convergence condition of the proposed algorithm. First, we introduce space-adaptive regularization operators which change according to edge characteristics of local images in order to effectively prevent the restored edges and boundaries from reblurring. And, pseudo projection operator is used to reduce the ringing artifact which results from extensive amplification of noise components in the restoration process. The analysis of convergence condition shows that the proposed algorithm is stable convergent to the fixed point. According to the experimental results for various signal-to-noise ratios(SNR) and blur models, the proposed algorithms outperform other methods and is robust to noise effects and edge reblurring by regularization especially.

1. INTRODUCTION

Image signals are considered as the most important ones among various signals, and many kinds of image processings have been studied to represent image information accurately. However, due to the limitation of the image recording or formation systems, the recorded images are inevitably degraded. In addition, noise is added to the transmitted image signal. Because this degraded image not only is different from original one but also is unsuitable for the human visual system, it is necessary to obtain the image which is restored from the degraded image and is suitable for the human visul system. The purpose of image restoration is to remove those degradations so that the restored image becomes as close as possible to the original one.

In general, image degradation can be linearly modelled as follows.

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

Where, $\mathbf{H}$ is the point spread function(PSF), $\mathbf{x}$ is a original image, and $\mathbf{n}$ is noise. Because we do not have any exact information of PSF and noise practically, we should estimate them and cannot help obtaining the image similar to the original one by using the estimated information. Also, due to the *ill-conditioned problem* of PSF, we cannot realize the inverse of the PSF and cannot recover the image only with inverse process of the PSF. Many kinds of image restoration algorithms have been researched in order to solve these difficulties. Among various restoration algorithms[1, 2], the regularized constrained iterative image restoration (RCIIR) algorithm has been widely researched with its many advantages. It allows the incorporation of various types of prior knowledge about the class of feasible solution and can eliminate nonstationary blurs. In addition, it can apply the characteristics of the local images to restoration process more effectively than other resotration algorithms. On the other hand, it has some problems such as ringing artifact, salt-pepper phenomenon with respect to noise level(SNR), and the sensitivity to the PSF. To overcome these difficulties, Legendijk[3] analyzed the ringing artifact and eliminated it. Katsaggelos[4, 5] adopted noise visibility function and CHOY[6] applied space-variable weights to the regularization. Recently, new algorithms which combine the PSF estimation and restoration[7, 8] were proposed. All these adopted the space-variant smoothing operators commonly in order to suppress noise amplification and solve nonstationary blurs. In this paper, we propose a new space-adaptive regularized constrained iterative image restoration algorithm. It adopts new directional regularization operators to preserve the high frequency regions and applies the space-variant filters which are robust to noise effects especially. This paper consists of six sections. In the section 2, we introduce briefly the theory of the RCIIR algorithm, and in the section 3, we propose a new space-adaptive regularized constrained iterative image restoration algorithms. In the section 4, we analyze the convergence condition of the proposed algorithms and in the section 5 and 6, experimental results and conclusions are presented respectively.

2. RCIIR ALGORITHM

The RCIIR algorithm restores a degraded image by minimizing the energy of output image of regularization operator with constraint as follows,

$$\begin{aligned} &minimize \quad \|\mathbf{Cx}\| \,, \\ &subject\ to \quad \|\mathbf{Hx}-\mathbf{y}\| \leq \varepsilon. \end{aligned} \qquad (2)$$

Where, $\mathbf{C}$ is the regularization operator which is a kind of high pass filter. By minimizing $\|\mathbf{Cx}\|$, high frequency regions are maximally suppressed in the restoration process with the constraint of restriction of noise energy. So, $\mathbf{C}$ operator suppresses not only noise amplification but also the recovery of edges or boundaries in the restoration process. To solve *eq* (2), a *Lagrange Multiplier*, λ is applied and the cost function to be minimized is constructed.

$$J(\mathbf{x}) = \|\mathbf{Cx}\|^2 + \lambda(\|\mathbf{y}-\mathbf{Hx}\|^2 - \|\mathbf{n}\|^2). \qquad (3)$$

Since image signals are sensitive to the human visual system, noise visibility function which considers the noise level and the characteristics of the local images is introduced to restore images suitable for human visual system. Noise visibility function, $f(i,j)$ is defined with local mean and variance, $M(i,j)$ as follows,

$$f(i,j) = \frac{1}{\theta M(i,j)+1}. \qquad (4)$$

By minimizing *eq* (3) with *eq* (4), we can finally formulate the restoration process.

$$\begin{aligned} \mathbf{x}_0 &= \beta\mathbf{H}^T\mathbf{y}, \\ \mathbf{x}_{k+1} &= (\mathbf{I}-\beta\alpha\mathbf{C}^T\mathbf{F}^T\mathbf{F}\mathbf{C})\mathbf{x}_k + \beta\mathbf{H}^T(\mathbf{y}-\mathbf{H}\mathbf{x}_k), \\ &= \beta\mathbf{H}^T\mathbf{y} + (\mathbf{I}-\beta\alpha\mathbf{C}^T\mathbf{F}^T\mathbf{F}\mathbf{C}-\beta\mathbf{H}^T\mathbf{H})\mathbf{x}_k \end{aligned} \qquad (5)$$

In *eq* (5), $\mathbf{F}$ is the noise visibility function and β is the weight for gradient between restored images successively in the steepest descent formulation.

3. THE PROPOSED ALGORITHMS

In restoring a degraded image with *eq* (5), Some problems to be solved exist in the restored image. First, due to the regularization operator, not only noise amplification but also edges or boundaries to be restored sharply are suppressed and blurred in the restored image. Our paper handles this problem as the most important point. Second, the ringing artifact appears in the plain region which is adjacent to abrupt boundary. And finally, in case of low SNR, noise is amplified so extremely that salt-pepper phenomenon is caused. The proposed algorithm solves these problems with directional operators, pseudo-projection operator, and adaptive noise reduction filters respectively.

3.1 Directional regularization operators

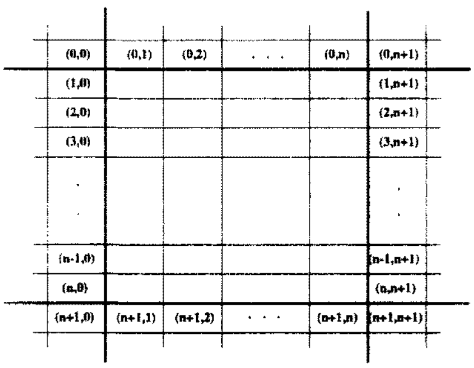
In order to prevent edges or boundaries from reblurring in the restoration process, we introduce new directional regularization operators which reflect the directions of edges or boundaries. Because the conventional regularization operator has no directivity in the frequency response, its output images in the restoration process are uniformly smoothed in any directions. We first estimate the direction of edges or boundaries to be restored by quadtree decomposition[9] and projection data of each decomposed block where includes the edges or boundaries to be restored, and apply the directional regularization operator *parallel to* the estimated direction. By applying the *parallel* regularization operator, the output value of the operator is small and this means the small regularization at the edges or boundaries. In estimating the directions of edges or boundaries, blockwise estimation which is based on quadtree decomposition diminishes the effect of noise in comparison with pointwise estimation. After quadtree decomposition, projection data of each decomposed block is used to decide dominant direction. Figure 1. (a) represents the projection data of one block which are indexed. With these projection data, we can decide the dominant direction of each block as follows,

$$\begin{aligned} diff(P_\angle) &= \sum_{k,l=0}^{n+1} |P^1_\angle(k,l) - P^2_\angle(k,l)|, \qquad (6) \\ direction &= \mathrm{Max}(diff(P_\angle),\ Threshold). \qquad (7) \end{aligned}$$

In case of *Threshold* is the maximum, no direction is selected and the conventional regularization operator is used. Figure 1. (b) shows the direction of quadtree decomposed blocks by different gray levels.

3.2 Pseudo projection operator

Since ringing artifact is caused by the *ill-conditioned*



(a)projection data

(b)information of directions

Figure 1:

of PSF, ringing artifact appears in the restored image as the impulsive noise different extremely from neighboring pixel values. In order to reduce the impulsive output of restoration process, we adopt pseudo projection operator which is adopted in various scaled windows. We summarize this operator as five steps.

Step 0 : ws = 1

Step 1 : **FIND** x_{med}, x_{max}, $x_{min} \in \mathrm{W}_{\mathrm{ws}\times\mathrm{ws}}$

Step 2 : $T^- = x_{med} - x_{min}$, $T^+ = x_{med} - x_{max}$

Step 3 : **IF** $T^- > 0$ *and* $T^+ < 0$ *and* ws $<$ ws$_{max}$
THEN *goto Step* 4,
ELSE $++$ws *and goto Step* 1

Step 4 : $U^- = x(i,j) - x_{min}$, $U^+ = x(i,j) - x_{max}$

Step 5 : **IF** $U^- \leq 0$ or $U^+ \geq 0$
THEN $x(i,j) = w \cdot x(i,j) + (1-w) \cdot x_{med}$
ELSE $x(i,j) = x(i,j)$

3.3 Adaptive noise reduction filters

In general, it is difficult to apply the local image characteristics to restoration processes accurately since the region of supports(ROS) of PSF or noise visibility function are different from those of regularization operators. To restore a degraded image more adaptively, we detect edges which are classified and apply weights corresponding to the class to the regularizarion operation. If robust edge is detected, the regularized weight is large, and vice versa. Finally, in the last step of restoration, low pass filter and momentum are used to guarantee convergence condition of restored image.

4. ANALYSIS OF CONVERGENCE CONDITION

We can formulate the restoration process described before as like *eq* (5),

$$
\begin{aligned}
\mathbf{x}_0 &= \beta\mathbf{H}^T\mathbf{y}, \\
\tilde{\mathbf{x}}_{k+1} & \\
&= (\mathbf{I} - \beta\mathbf{C}^T\mathbf{F}^T\mathbf{W}\mathbf{F}\mathbf{C})\mathbf{x}_k + \beta\mathbf{H}^T(\mathbf{y} - \mathbf{H}\mathbf{x}_k) + \gamma\Delta\mathbf{x}_k, \\
&= \beta\mathbf{H}^T\mathbf{y} + (\mathbf{I} - \beta\mathbf{C}^T\mathbf{F}^T\mathbf{W}\mathbf{F}\mathbf{C} - \beta\mathbf{H}^T\mathbf{H})\mathbf{x}_k + \gamma\Delta\mathbf{x}_k, \\
\mathbf{x}_{k+1} &= \mathbf{L}\mathbf{P}\tilde{\mathbf{x}}_{k+1}. \qquad (8)
\end{aligned}
$$

In *eq* (8), **C** is the directional regularization operator, **W** is adaptive noise reduction filter, **P** is the pseudo projection operator, and **L** is the low pass filter. The last term, $\gamma\Delta\mathbf{x}_k$ is the momentum. This iterative procedure is convergent to a fixed point if it is contractive[10]. Contractive mapping is the mapping that satisfies the condition such as

$$d(\mathbf{x}_{k+1}, \mathbf{y}_{k+1}) \leq s d(\mathbf{x}_k, \mathbf{y}_k), \quad s \leq 1. \qquad (9)$$

In order to analyze the convergence condition, first let us define some symbols from in *eq* (8) as

$$\mathbf{T} \stackrel{\text{def}}{=} \mathbf{LP}(\mathbf{I} - \beta\mathbf{C}^T\mathbf{F}^T\mathbf{W}\mathbf{F}\mathbf{C} - \beta\mathbf{H}^T\mathbf{H}), \quad (10)$$

$$d_k \stackrel{\text{def}}{=} d(\mathbf{x}_k, \mathbf{y}_k). \qquad (11)$$

By substituting *eq* (8) for *eq* (9), we can formulate the recursive equation as *eq* (12) and solve this equation as *eq* (13).

$$d_{k+1} \leq ||\mathbf{T}|| \cdot d_k + d_0 \qquad (12)$$

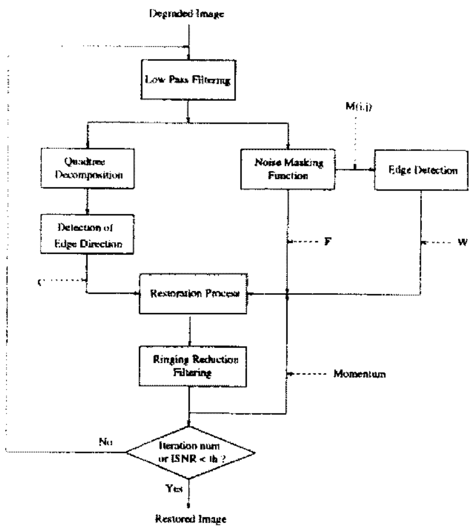


Figure 2: Flow chart of proposed algorithm

$$d_k \leq \frac{d_0}{1-\|\mathbf{T}\|} + (d_0 - \frac{d_0}{1-\|\mathbf{T}\|}) \cdot \|\mathbf{T}\|^k \qquad (13)$$

Finally, the convergence condition of *eq* (8) is simplified by the condition of **LP** as

$$\|\mathbf{LP}\| < \frac{1}{1+\beta(18 \cdot w_{max} \cdot f_{max} + \frac{1}{KL})}, \qquad (14)$$

where, the number 18 is the norm of **C**, and w_{max} and f_{max} are the maximum of **W** and **F**, respectively. As you see in *eq* (14), low pass filter and pseudo projection operator determine the convergence condition and they are necessary to be designed optimally.

5. EXPERIMENTAL RESULTS

Our experiment was executed as shown in Figure 2. We blurred the test image with 15×15 uniform or 15×1 motion and added gaussian noise to the blurred image. The directional regularization operators were used in the *four* directions such as horizontal, vertical, and two diagonals. To compare with other algorithms, the measure of performance is the ratio of ISNR's(Improved SNR) as follows,

$$R_{\text{ISNR}} = \text{ISNR}_{proposed}/\text{ISNR}_{method\ of\ [5]} \qquad (15)$$

images	ratio of ISNR's [dB]			
	uniform blur		motion blur	
	30dB	free	20dB	30dB
Lena	1.25	1.18	1.70	1.03
bank	1.24	1.16	1.58	1.05

Table 1: The ratios of ISNR's

The results for various SNR's and blurs are summarized in Table 1. As you see in Table 1, the proposed algorithm outperforms the method of [5] and lower SNR, better performance. Compared with the results of [6] and [8], the proposed algorithm outperforms, too. In addition, the restored images have less ringing artifact and salt-pepper phenomenon. This means that the pseudo projection operator and noise reduction filters performed their functions very excellently. Figure 3 shows the restored images by two methods and Figure 4 compares the performance of directional regularization operators at the edges with the conventional regularization operator. And the momentum improves the convergence performance in both the speed and accuracy by 0.1dB.

6. CONCLUSIONS

In this paper, we have proposed a new space-adaptive regularized constrained iterative image restoration algorithm and have analyzed the convergence condition of proposed algorithm. According to the experimental results, we found that directional regularization operators restored edges and boundaries more accurately than other algorithms did. The adaptive noise reduction filter and pseudo projection operator reduced salt-pepper phenomenon and ringing artifact effectively. Momentum in the steepest descent formulation improved the performance of convergence by 0.1dB. And the proposed algorithm guaranteed more excellent convergence of the iterative restoration process and was robust to noise effects especially. Further researches will be focused on the combination of image restoration and PSF estimation and application to the motion blurs of video sequences. Also, we will continue the research on the optimization of parameters in the proposed algorithm so that it can show more robust performance to PSF estimation error.

7. REFERENCES

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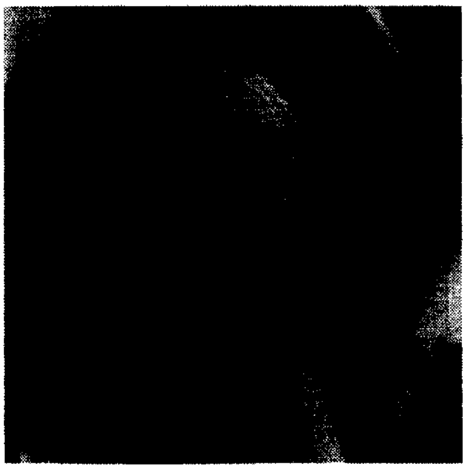
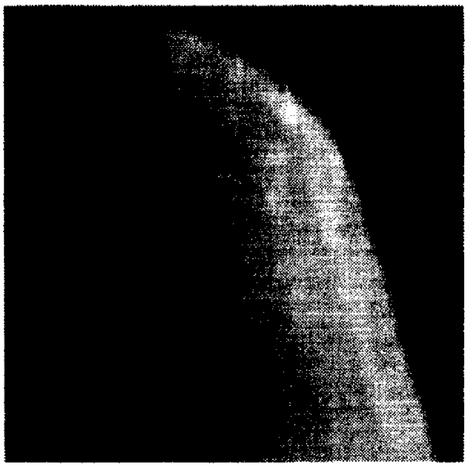


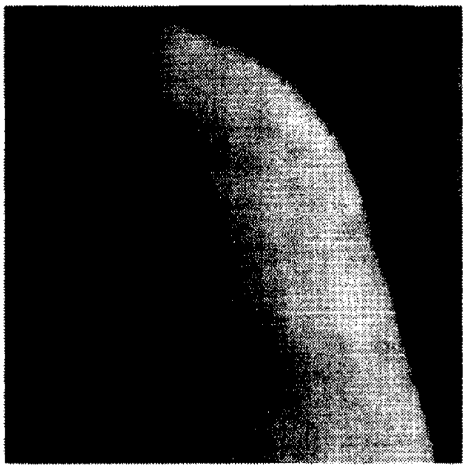
(a)method of [5]

(b)proposed method

Figure 3: Comparison of restored images



(a)method of [5]



(b)proposed method

Figure 4: Comparison of restored boundary

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