Proceedings of the 12th
KACC, October 1997

FDI Observer Design for Linear System via STWS

○ Pius Ahn*, Min Hyung Kim**, Jae Il Kim***, Moon Hee Lee‡ and Doo Soo Ahn*

* Dept. of Electrical Eng., SKKU, 300 Chunchun-dong, Jangan-gu, Suwon 440-746, Korea
○ Tel: +82-331-290-7167; Fax: +82-331-290-7179; E-mail: piusahn@yurim.skku.ac.kr

** Automation Team of Water Resources Research Institute, Korea Water Resources Corp., Taejon, Korea
Tel: +82-42-860-0344; Fax: +82-42-860-0368; E-mail: kmh@kowaco.or.kr

*** Dept. of Computer Science, DETC, #72 Yangjung-dong, Jin-gu, Pusan, 614-715, Korea
Tel : +82-51-860-3210 ; Fax : +82-51-860-3334; E-mail : kji@detc.dongeui-tc.ac.kr

‡ Daan Electronics Co., Ltd., Daan B/D, 3F 22-1 Shingye-dong, Yongsan-gu, Seoul, Korea
Tel : +82-2-701-2711 ; Fax : +82-2-701-2714; E-mail : mhlee@daan.co.kr

Abstracts This paper deals with an algebraic approach to FDI observer design procedure. In general, FDI observer can be designed as Luenberger-type and equations for unknown input and actuator fault estimation include derivation of system outputs which is not available from the measurement directly. At this point, this paper presents STWS approach which can convert the derivation procedure to the recursive algebraic form by using its orthogonality and disjointness to alleviate such problems.

Keywords FDI(fault detection and isolation) observer, STWS(single term Walsh series), Algebraic approach

1. INTRODUCTION

It is possible to apply the Walsh function in the system analysis and control area since Corrinton[1] introduced Walsh function's integral operation matrix. The advantage of Walsh function approach to the system analysis and control is that it can convert the differential equation to the algebraic form using its integral operation matrix. Recently, the development of micro processor realized the Walsh function's practical applications and it is proved by many researchers. For the last decade, however, a few researchers[2-5] have the interest in orthogonal function's differential operation to apply UIO(unknown input observer) and FDI(fault detection and isolation) observer design procedure. It is reason that the application of integral operation matrix is not suitable to apply to the UIO and FDI observer design. In the previous works[2,3], orthogonal function's differential operation is studied at the point of view of calculus of variation's approximation. But this is not practical form by reason that it is difficult to find generalized recursive form. Some works[4,5] present a novel approach to orthogonal function's differential operation by using its orthogonality and disjointness. The benefit of the approach is that it can possible to find a generalized recursive algebraic form for the derivative procedure. In general, FDI observer which is derived as a form of Luenberger-type includes system output's differential value. Therefore, this paper presents STWS(single term Walsh series) approach to FDI observer to eliminate the differential of system output in the unknown input and actuator fault estimation equations. Presented example checked out that proposed algebraic approach is more useful when the system output includes unexpected measurement noises.

2. DERIVATION OF STWS' DIFFERENTIAL OPERATION

Walsh function is an orthonomal function which is defined in the interval $[0, 1)$. Hence, time scaling is needed to apply Walsh function for an arbitrary continuous function $f(t)$ which is defined in the interval $[0, t_f)$. Let's define an interval $\tau \in [0, 1)$ and substitute $t = \frac{t_f}{m}\tau$ then $f(t)$ can be approximated by using STWS as the following eq.(1)

$$f(\tau) = F_1\phi_0(\tau) \tag{1}$$

Assume that $\frac{d}{d\tau}f(\tau)$ can be approximated by using STWS as the following eq.(2)

$$\frac{d}{d\tau} f(\tau) = \overline{F}_1 \phi_0(\tau) \quad (2)$$

Integrating eq.(2) and applying STWS' integral operation which is $\frac{1}{2}$, eq.(2) can be represented as eq.(3).

$$F_1 = \frac{1}{2} \overline{F}_1 + f(0) \quad (3.a)$$

$$\overline{F}_1 = 2[F_1 - f(0)] \quad (3.b)$$

And eq.(4) can be obtained from eq.(2) by taking least square error method.

$$\overline{F_1} = \int_0^1 f(\tau)\phi_0(\tau)d\tau \quad (4.a)$$

$$f(1) = \overline{F_1} + f(0) \quad (4.b)$$

Consequently, shifting the interval to the positive direction by $\frac{t_f}{m}$ and calculating $\dot{f}(\tau)$'s STWS coefficient is

$$\overline{F}_2 = 2[F_2 - f(1)] = 2[F_2 - \overline{F}_1 - f(0)]$$

$$\vdots$$

$$\overline{F}_i = 2[F_i - \overline{F}_{i-1} - \overline{F}_{i-2} \cdots - \overline{F}_1 - f(0)]$$

$$\overline{F}_{i+1} = 2[F_{i+1} - \overline{F}_i - \overline{F}_{i-1} \cdots - \overline{F}_1 - f(0)]$$

$$\vdots$$

$$\overline{F}_m = 2[F_m - \overline{F}_{m-1} - \overline{F}_{m-2} \cdots - \overline{F}_1 - f(0)] \quad (5)$$

Finally, one can obtain a recursive algorithm for $\overline{F}_i$ as the following eq.(6).

$$\overline{F}_1 = 2[F_1 - f(0)]$$

$$\overline{F}_i = 2[F_i - F_{i-1}] - \overline{F}_{i-1} \qquad i = 2, 3, \cdots, m \quad (6)$$

3. FDI OBSERVER DESIGN PROCEDURE

3.1 FDI observer design

Consider the following system which includes unknown input $d(t)$, actuator fault $f(t)$ and sensor failure $v(t)$.

$$\dot{x}(t) = Ax(t) + Bu(t) + Dd(t) + Ff(t) \quad (7.a)$$

$$y(t) = Cx(t) + Ev(t) \quad (7.b)$$

where, $x \in \mathcal{R}^{n\times 1}$, $u \in \mathcal{R}^{r\times 1}$, $d \in \mathcal{R}^{p\times 1}$, $f \in \mathcal{R}^{q\times 1}$, $y \in \mathcal{R}^{m\times 1}$ and $v \in \mathcal{R}^{l\times 1}$

In this paper, assuming that $v(t)$ is constant.

Let's represent eq.(7) as $n+l$ th dynamical system eq.(8) which includes $v(t)$ in the state vector.

$$\dot{x}_v(t) = \overline{A}x_v(t) + \overline{B}u(t) + \overline{D}d(t) + \overline{F}f(t) \quad (8.a)$$

$$y(t) = \overline{C}x_v(t) \quad (8.b)$$

where, $x_v(t) \triangleq \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}$, $\overline{A} \triangleq \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}$, $\overline{B} \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix}$

$\overline{D} \triangleq \begin{bmatrix} D \\ 0 \end{bmatrix}$, $\overline{F} \triangleq \begin{bmatrix} F \\ 0 \end{bmatrix}$ and $\overline{C} \triangleq [C \ E]$

Proposed observer has the following form

$$\dot{z}(t) = Lz(t) + Hu(t) + Py(t) \quad (9.a)$$

$$\hat{x}_v(t) = z(t) + Ny(t) \quad (9.b)$$

In eq.(9), each matrix is determined as the following[6].

$$L = R\overline{A} - K\overline{C} ,\ H = R\overline{B} ,\ P = K + LN \quad (10)$$

$$N = [\overline{D}\ \overline{F}] \begin{bmatrix} M_p \\ M_q \end{bmatrix} \quad (11)$$

$$R = I - [\overline{D}\ \overline{F}] \begin{bmatrix} M_p \\ M_q \end{bmatrix} \overline{C} \quad (12)$$

$$\begin{bmatrix} M_p \\ M_q \end{bmatrix} = [CD\ CF]^+ \quad (\text{+ means left pseudo-inverse}) \quad (13)$$

<u>**Theorem 1**</u>

If the pair $(R\overline{A}, \overline{C})$ is observable and $\rho[CD\ CF] = p + q$, there exists an observer eq.(9) which satisfies $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} [x_v(t) - \hat{x}_v(t)] = 0$

■

▶ Its proof is presented in ref.[6].

From the observer eq.(9), estimated unknown input $d(t)$, actuator fault $f(t)$ and sensor failure $v(t)$ calculated through eq.(14)~(16) and estimated state of the system eq.(7) obtained from eq.(17). See also ref.[6] for its derivation procedure.

$$\hat{d}(t) = M_p(\dot{y}(t) - \overline{C}(\overline{A}z(t) + \overline{B}u(t) + \overline{A}Ny(t))) \quad (14)$$

$$\hat{f}(t) = M_q(\dot{y}(t) - \overline{C}(\overline{A}z(t) + \overline{B}u(t) + \overline{A}Ny(t))) \quad (15)$$

$$\hat{v}(t) = [0 \ \ I_l]\ \hat{x}_v(t) \quad (16)$$

$$\hat{x}(t) = [\ I_n\ \ 0]z(t) + (D\,M_p + F\,M_q)y(t) \quad (17)$$

3.2 Applying STWS for FDI observer design

In eq.(14) and (15), differential of the system output is required to obtain the estimated value of unknown input and actuator fault. To alleviate such problem, this paper proposes an algebraic approach to FDI observer design procedure via STWS. To achieve this, solution of eq.(9) is needed in the first place. One can represent the eq.(9) in the interval $t \in [0, \frac{t_f}{m})$ by using STWS.

$$\overline{Z}_1\phi_0(\tau) = \frac{t_f}{m} LZ_1\phi_0(\tau) + \frac{t_f}{m} HU_1\phi_0(\tau) + \frac{t_f}{m} PY_1\phi_0(\tau) \quad (18)$$

where,

$\dot{z}(\tau) \simeq \bar{Z}_1\phi_0(\tau)$, $z(\tau) \simeq Z_1\phi_0(\tau)$, $u(\tau) \simeq U_1\phi_0(\tau)$

$y(\tau) \simeq Y_1\phi_0(\tau)$ and $t = \frac{t_f}{m}\tau$, $\tau \in [0, 1)$

Similar to eq.(3.a), $\bar{Z}_1$ and Z_1 have the following relation

$$Z_1 = \frac{1}{2}\bar{Z}_1 + z(0) \tag{19}$$

By substituting eq.(19) to eq.(18) and rearranging eq.(18) for $\bar{Z}_1$, it is easy to obtain eq.(20).

$$\bar{Z}_1 = \left[I - \frac{t_f}{2m}L\right]^{-1} + \left[\frac{t_f}{m}Lz(0) + \frac{t_f}{m}HU_1 + \frac{t_f}{m}PY_1\right] \tag{20}$$

Let's shift the interval to the positive direction by $\frac{t_f}{m}$ sequentially. Eq.(21) is an generalized recursive algorithm for Z_i which is a solution of eq.(9.a)

$$\bar{Z}_i = \left[I - \frac{t_f}{2m}L\right]^{-1} + \left[\frac{t_f}{m}Lz(i-1) + \frac{t_f}{m}HU_i + \frac{t_f}{m}PY_i\right] \tag{21.a}$$

$$Z_i = \frac{1}{2}\bar{Z}_i + z(i-1) \tag{21.b}$$

$$z(i) = \bar{Z}_i + z(i-1) \tag{21.c}$$

Now, solve the eq.(14) and (15) by using STWS. Eq.(14) and (15) can be represented as eq.(22) and (23) in the interval $t \in [0, \frac{t_f}{m})$.

$$\frac{t_f}{m}\hat{D}_1\phi_0(\tau) = M_p\Big[\bar{Y}_1\phi_0(\tau) - \frac{t_f}{m}\bar{C}[\bar{A}Z_1\phi_0(\tau) + \frac{t_f}{m}\bar{B}U_1\phi_0(\tau) + \frac{t_f}{m}\bar{A}NY_1\phi_0(\tau)]\Big] \tag{22}$$

where, $\hat{d}(\tau) \simeq \hat{D}_1\phi_0(\tau)$, $y(\tau) \simeq \bar{Y}_1\phi_0(\tau)$, $t = \frac{t_f}{m}\tau$ and $\tau \in [0, 1)$

$$\frac{t_f}{m}\hat{F}_1\phi_0(\tau) = M_q\Big[\bar{Y}_1\phi_0(\tau) - \frac{t_f}{m}\bar{C}[\bar{A}Z_1\phi_0(\tau) + \frac{t_f}{m}\bar{B}U_1\phi_0(\tau) + \frac{t_f}{m}\bar{A}NY_1\phi_0(\tau)]\Big] \tag{23}$$

where, $\hat{f}(\tau) \simeq \hat{F}_1\phi_0(\tau)$

Shifting the interval to the positive direction by $\frac{t_f}{m}$ sequentially and using the relation eq.(6), it is easy to obtain recursive algorithm for $\hat{D}_i$ and $\hat{F}_i$.

$$\hat{D}_1 = M_p\Big[\frac{2m}{t_f}[Y_1 - y(0)] - \bar{C}[\bar{A}Z_1 + \bar{B}U_1 + \bar{A}NY_1]\Big]$$

$$\hat{D}_i = M_p\Big[\frac{2m}{t_f}[Y_i - Y_{i-1}] - \frac{m}{t_f}\bar{Y}_{i-1} - \bar{C}[\bar{A}Z_i + \bar{B}U_i + \bar{A}NY_i]\Big] \tag{24}$$

$$\hat{F}_1 = M_q\Big[\frac{2m}{t_f}[Y_1 - y(0)] - \bar{C}[\bar{A}Z_1 + \bar{B}U_1 + \bar{A}NY_1]\Big]$$

$$\hat{F}_i = M_q\Big[\frac{2m}{t_f}[Y_i - Y_{i-1}] - \frac{m}{t_f}\bar{Y}_{i-1} - \bar{C}[\bar{A}Z_i + \bar{B}U_i + \bar{A}NY_i]\Big] \tag{25}$$

From the eqs.(21),(24) and (25), one can know that proposed method is useful for its algebraic form. If the unexpected measurement noise exists in eq.(14) and (15), proposed method will have better performance than the case such that differentiator is included to obtain a system output's differential value. This paper verifies the such usefulness of proposed STWS approach through the example.

4. EXAMPLE

Let's consider the system to examine the proposed STWS approach to FDI observer design procedure.

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 & 1 \\ 0 & -0.5 & -1 \\ 0 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} f(t) \tag{26.a}$$

$$y(t) = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v(t) , \quad x(0) = [1 \ -1 \ 2]^T \tag{26.b}$$

Assume that input $u(t)$ is 1, unknown input $d(t)$ is $0.5\sin(t)$, actuator fault $f(t)$ occurs 0.8 at 5 sec., and sensor fault $v(t)$ occurs -0.1 at 7 sec.. One can choose observer gain matrix K to assign arbitrary observer poles at -4, -5, -5 and -6

$$K = \begin{bmatrix} -0.72 & 3.27 & -4.00 \\ 0.73 & 1.73 & 4.00 \\ -5.73 & 3.27 & -4.00 \\ 8.11 & -3.89 & 12.00 \end{bmatrix}$$

Therefore, derived observer equation is

$$\dot{z}(t) = \begin{bmatrix} -2.55 & -2.27 & 2.27 & 4.00 \\ -2.46 & -2.73 & -2.27 & -4.00 \\ 2.46 & -2.28 & -2.73 & 4.00 \\ -4.22 & 3.89 & -3.89 & -12.00 \end{bmatrix} z(t) + \begin{bmatrix} -1 & 1 & -4 \\ 1 & -1 & 4 \\ -1 & 1 & -4 \\ 0 & 0 & 12 \end{bmatrix} y(t) \tag{27}$$

and one can obtain the estimation equation for unknown input, actuator fault and sensor fault as eqs.(28), (29) and (30).

$$\hat{d}(t) = y_1(t) + 2z_1(t) + z_2(t) - 2z_3(t) + y_1(t) - y_2(t) \tag{28}$$

$$\hat{f}(t) = -\dot{y}_1(t) + \dot{y}_2(t) - 0.5\, z_2(t) + 2\, z_3(t) + u(t) + 0.5\, y_1(t) - 0.5\, y_2(t) \quad (29)$$

$$\hat{v}(t) = z_4(t) \quad (30)$$

Fig. 1 is an additional noise in the system output $y_1(t)$ to verify the advantage of STWS approach. Fig. 2~5 are the comparison between STWS approaches and the case such that differentiator is included in eq.(28) and (29).

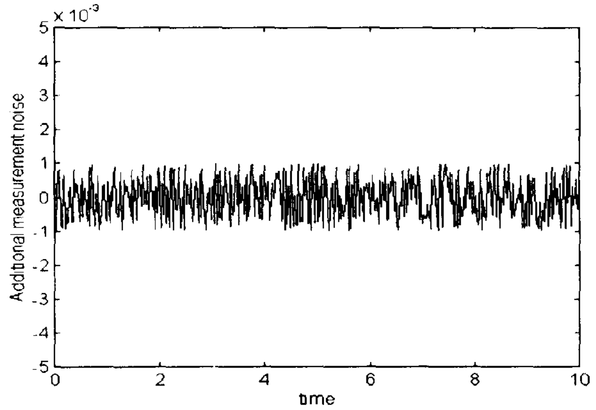

Fig. 1. Additional measurement noise

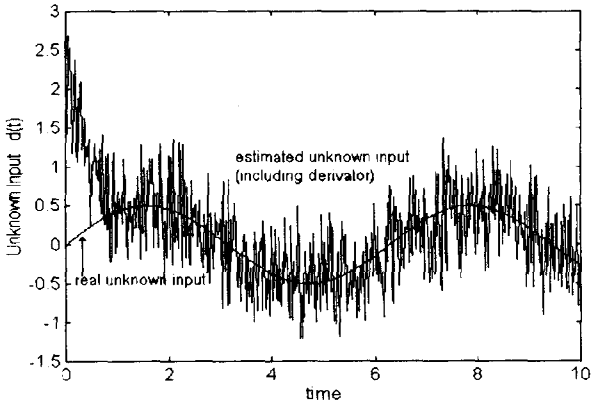

Fig. 2. Estimated unknown input (differentiator included case)

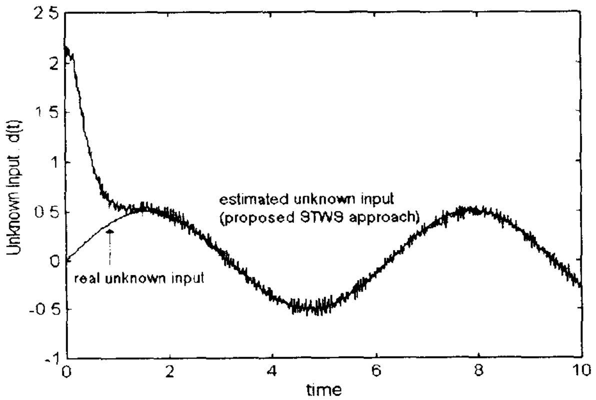

Fig. 3. Estimated unknown input (proposed STWS approach)

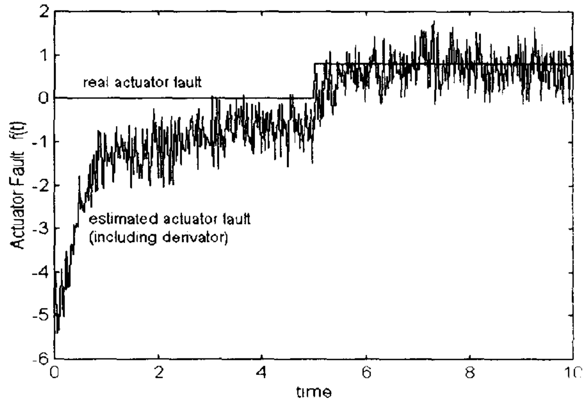

Fig. 4. Estimated actuator fault (differentiator included case)

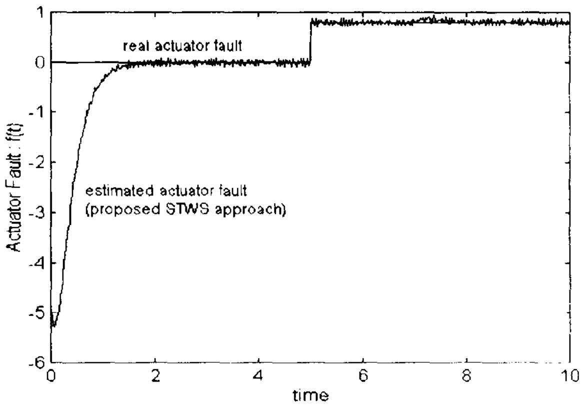

Fig. 5. Estimated actuator fault (proposed STWS approach)

5. CONCLUSION

This paper presents an algebraic approach to FDI observer design procedure. It is proved in the example that presented method is more useful when the unexpected additional measurement noise exists in the system output. It is reason why presented STWS approach approximates the system output's differential to algebraic form which is composed of system output's STWS coefficient and its initial value only. In practical case, there are a lot of unexpected noises and it is harmful to estimate system fault. Therefore, we can expect presented method is more effective than the other method in such cases.

REFERENCES

[1] M.S. Corrington, "Solution of differential equations with Walsh functions," IEEE Trans. Cir. Theory, vol. 20, pp. 470-476, 1973.

[2] F. Kraus and W. Schaufelberger, "Identification with block-pulse functions, modulating functions and differential operators," Int. J. Cont., vol. 51, pp. 931-942, 1990.

[3] Pius Ahn, M. H. Kim and D. S. Ahn, "An algebraic method to design of unknown input observer," Proc. of ICARCV'97, pp. 2351-2355, 1996.
[4] Pius Ahn, M. K. Lee and D. S. Ahn, "UIO design for linear time invariant systems via STWS," Trans. on KIEE, vol. 46, no. 1, pp. 128-132, 1997.
[5] S. I. Han, Pius Ahn, M. H. Kim and D. S. Ahn, "An algebraic approach to unknown inputs observer design using block pulse function," Trans. on KIEE, vol. 46, no. 6, pp. 950-957, 1997.
[6] M. H. Kim, Pius Ahn, J. H. Jung and D. S. Ahn, " Design of FDI observer for linear system," Joint Conference of ICASE, KIEE, and KITE on Control, Automation and Robotics, pp.379-382, 1997.