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A new derivation method of the Generalized Jacobian Matrix of a space robot and its application to a multi-robot system

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Abstract This paper deals with a new method to derive the Generalized Jacobian Matrix of a space robot. In a conventional method to derive the Generalized Jacobian Matrix, generalized coordinates select joint angles and a space robot body's position and attitude angle. But, in this paper, we select position and attitude angle of the end-effector or the handled floating object as generalized coordinates. Then, we can derive the Generalized Jacobian Matrix of the system which consists of several space robots and a handled floating object. Moreover, control methods operated by only one space robot can be easily extended to the cases of cooperation task by several space robots. Computer simulations show that the Generalized Jacobian Matrix derived here is effective.
Keywords Space Robot, Generalized Jacobian Matrix, Multi-Robot system, Resolved Motion Rate Control

1 Introduction

Operations in space are high risk activities for astronauts and not efficiency, because the environment of space is different from that of the terrestrial, for example, micro gravity, high vacuum, and so on. Therefore, space robots are expected to replace astronauts in future space missions.

Operational ability of a space robot depends on its scale, and the scale of a space robot is restricted by the capacity of a rocket, which is used to launch the space robot into space. For this reason, the cooperation by several smaller space robots is necessary in space development.

Many control methods for space robot have been reported [1] [2], most of them are designed based on the Generalized Jacobian Matrix and conventional control methods, for example, Resolved Motion Rate Control and Resolved Acceleration Control and so on. However, these methods are only effective for the cases that a single robot is used in operation.

We propose a new method to derive the Generalized Jacobian Matrix for space robot. In a conventional method of deriving the Generalized Jacobian Matrix, joint angles and space robot body's position and attitude angle are selected as generalized coordinates of the space robot. But in this paper, we select the end-effector or the handled floating object's position and attitude angle as generalized coordinates. Then, we can derive the Generalized Jacobian Matrix of the system which consists of several space robots and a handled floating object. Moreover, control methods operated by only one space robot can be easily extended to the cases of the cooperation task by several space robots.

2 Modeling

In this paper, we consider the space robot systems shown in Fig.(1) and Fig.(2). The system shown in Fig.(1) is called "R-O system". The system called "R-O-R system" is shown in Fig.(2). 'R', 'O' and '-' mean a robot, a object and a manipulator respectively.

Although these model are represented in plane, we consider the operational space is 3 dimension, the D.O.F. of Robot1, Robot2 is n_1, n_2 respectively.

Assumptions and symbols used in this paper are defined as follows.

[Assumptions]

1. All elements of the system are rigid.
2. No external force acts on the system, i.e., the conservation laws of momentum and angular momentum stand up.
3. The initial momentum and angular momentum of the sytem are zero.

[Symbols]

$\boldsymbol{p}_{int}$	:	*position vector of the interested point*
$\boldsymbol{p}_i^k$	:	*position vector of the joint i of the robot k*
$\boldsymbol{r}_i^k$	:	*position vector of the center of mass of the link i of the robot k*
$\boldsymbol{r}_g$	:	*position vector of the center of the mass of the system*
ϕ_0	:	*attitude angle of the floating object*
ϕ_0^k	:	*angle of the point grasped by robot k*
ϕ_i^k	:	*angle of the joint i of robot k*
$\boldsymbol{E}$	:	*unit matrix*

$$\boldsymbol{r}_{0i}^k \triangleq \boldsymbol{r}_i^k - \boldsymbol{r}_0, \quad \boldsymbol{r}_{0g} \triangleq \boldsymbol{r}_g - \boldsymbol{r}_0$$

$$\tilde{a} \triangleq \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix}$$
$for\ vector\ \boldsymbol{a} = (a_x, a_y, a_z)^T$

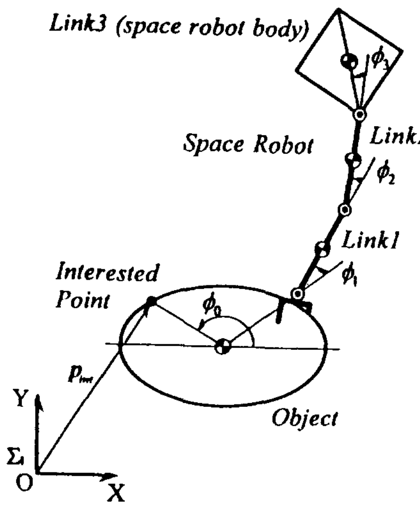

Figure 1: R-O System Model

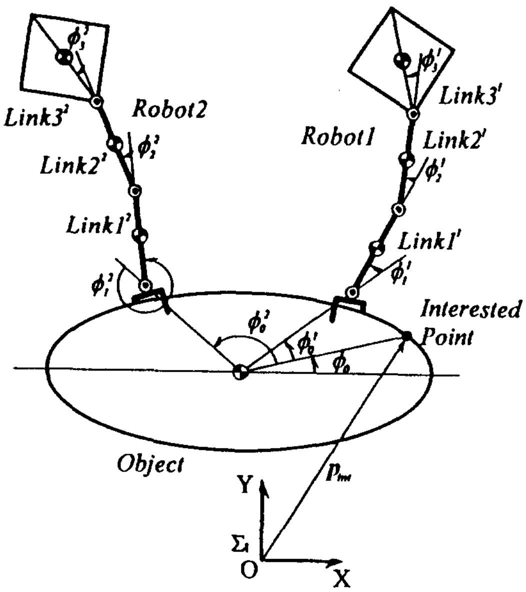

Figure 2: R-O-R System Model

3 Derivation of Kinematic Relation (Generalized Jacobian Matrix : GJM)

A derivation method of the Generalized Jacobian Matrix of a space robot is already proposed [2]. In his method, generalized coordinates select joint angles and a space robot body's position and attitude angle.

In this paper, we select position and attitude angle of the end-effector or the handled floating object as generalized coordinates.

First, we define a manipulation variable vector of the interested point as follows:

$$\boldsymbol{\nu} \triangleq \begin{pmatrix} \dot{\boldsymbol{p}}_{int} \\ \boldsymbol{\omega}_0 \end{pmatrix} \in \Re^{m}, \dot{\boldsymbol{p}}_{int} \in \Re^{m_1}, \boldsymbol{\omega}_0 \in \Re^{m_2}. \qquad (1)$$

Then, following equation stand up,

$$\boldsymbol{\nu} = \boldsymbol{J}_s \begin{pmatrix} \boldsymbol{v}_0 \\ \boldsymbol{\omega}_0 \end{pmatrix}, \qquad (2)$$

where

$$\boldsymbol{J}_s \triangleq \begin{pmatrix} \boldsymbol{E} & -(\widetilde{\boldsymbol{p}_{int} - \boldsymbol{r}_0}) \\ 0 & \boldsymbol{E} \end{pmatrix} \in \Re^{m\times 6}$$

is Jacobian Matrix of the manipulation variable vector $\boldsymbol{\nu}$ for $(\boldsymbol{v}_0^T, \boldsymbol{\omega}_0^T)^T$. Now, $\boldsymbol{v}_0$ is a velocity vector of the floating object, $\boldsymbol{\omega}_0$ is an angular velocity vector of the floating object.

The conservation laws of momentum and angular momentum of R-O system can be described as follows:

$$\begin{pmatrix} \boldsymbol{P} \\ \boldsymbol{L} \end{pmatrix} = \boldsymbol{H}_s \begin{pmatrix} \boldsymbol{v}_0 \\ \boldsymbol{\omega}_0 \end{pmatrix} + \boldsymbol{H}_m \dot{\boldsymbol{\phi}} = \boldsymbol{0}, \qquad (3)$$

$$\boldsymbol{H}_s \triangleq \begin{pmatrix} w\boldsymbol{E} & -w\tilde{\boldsymbol{r}}_{0g} \\ w\tilde{\boldsymbol{r}}_g & \boldsymbol{I}_\omega \end{pmatrix},$$
$$\boldsymbol{H}_m \triangleq \begin{pmatrix} \boldsymbol{J}_{Tw} \\ \boldsymbol{I}_\phi \end{pmatrix},$$
$$\boldsymbol{I}_\omega \triangleq \sum_{i=1}^{3}(\boldsymbol{I}_i - m_i\tilde{\boldsymbol{r}}_i\tilde{\boldsymbol{r}}_{0i}) + \boldsymbol{I}_0,$$
$$\boldsymbol{J}_{Tw} \triangleq \sum_{i=1}^{3} m_i \boldsymbol{J}_{Ti},$$
$$\boldsymbol{I}_\phi \triangleq \sum_{i=1}^{3}(\boldsymbol{I}_i\boldsymbol{J}_{Ri} + m_i\tilde{\boldsymbol{r}}_i\boldsymbol{J}_{Ti}),$$

$$\boldsymbol{J}_{Ti} \triangleq (\boldsymbol{k}_1 \times (\boldsymbol{r}_i - \boldsymbol{p}_1), \boldsymbol{k}_2 \times (\boldsymbol{r}_i - \boldsymbol{p}_2), \cdots, \boldsymbol{k}_i \times (\boldsymbol{r}_i - \boldsymbol{p}_i), 0, \cdots, 0) \in \Re^{3\times n},$$
$$\boldsymbol{J}_{Ri} \triangleq (\boldsymbol{k}_1, \boldsymbol{k}_2, \cdots, \boldsymbol{k}_i, 0, \cdots, 0) \in \Re^{3\times n},$$
$$\dot{\boldsymbol{\phi}} \triangleq (\dot{\phi}_1, \dot{\phi}_2, \cdots, \dot{\phi}_n)^T \in \Re^n,$$

where $\boldsymbol{P}$, $\boldsymbol{L}$ is momentum and angular momentum of the system respectively.

From Eq.(3), the following equation is given.

$$\begin{pmatrix} \boldsymbol{v}_0 \\ \boldsymbol{\omega}_0 \end{pmatrix} = -\boldsymbol{H}_s^{-1}\boldsymbol{H}_m\dot{\boldsymbol{\phi}} \qquad (4)$$

Substituting Eq.(4) for $(\boldsymbol{v}_0{}^T, \boldsymbol{\omega}_0^T)^T$ in Eq.(2), Kinematic Relation of R-O system, we have

$$\boldsymbol{\nu} = \boldsymbol{J}_{int}^{*}\dot{\boldsymbol{\phi}}, \qquad (5)$$

where

$$\boldsymbol{J}_{int}^{*} \triangleq -\boldsymbol{J}_s\boldsymbol{H}_s^{-1}\boldsymbol{H}_m\dot{\boldsymbol{\phi}} \in \Re^{m\times n}$$

is Generalized Jacobian Matrix of $\boldsymbol{\nu}$ for $\boldsymbol{\phi}$.

4 Application to a multi-robot system

In this chapter, we derive the Kinematic Relation (GJM) of R-O-R system by the above method.

For R-O-R system, Eq.(2) stand up, too. Similarly to R-O system, the conservation laws of momentum and angular momentum of R-O-R system can be described as follows:

$$\begin{pmatrix} P \\ L \end{pmatrix} = H_s \begin{pmatrix} v_0 \\ \omega_0 \end{pmatrix} + H_m \dot{\phi} = 0, \quad (6)$$

where

$$H_s \triangleq \begin{pmatrix} wE & -w\tilde{r}_{0g} \\ w\tilde{r}_g & I_\omega \end{pmatrix},$$

$$H_m \triangleq \begin{pmatrix} J_{Tw} \\ I_\phi \end{pmatrix},$$

$$I_\omega \triangleq \sum_{k=1}^{2}\sum_{i=1}^{3}(I_i^k - m_i^k \tilde{r}_i^k \tilde{r}_{0i}^k) + I_0,$$

$$J_{Tw} \triangleq \sum_{k=1}^{2}\sum_{i=1}^{3} m_i^k J_{Ti}^k,$$

$$I_\phi \triangleq \sum_{k=1}^{2}\sum_{i=1}^{3}(I_i^k J_{Ri}^k + m_i^k \tilde{r}_i^k J_{Ti}^k),$$

$$J_{Ti}^1 \triangleq (k_1^1 \times (r_i^1 - p_1^1), k_2^1 \times (r_i^1 - p_2^1), \cdots, k_i^1 \times (r_i^1 - p_i^1), 0, \cdots, 0, 0, \cdots, 0) \in \Re^{3\times n},$$

$$J_{Ti}^2 \triangleq (0, \cdots, 0, k_1^2 \times (r_i^2 - p_1^2), k_2^2 \times (r_i^2 - p_2^2), \cdots, k_i^2 \times (r_i^2 - p_i^2), 0, \cdots, 0) \in \Re^{3\times n},$$

$$J_{Ri}^1 \triangleq (k_1^1, k_2^1, \cdots, k_i^1, 0, \cdots, 0, 0, \cdots, 0) \in \Re^{3\times n},$$

$$J_{Ri}^2 \triangleq (0, \cdots, 0, k_1^2, k_2^2, \cdots, k_i^2, 0, \cdots, 0) \in \Re^{3\times n},$$

$$\dot{\phi} \triangleq \begin{pmatrix} \dot{\phi}^1 \\ \dot{\phi}^2 \end{pmatrix} \in \Re^n,$$

$$\dot{\phi}^1 \triangleq (\dot{\phi}_1^1, \dot{\phi}_2^1, \cdots, \dot{\phi}_{n_1}^1)^T \in \Re^{n_1},$$

$$\dot{\phi}^2 \triangleq (\dot{\phi}_1^2, \dot{\phi}_2^2, \cdots, \dot{\phi}_{n_2}^2)^T \in \Re^{n_2}.$$

However, I_ω, J_{Tw}, I_ϕ, J_{Ti} and J_{Ri} are different from those in R-O system.

Therefore, similarly to R-O system, the Kinematic Relation of R-O-R system becomes

$$\nu = J_{int}^* \dot{\phi}. \quad (7)$$

5 Computer Simulation

In this chapter, we verify the validity of the GJM derived in the preceding section by the computer simulation. We design Resolved Motion Rate Control for R-O system and R-O-R system.

5.1 Resolved Motion Rate Control

From Eq.(5) or Eq.(7), the Resolved Motion Rate Control law is obtained as follows:

$$\dot{\phi}_{cd} = [J_{int}^*]^\dagger \nu_d + (I - [J_{int}^*]^\dagger J_{int}^*) k, \quad (8)$$

where

$\dot{\phi}_{cd}$	:	*joint angular velocity command*
ν_d	:	*desired manipulation variable*
k	:	*any n order vector*
$[J_{int}^*]^\dagger$	:	*pseudo inverse matrix of* J_{int}^*.

To implement the RMRC by digital computers, RMRC law must be discretized. For R-O system, we discretized Eq.(8) as follows:

$$\dot{\phi}_{cd}(k) = [J_{int}^*(k)]^\dagger \{\nu_d(k+1) - \Lambda e(k)\}. \quad (9)$$

For R-O-R system, we have

$$R1 : \dot{\phi}_{cd}(k) = [J_{int}^*(k, k-1)]^\dagger \{\nu_d(k+1) - \Lambda e(k)\}, \quad (10)$$

$$R2 : \dot{\phi}_{cd}(k) = [J_{int}^*(k-1, k)]^\dagger \{\nu_d(k+1) - \Lambda e(k)\}, \quad (11)$$

where

$$\nu_d = \begin{pmatrix} \{P_d(k+1) - P_d(k)\}/T \\ \{\phi_0(k+1) - \phi_0(k)\}/T \end{pmatrix},$$

$$e(k) = \begin{pmatrix} P_{intd}(k) - P_{int}(k) \\ \phi_{0d}(k) - \phi_0(k) \end{pmatrix},$$

$$\Lambda = diag\{\lambda_1, \lambda_2, \lambda_3\}.$$

Now, vector k is set up zero vector. $J_{int}^*(m, n)$ is the GJM that contained the joint angles of Robot1 at the discrete time mT and the joint angles of Robot2 at the discrete time nT.

Eq.(10),Eq.(11) is the discrete time RMRC law for the Robot1, Robot2 respectively.

We consider one sampling period as the computational time delay in Eq.(9), Eq.(10) and Eq.(11). Moreover, in Eq.(10) and Eq.(11), one sampling period of the communication time delay is considered. In order to compensate tracking errors of position and attitude angle, feedback $-\Lambda e(k)$ is added to the control input.

5.2 Simulation Conditions

Computer simulations were carried out under the following conditions. Space Robots rotate a floating object to 30.0[degree] from the initial position, and fix the center of mass of the object.

Physical parameters of the space robots and the floating object are shown in Table1 and Table2.

Table1 Physical Parameters of the space robots

	Link1, 2	Link3
Mass [kg]	100.0	1000
Moment of Inertia [kgm^2]	33.33	333.3
Length [m]	2.0	2.0

Table2 Physical Parameters of the floating object

	Object
Mass [kg]	1000
Moment of Inertia [kgm^2]	333.3
Length [m]	2.0

The sampling period T is $0.01[sec.]$, and the feedback gain λ_i is -0.3.

5.3 Simulation Results

Fig.(3) ~ Fig.(6) show simulation results. Simulation results show that the GJM derived by the proposed method is effective for the multi-robot system.

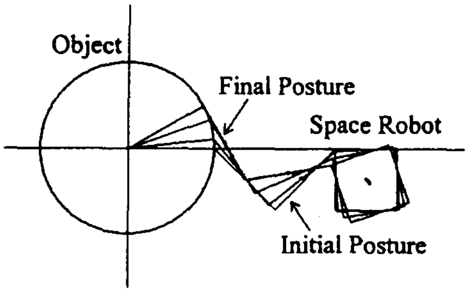

Figure 3: Simulation Result (R-O)

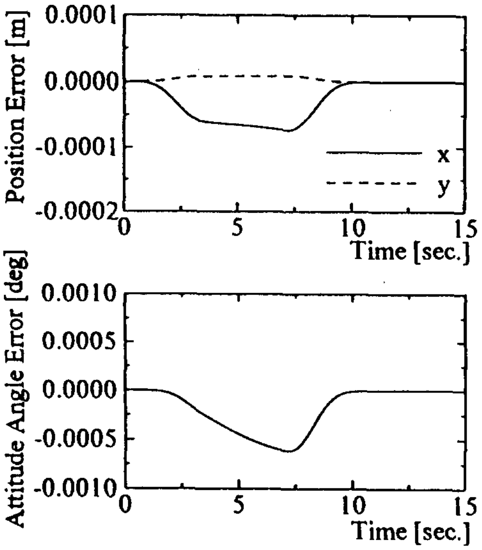

Figure 4: Time History of Tracking Error (R-O)

6 Conclusion

We proposed a new method to derive the Generalized Jacobian Matrix of a space robot. In the new method, the end-effetor or the handled floating object's position, attitude angle and joint angles are selected as generalized coordinates of the space robot. Using the proposed method, we can easily derive the Generalized Jacobian Matrix of the system which consists of several space robots and a handled floating object. Computer simulation results showed that the Generalized Jacobian Matrix derived by the proposed method is effective for the multi-robot system.

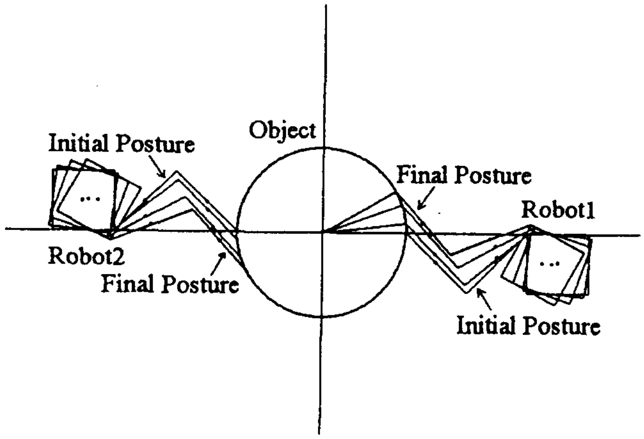

Figure 5: Simulation Result (R-O-R)

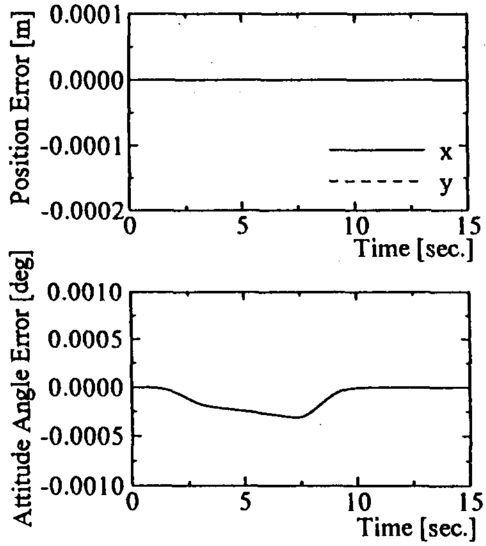

Figure 6: Time History of Tracking Error (R-O-R)

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