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A NEW DISCRETE-TIME ROBOT MODEL AND ITS VALIDITY TEST

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Abstracts Digital control of robot manipulator employs discrete-time robot models. It is important to explore effective discrete-time robot models and to analyze their properties in control system designs. This paper presents a new type discrete-time robot model. The model is derived by using trapezoid rule to approximate the convolution integral term, then eliminating nonlinear force terms from robot dynamical equations. The new model obtained has very simple structure, and owns the properties of independence to the nonlinear force terms. According to evaluation criteria, three aspects of the model properties: model accuracy, model validity range and model simplicity are examined and compared with commonly used discrete-time robot models. The validity of the proposed model and its advantages to control system designs are verified by simulation results.

Keywords robot manipulator, discrete-time model, model validity, digital control, robust

1. INTRODUCTION

The dynamical equations of robot manipulator have been well known, and a lot of systematic methods are developed to derive them [1]. Since the robot dynamics are expressed as nonlinear, highly coupled multivariable differential equations, it is mandatory to use digital computer in dynamic simulations and in control system designs. Traditionally, control schemes are designed in continuous time domain, then implemented on digital computer with an assumption of a very high sampling rate [2, 3]. However, the properties of a continuous system are often quite different with its "equivalent" system in discrete-time domain, e.g. the stability of the system may be deteriorated after discretization. It is therefore more reasonable to explore computer oriented discrete-time robot models and design digital control systems which can be realized directly on computers.

There are several papers discussing methods for obtaining discrete-time robot model [4, 5, 6, 7]. Neuman and Tourassis design their model to guarantee conservation of energy at each sampling instant. Nicosia et al. derive their models by applying numerical discretization techniques to the minimization problem of the Lagrange functional. The discrete-time models they obtained have similar formulation with the robot differential equation, in which the inertial term, nonlinear force terms and the input force term are included. Therefore, these models are still complex in structure and heavy in computation.

Recently, Ohkawa proposes a discrete-time approximated model for mechanical system [8]. This study extends that work to robot manipulators. The obtained discrete-time robot model has very simple formulation, and structurally independent to the nonlinear force terms of the robot dynamical equations.

The outline of this paper is as follows: The derivation of the new discrete-time robot model is given in section 2. Section 3 discusses robot model validity issues. According to the requirement of robot control systems, model validity criteria are chosen, corresponding simulation schemes are devised, and simulation results are discussed. Section 4 provides a summary.

2. A NEW DISCRETE-TIME ROBOT MODEL

In this section, we present the derivation of the new discrete-time robot model.

The n-link robot dynamical equation can be expressed as:

$$M(\theta)\ddot{\theta} + V(\theta,\dot{\theta}) + G(\theta) + F(\dot{\theta}) = \tau \qquad (1)$$

where $\theta \in \Re^n$ is the generalized joint position vector, $\tau \in \Re^n$ is the generalized input force vector, $M(\theta) \in \Re^{n\times n}$ is the positive definite inertial matrix, $V(\theta,\dot{\theta}) \in \Re^n$ is a vector representing Coriolis/centrifugal force effect, $G(\theta) \in \Re^n$ is gravity vector, while $F(\dot{\theta}) \in \Re^n$ represents the friction force vector.

Define $H(\theta,\dot{\theta})$ as nonlinear force term, which includes Coriolis/centrifugal force, gravity and friction force terms.

$$H(\theta,\dot{\theta}) = V(\theta,\dot{\theta}) + G(\theta) + F(\dot{\theta}) \qquad (2)$$

Equation (1) can be rewritten as:

$$M(\theta)\ddot{\theta} + H(\theta,\dot{\theta}) = \tau \qquad (3)$$

Represent (3) into state-space formulation as:

$$\frac{d}{dt}\begin{bmatrix}\theta(t)\\ \dot{\theta}(t)\end{bmatrix} = \begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta(t)\\ \dot{\theta}(t)\end{bmatrix} + \begin{bmatrix}0\\ I\end{bmatrix} M^{-1}(\theta)[\tau(t) - H(\theta,\dot{\theta})] \qquad (4)$$

The state variables of the equation are generalized joint position θ, and joint velocity $\dot{\theta}$, whereas $I \in \Re^{n\times n}$ is an identity matrix.

Integrating both sides of (4), we get:

$$\begin{bmatrix} \theta(T) \\ \dot{\theta}(T) \end{bmatrix} = e^{AT} \begin{bmatrix} \theta(0) \\ \dot{\theta}(0) \end{bmatrix} + e^{AT} \int_0^T e^{-At} \begin{bmatrix} 0 \\ I \end{bmatrix} M^{-1}(\theta)[\tau(t) - H(\theta,\dot{\theta})]dt \quad (5)$$

where $A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$, and

$$e^{AT} = \mathcal{L}^{-1}[sI - A]^{-1}|_{t=T} = \begin{bmatrix} I & TI \\ 0 & I \end{bmatrix} \quad (6)$$

Therefore, (5) becomes:

$$\begin{bmatrix} \theta(T) \\ \dot{\theta}(T) \end{bmatrix} = \begin{bmatrix} I & TI \\ 0 & I \end{bmatrix} \begin{bmatrix} \theta(0) \\ \dot{\theta}(0) \end{bmatrix} + \begin{bmatrix} I & TI \\ 0 & I \end{bmatrix} \cdot \int_0^T \begin{bmatrix} -tI \\ I \end{bmatrix} M^{-1}(\theta)[\tau(t) - H(\theta,\dot{\theta})]dt \quad (7)$$

Using notation $\mathcal{S}$ to represent the integration part in (7) as:

$$\mathcal{S} = \int_0^T \begin{bmatrix} -tI \\ I \end{bmatrix} M^{-1}(\theta)[\ \tau(t) - H(\theta,\dot{\theta})\ ]dt \quad (8)$$

Since it is difficult to calculate the exact value of $\mathcal{S}$ for arbitrary $H(\theta,\dot{\theta})$, we use trapezoid rule to do the approximation:

$$\mathcal{S} = \int_0^T s(t) \approx \frac{T}{2}\ [\ s(0) + s(T)\ ] \quad (9)$$

Therefore, $\mathcal{S}$ can be calculated and approximated using trapezoid rule as:

$$\mathcal{S} = \begin{bmatrix} \int_0^T \{-tM^{-1}(\theta)[\tau(t) - H(\theta,\dot{\theta})]\}dt \\ \int_0^T M^{-1}(\theta)[\tau(t) - H(\theta,\dot{\theta})]dt \end{bmatrix} \approx \begin{bmatrix} -\frac{T^2}{2}M^{-1}(\theta(T))[\tau(0) - H(\theta(T),\dot{\theta}(T))] \\ \cdots\cdots\cdots\cdots \\ \frac{T}{2}\{M^{-1}(\theta(T))[\tau(0) - H(\theta(T),\dot{\theta}(T))] \\ +M^{-1}(\theta(0))[\tau(0) - H(\theta(0),\dot{\theta}(0))]\} \end{bmatrix} \quad (10)$$

Note that the control input $\tau(t)$ is piece-wise constant.

Substitute (10) for (7), and replace time instant 0 by k, T by $(k+1)$ respectively. Then following difference equations are obtained:

$$\theta(k+1) = \theta(k) + T\dot{\theta}(k) + \frac{T^2}{2}M^{-1}(\theta(k))[\tau(k) - H(\theta(k),\dot{\theta}(k))] \quad (11)$$

$$\dot{\theta}(k+1) = \dot{\theta}(k) + \frac{T}{2}M^{-1}(\theta(k))[\tau(k) - H(\theta(k),\dot{\theta}(k))] + \frac{T}{2}M^{-1}(\theta(k+1))[\tau(k) - H(\theta(k+1),\dot{\theta}(k+1))] \quad (12)$$

By substituting (11) for (12), we eliminate the nonlinear forces term H from above equations:

$$\frac{2}{T^2}M(\theta(k))[\theta(k+1) - 2T\dot{\theta}(k) - \theta(k-1)] + \tau(k-1) = \tau(k) \quad (13)$$

Obviously, (13) has the novel structure to commonly used robot models: firstly, it excludes the nonlinear force terms from robot dynamical equation; secondly, the observation of the velocity signal is necessary.

Remark 1. The fundamental characteristics of the new discrete-time robot model of (13) is its eliminating the nonlinear force terms from robot equations. This is realized by using trapezoid rule to approximate the integration part in robot equation, and then successfully eliminating the nonlinear force term $H(\theta,\dot{\theta})$. The obtained model has very simple structure. This assumes the control algorithms based on this model can also be simplified.

Remark 2. From (12) we could see, the velocity signal includes the information of nonlinear forces, therefore, if only we could observe output velocity, or approximately calculate the velocity from position signals, the effect of nonlinear forces are considered implicitly in the model. But it saves tedious computation for nonlinear force terms.

3. MODEL VALIDITY TEST

In this section, we verify the validity of the discrete-time robot model proposed in the preceding section by computer simulation.

A 2-link vertical manipulator with revolution joints is considered. Its physical parameters are compiled in Table 1.

TABLE 1. Physical parameters of 2-link manipulator

manipulator	Link 1	Link 2
mass (kg)	3	0.6
length (m)	0.35	0.28
inertia(kg-m)	0.262	0.0313
friction coefficient	0.2	0.2

Comparisons were carried out among Lagrange forward model, Nicosia's backward model [5] and the proposed discrete-time robot model of (13).

Lagrange forward model can be expressed as:

$$M(\theta(k))[\frac{\theta(k+2) - 2\theta(k+1) + \theta(k)}{T^2}] + H(k) = \tau(k) \quad (14)$$

Nicosia's backward model can be written as:

$$M(\theta(k+1))[\frac{\theta(k+1) - \theta(k)}{T^2}] - M(\theta(k))[\frac{\theta(k) - \theta(k-1)}{T^2}] + H(k) = \tau(k) \quad (15)$$

From the control design point of view, three aspects of the model properties are considered as evaluation criteria.

3.1 Model Accuracy

To examine the model accuracy, we input reference signals to both continuous time robot model and discrete-time robot modeld, and then measure their output errors to see how good the fit is. Simulation scheme is described in Fig.1.

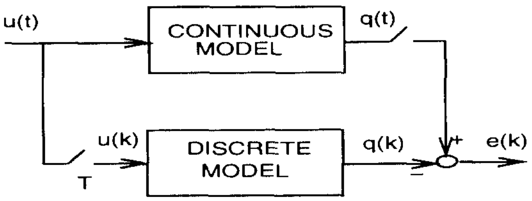
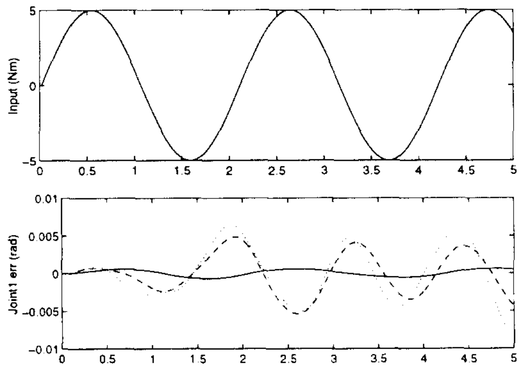


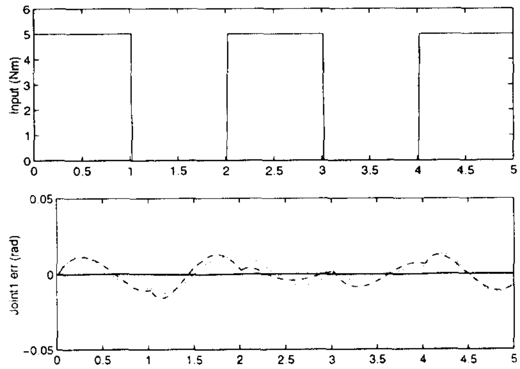
Fig.1 Simulation I — Model Accuracyd

Sampling period is $T = 0.01s$. The measurement of the velocity signal is assumed available. First, we use sine wave as input signal $\tau_1(t) = 5sin3t$, $\tau_2(t) = 0$. The output errors of joint 1 are shown in Fig.2. From simulation results, we can see the proposed discrete-time model shows better accuracy.



- - - Lagrange model ...... Nicosia model
—– Proposed model

Fig.2 Output error under sine input (T=0.01s)



- - - Lagrange model ...... Nicosia model
—– Proposed model

Fig.3 Output error under square input (T=0.01s)

Then, we use square wave as input signal, as shown in Fig.3. It can be seen that since the new model making use of the velocity signal directly, even the input signal changes rapidly, the output of the new model can still follow the continuous model in very small error. Therefore, we could say if the output velocity can be observed correctly, the new model shows better accuracy than other commonly used robot models.

3.2 Model Validity Range Towards T

It is neither easy nor necessary to derive universally valid robot models. The validity of a discrete-time robot model depends very much on the sampling periods. To what extent the sampling period limits the use of a discrete-time robot model has to be considered.

If a discrete-time robot model can endure relatively greater sampling periods, and still work well, we say that model has broader validity range towards sampling period T, or in other words, it owns larger stable region to the sampling periods.

In stead of making a mathematical analysis, we devise a simulation scheme, as shown in Fig.4, to carry out the examination. In Fig.4, the discrete-time robot model is taken as an adaptive observer. Parameters of the model are adjusted by adaptive algorithms. Increasing sampling period until the output error $e(k)$ exceeds a certain bound e_{lim}, the maximum sampling period T_{max} could be determined.

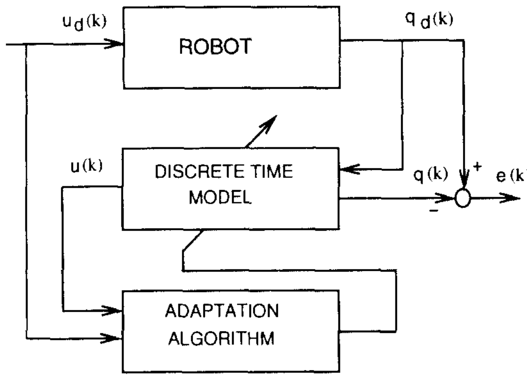


Fig.4 Simulation II — Adaptive Observer

Table 2 shows the maximum of sampling period T_{max} each model can work with. The output error limitations are set to be $e_{lim1} = 1(rad)$ and $e_{lim2} = \infty$ respectively.

TABLE 2 *Tmax*

Model\ T(s)	Tmax1 (e_{lim1})	Tmax2 (e_{lim2})
Lagrange	0.10	0.10
Nicosia	0.12	0.14
Proposed	0.16	0.40

It can be seen that the proposed model can endure greater sampling time, which implies that the new model has relatively broader validity range towards sampling periods.

3.3 Model Simplicity

Discrete-time robot models can also be expressed as:

$$\tau(k) = Y(k+1)\,\Sigma \tag{16}$$

where where $Y(k+1) \in \Re^{n\times m}$ is a regressive matrix of known function of joint positions and velocities; $\Sigma \in \Re^{m}$ is a vector of physical parameters.

For a 2-link manipulator, $Y(k)$ and Σ of the proposed model can be calculated as:

$$Y(k) = \begin{bmatrix} Y_{11}(k) & Y_{12}(k) & Y_{13}(k) & Y_{14}(k) \\ Y_{21}(k) & Y_{22}(k) & Y_{23}(k) & Y_{24}(k) \end{bmatrix} \tag{17}$$

$$\begin{aligned}
Y_{11}(k) &= \theta_1(k) - 2T\dot{\theta}_1(k-1) - \theta_1(k-2) \\
Y_{12}(k) &= [\theta_1(k) - 2T\dot{\theta}_1(k-1) - \theta_1(k-2)]\cdot \\
&\quad cos(\theta_2(k-1)) \\
Y_{13}(k) &= \theta_2(k) - 2T\dot{\theta}_2(k-1) - \theta_2(k-2) \\
Y_{14}(k) &= [\theta_2(k) - 2T\dot{\theta}_2(k-1) - \theta_2(k-2)]\cdot \\
&\quad cos(\theta_2(k-1)) \\
Y_{21}(k) &= 0 \\
Y_{22}(k) &= 0 \\
Y_{23}(k) &= \theta_1(k) - 2T\dot{\theta}_1(k-1) - \theta_1(k-2) + \\
&\quad \theta_2(k) - 2T\dot{\theta}_2(k-1) - \theta_2(k-2) \\
Y_{24}(k) &= [\theta_1(k) - 2T\dot{\theta}_1(k-1) - \theta_1(k-2)]\cdot \\
&\quad cos(\theta_2(k-1)),
\end{aligned}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{bmatrix}^T \tag{18}$$

$$\begin{aligned}
\sigma_1 &= \frac{2}{T^2}[m_1 lg_1^2 + m_2 l_1^2 + m_2 lg_2^2 + I_1 + I_2)] \\
\sigma_2 &= \frac{2}{T^2}[2m_2 l_1 lg_2] \\
\sigma_3 &= \frac{2}{T^2}[m_2 lg_2^2 + I_2] \\
\sigma_4 &= \frac{2}{T^2}[m_2 l_1 lg_2]
\end{aligned}$$

In adaptive control methods, when the updating algorithms are used for calculating adaptation gains, the number of estimated parameters of the robot system determine the computational efficiency. Since the proposed model excluded nonlinear force terms, the number of the parameters been identified on-line is only four, while for both Lagrange and Nicosia's models the estimated parameters are seven.

4. CONCLUSIONS

Discrete-time robot models play an important role in digital control of robot systems. In this paper, we derive a new type of discrete-time robot model. The new model obtained is simple in structure and possesses good properties in accuracy, validity range and simplicity. Its effectiveness is demonstrated by computer simulation.

Future research work will be incorporating the proposed discrete-time robot model in control applications.

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