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Iterative Identification Methods for Ill-conditioned Processes

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Abstract: Some ill-conditioned processes are very sensitive to small element-wise uncertainties arising in classical element-by-element model identifications. For such processes, accurate identification of singular values and right singular vectors are more important than those of the elements themselves. Singular values and right singular vectors can be found by iterative identification methods which implement the input and output transformations iteratively. Methods based on SVD decomposition, QR decomposition and LU decomposition are proposed and compared with the Kuong and MacGregor's method. Convergence proofs are given. These SVD and QR methods use normal matrices for the transformations which cannot be calculated analytically in general and so they are hard to apply to dynamic processes, whereas the LU method uses simple analytic transformations and can be directly applied to dynamic processes.

Keywords: Iterative Identification, Ill-conditioned Process, SVD decomposition, LU decomposition

Introduction

Interaction between methods of identification and control is a recent topic of interest. Classical identification approaches often do not provide information appropriate for robust controller design methods. Control-relevant identification methods which will give better models to design control systems are required (Gevers, 1991). These include identification of uncertainty regions in addition to the nominal model, worst case identifications matching the framework of H_∞ control and iterative closed-loop identification leading to successively better design of controllers (Kuong and MacGregor, 1994).

For ill-conditioned multivariable processes, additional attention should be paid to identification. Inversion of some ill-conditioned processes are very sensitive to modeling error. That is, small element-wise error arising in the classical identification can be amplified. Explicit and implicit model-based control systems that use inversion can exhibit poor robust performances. In such cases, it is very important to accurately determine the small singular values which become large in inversion and the corresponding singular vectors (Li and Lee, 1996).

Usually equal input perturbations do not excite the small singular values and their singular vectors sufficiently. Andersen et al. (1989) have shown that closed-loop identification can excite the small singular values and their corresponding singular vectors more completely than classical open-loop identification methods and provide a better model to design control systems. With input transformations via right singular vectors assumed to be known a priori, Kuong and MacGregor (1993) have proposed an identification method which can excite each singular value and its singular vector separately. Later, they also proposed an iterative method which refines the right singular vectors (Kuong and MacGregor, 1994).

Li and Lee (1996) have proposed an identification method for ill-conditioned multivariable processes which fits both the process model and its inverse. They found the inverse of process from an experimentally determined relative gain array (Seborg et al., 1989). Achieving a small error in the process gain matrix guarantees accurate large singular values and singular vectors and a small error in the inverse of process gain matrix guarantees accurate small singular values and singular vectors. For higher order processes, stable control systems must be designed with integral action to obtain the relative gain array experimentally, which may not be easy.

Ill-conditioned processes consist of large elements due to large singular values and small elements due to small singular values. For the purpose of control system design, it is more important to determine accurately the small elements due to small singular values. The small elements cannot be found accurately unless they are separated from the large elements. Components due to large singular values and small singular values should be separated and then identified independently.

Kuong and MacGregor's method (1993) can do this when the right singular vectors are known. They also proposed an iterative method (1994) which can be applied for poorly known right singular vectors. However, a convergence proof is not established for higher order processes. In this paper three iterative identification methods for ill-conditioned multivariable processes are proposed and compared with the Kuong and MacGregor method (1994). Convergence proofs are given with small identification error at each step. Properties of each method are explained.

Iterative Identification Methods

Structure: The uncertainty condition numbers $\gamma_p(\bullet)$ and $\gamma^*_p(\bullet)$ are small for diagonal matrices and some triangular matrices. If the process can be converted to the diagonal (or triangular) form with appropriate transformations, robust models can be obtained. Methods which diagonalize (or triangularize) the process successively with identified models having errors are investigated.

Transformations in inputs and outputs are applied. Let

$$G_k \equiv Y_k \cdots Y_0 G X_0 \cdots X_k$$

It can be identified as

$$\overline{G}_k = G_k + \varepsilon G_k \circ \Delta_k$$

Then we have an identified model of the process gain matrix as

$$\overline{G} = Y^{-1}\overline{G}_k X^{-1} = G + \varepsilon Y^{-1}(G_k \circ \Delta_k) X^{-1}$$

where $X = X_1 \cdots X_k$ and $Y = Y_k \cdots Y_1$. For this model,

$$\left\|\overline{G}G^{-1} - I\right\|_p = \varepsilon \left\|Y^{-1}(G_k \circ \Delta) G_k Y\right\|_p$$

$$\leq \varepsilon \gamma_p(G_k) \|Y\|_p \left\|Y^{-1}\right\|_p \|\Delta\|_p$$

Hence, if Y is well-conditioned and $\gamma_p(G_k)$ is small, the model $\overline{G}$ at the k-th step will be adequate to design a control system having robust performance and robust stability.

Iterative methods which derive G_k to be diagonal (or triangular) with the input and output transformations are considered. The procedure is as follows;

Step 1: Set $X_0=Y_0=I$ and identify G_0.

Step 2: Find X_1 and Y_1 such that $Y_1\overline{G}_0 X_1$ has a given diagonal (or triangular) structure and identify G_1.

Step 3: Repeat Step2 until the identified model is not much different from the given structure.

Conditions for convergence of the above iterative identification methods assuming a small identification error ε at each step can be obtained. For this, we use the notation:

Ω = set of diagonal (or triangular) matrices

Ω_k = set of matrices whose elements are such that

$$\Theta + \varepsilon^k \widetilde{\Theta},\ \Theta \in \Omega.$$

Since Ω is the set of diagonal (or triangular) matrices, if

$$\Theta \in \Omega,\ \Theta + \varepsilon \Theta \circ \Delta \in \Omega.$$

Lemma 1: Assume that there exists matrices

$$X = I + O(\varepsilon^k) \text{ and } Y = I + O(\varepsilon^k)$$

such that, for $\Theta \in \Omega$, $Y(\Theta + \varepsilon^k \widetilde{\Theta})X \in \Omega$. Then the estimate at the k-th step is $\overline{G}_k \in \Omega_k$.

This Lemma proves that, for sufficiently small ε, $\overline{G}_k$ converges to a diagonal (or triangular) form. The radius of convergence of ε is dependent on the specific choices of X_k and Y_k. Convergence of $\overline{G}_k$ does not mean that $\left\|\overline{G}G^{-1} - I\right\|$ converges to zero or singular values converge to true values. However, these can be quite small.

Next we derive three specific methods with respective convergence analysis:

(1) SVD method: The uncertainty condition number is small for the diagonal matrices. The successive design of a full decoupler with Y_k=I and $X_k = \overline{G}_k^{-1}$ is the simplest way for diagonalization. However, it is not practical because X_k is very sensitive to the model identification error and so the convergence radius of ε is usually small. The singular value decomposition (SVD) method can be used to diagonalize the process iteratively. Apply the singular value decomposition to the k-th estimates $\overline{G}_k$ as:

$$\overline{G}_k = W_k \Sigma_k V_k^T$$

where W_k and V_k are both normal matrices (that is, $W_k^T W_k$=I and $V_k^T V_k$=I) and Σ_k is a diagonal matrix. Then

$$X_{k+1} = V_k^T$$
$$Y_{k+1} = W_k$$

will diagonalize the process. This choices of X_k and Y_k can be shown to satisfy the assumptions of Lemma 1.

This method uses both the input transformation X_k and the output transformation Y_k. During the output transformation, arithmetic operations are applied to the measured output which has a finite word length such as 12 bit. This may result in less accurate estimates.

(2) Kuong and MacGregor (KM) method: For ill-conditioned processes, an iterative identification method based on the SVD decomposition has been proposed by Kuong and MacGregor (1994). Instead of using both transformations of input and output, only input transformation was utilized;

$$X_{k+1} = V_k^T$$
$$Y_{k+1} = I$$

Convergence of this method has not been proved yet. But simulations show that it is also convergent for wide range of the relative identification error ε.

(3) QR method: An obstacle in proving the convergence of the KM method with Lemma 1 is that it does not transform the process to a given structure such as a diagonal and a triangular form. To overcome this, a slightly different method using QR decomposition has been derived. We decompose the k-th estimates as

$$\overline{G}_k = P_k R_k^T Q_k^T$$

where P_k is a permutation matrix, Q_k is a normal matrix and R_k is a lower triangular matrix. Then

$$X_{k+1} = Q_k^T$$
$$Y_{k+1} = P_k$$

will triangularize the process. For the triangular matrix , the uncertainty condition number for stability is very low and the uncertainty condition number for performance is usually much less than that of the original matrix.

(4) LU method: The first three methods all use complicated transformations and extension to dynamical systems may not be easy. A method which can be readily extended to dynamic systems is now derived. We decompose the k-th estimates as

$$\overline{G}_k = P_k L_k U_k$$

where P_k is a permutation matrix, L_k is a lower triangular matrix and U_k is a upper triangular matrix. Then

$$Y_k = P_k$$
$$X_k = U_k$$

will triangularize the process.

This method provides a way to design a one-way decoupler iteratively. The input transformation X_k is given as analytic rational function of elements of $\overline{G}_k$. Hence it can be easily extended to the dynamical systems.

Conclusion

Iterative identification methods are proposed for ill-conditioned processes which are very sensitive for the element uncertainties. For such processes, element uncertainties can be amplified and classical element-by-element identification cannot be used. Methods based on SVD, QR and LU decompositions which diagonalize and triangularize the process iteratively with incompletely identified process model are investigated. Convergence proofs are given. Convergence rate is dependent on the magnitude of identification error and error pattern. Simulation has shown that the SVD method has the largest convergence ranges of identification error and other methods including the Kuong and MacGregor method (1994) have similar convergence ranges. The SVD, QR and KM methods are all based on transformations with normal matrices and hence cannot easily be extended to dynamic process transfer functions. On the other hand, the LU method uses analytical transformations and can be applied to dynamic processes. Actually, the LU method is just to design a one-way decoupler iteratively.

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