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A Method for Linearizing Nonlinear System by use of Polynomial Compensation

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Abstract

In this paper, the authors propose a new method for linearizing a nonlinear dynamical system by use of polynomial compensation. In this method, an M-sequence is applied to the nonlinear system and the crosscorrelation function between the input and the output gives us every crosssections of Volterra kernels of the nonlinear system up to 3rd order. We construct a polynomial compensation function from comparison between 1st order Volterra kernel and high order kernels. The polynomial compensation function is, in this case, of third order whose coefficients are variable depending on the amplitude of the input signal. Once we can get compensation function of nonlinear system, we can construct a linearization scheme of the nonlinear system. That is, the effect of second and third order Volterra kernels are subtracted from the output, thus we obtain a sort of linearized output. The authors applied this method to a saturation-type nonlinear system by simulation, and the results show good agreement with the theoretical considerations.

Keywords Nonlinear system, Identification, Volterra kernel, M-sequence, Crosscorrelation

1. Introduction

The authors proposed a method for identifying Volterra kernels of nonlinear system by use of pseudo-randam M-sequence [1][2][3][6][7]. And in KACC '96 we proposed a method of linearizing nonlinear systems in which the output is subtracted by the effect of higher order Volterra kernels[4]. But in this case, there exists a weak point that we can not apply the method to the nonlinear system whose characteristics is dependent on the amplitude of input signal.

In this paper, we propose a new linearizing method. First, the input signals having several amplitude are applied to the nonlinear system, and we calculate the relation between the amplitude of the input and the coefficient of so called compensation polynomial. Second, the output of the nonlinear system is subtracted by the estimated effect of 2nd and 3rd order Volterra kernels by using the relation between amplitude of input and coefficient of the polynomial.

2. Principle of identification of Volterra kernels

Let us denote the system input as $u(t)$ and the output $y(t)$. Then the output of a nonlinear system can be written in general as follows[1]:

$$y(t) = \sum_{i=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \ldots, \tau_i) \times u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_i)d\tau_1 d\tau_2 \ldots d\tau_i. \quad (1)$$

Here we call $g_i(\tau_1, \tau_2, \ldots, \tau_i)$ i-th oder Volterra kernel. When we take the crosscorrelation function between $u(t)$ and $y(t)$, we have,

$$\begin{aligned}\phi_{uy}(\tau) &= \overline{u(t-\tau)y(t)} \\ &= \sum_{i=1}^{\infty} \int_0^{\infty} \cdots \int_0^{\infty} g_i(\tau_1, \tau_2, \ldots, \tau_i) \\ &\quad \overline{\times u(t-\tau)u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_i)} \\ &\quad \times d\tau_1 d\tau_2 \ldots d\tau_i. \end{aligned} \quad (2)$$

Here $\overline{\quad}$ denotes time average. When we use a pseudo-random M-sequence as input $u(t)$, we get Eq.(3) by use of so-called "shift and add property" of the M-sequence. That is, there exists one and only one integer number k_{ii}^j (mod N) which satisfies the following equation.

$$\begin{aligned} u(t)u(t+k_{i1}^j\Delta t)u(t+k_{i2}^j\Delta t)\ldots u(t+k_{ii-1}^j\Delta t) \\ = u(t+k_{ii}^j\Delta t) \end{aligned} \quad (3)$$

Then, Eq.(2) becomes the next equation.

$$\begin{aligned}\phi_{uy}(\tau) &= \Delta t g_1(\tau) + F(\tau) \\ &+ \sum_{i=2}^{\infty} i!(\Delta t)^i \\ &\times \sum_{j=1}^{m_i} g_i(\tau - k_{i1}^j\Delta t, \ldots, \tau - k_{ii}^j\Delta t) \end{aligned} \quad (4)$$

where $F(\tau)$ is a function of τ.
If $k_{ii}^{(j)}$'s of $g_i(\tau, \ldots, \tau_i)$ are apart from each other sufficiently (say, 20 to 30 Δt apart), then we can obtain Volterra kernels separately from Eq.(4).

3. Coefficient compensation of approximated polynomial

In this study, we approximate the nonlinear system to be identified by polynomial type nonlinear system as shown in the next equation.

$$y(t) = z(t) + a_2 \cdot z^2(t) + a_3 \cdot z^3(t) \quad (a_2, a_3 \in \Re) \quad (5)$$

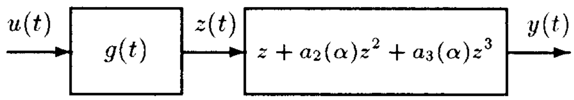

Figure 1: Nonlinear system having up to 3rd order Volterra kernels

Eq.(5) seems reasonable, because the amplitude of Volterra kernels becomes small when the degree of Volterra kernels becomes large. If we set impulse response of linear system as $g_1(\tau)$, then 2nd and 3rd order Volterra kernels can be described by the next equations,respectively[5].

$$g_2(\tau_1, \tau_2) = a_2 \cdot g_1(\tau_1) \cdot g_1(\tau_2) \quad (6)$$

$$g_3(\tau_1, \tau_2, \tau_3) = a_3 \cdot g_1(\tau_1) \cdot g_1(\tau_2) \cdot g_1(\tau_3) \quad (7)$$

When we know the impulse response of linear system, we can write dynamic characteristics of nonlinear system by use of coefficients a_2, a_3, instead of getting Volterra kernels. But, in case of the nonlinear system whose characteristics is dependent on amplitude of the input (for example: saturation type),we are not able to use constants a_2,a_3. In this case, the crosscorrelation function between the input $u(t)$ (whose amplitude is $\pm\alpha$) and the output can be written as

$$\begin{aligned} &\phi_{uy}(\tau, \alpha) \\ \simeq\ & \Delta t\{g_1(\tau) + (\Delta t)^2 F(\tau)\} \\ &+ 2(\Delta t)^2 a_2(\alpha) \sum_{j=1}^{m_2} g_1(\tau - k_{21}^{(j)}\Delta t) \cdot g_1(\tau - k_{22}^{(j)}\Delta t) \\ &+ 6(\Delta t)^3 a_3(\alpha) \sum_{j=1}^{m_3} g_1(\tau - k_{31}^{(j)}\Delta t) \\ &\cdot g_1(\tau - k_{32}^{(j)}\Delta t) \cdot g_1(\tau - k_{33}^{(j)}) \quad (8) \end{aligned}$$

Where $F(\tau)$ is described as in the next equation.

$$F(\tau) = 3\sum_{i=0}^{m} g_3(\tau, i, i) - 2g_3(\tau, \tau, \tau) \quad (9)$$

If a crosssection of 2nd Volterra kernel $g_2(\tau_1, \tau_2)$ appear in the crosscorrelation function at τ_k, we have

$$\phi_{uy}(\tau_k) = g_2(\tau_1, \tau_2) \quad (10)$$

Then,coefficient $a_2(\alpha)$ can be calculated by use of next equation just by using three points of the crosscorrelation function $\phi_{uy}(\tau_1, \alpha)$,$\phi_{uy}(\tau_2, \alpha)$,$\phi_{uy}(\tau_k, \alpha)$ .That is,

$$a_2(\alpha) = \frac{\phi_{uy}(\tau_k, \alpha)}{\phi_{uy}(\tau_1, \alpha) \cdot \phi_{uy}(\tau_2, \alpha)} \quad (11)$$

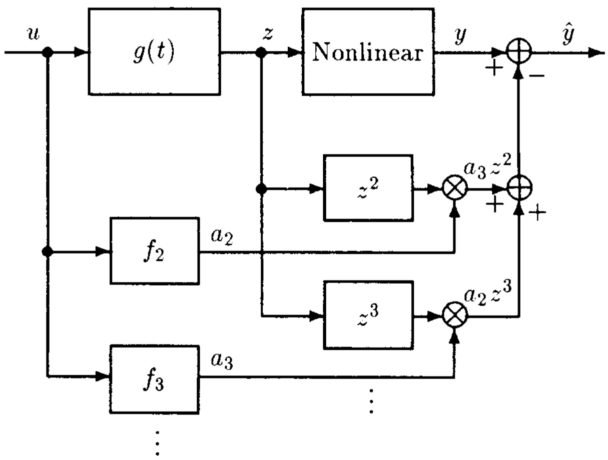

Figure 2: Linearization of nonlinear system

Similarly, $a_3(\alpha)$ is

$$a_3(\alpha) = \frac{\phi_{uy}(\tau_l, \alpha)}{\phi_{uy}(\tau_1, \alpha) \cdot \phi_{uy}(\tau_2, \alpha) \cdot \phi_{uy}(\tau_3, \alpha)} \quad (12)$$

The function $f_i = a_i(\alpha)$ is called here as polynomial compensation function.

4. A Method for linearization

In Eq.(1),the first term is considered as output of linear part of the nonlinear system.

$$\int_0^\infty g_1(\tau_1)u(t - \tau_1)d\tau_1$$

Therefore the linearized output can be obtained by subtracting the effect of nonlinearity from the system output. Then, linearized output $\hat{y}(t)$ can be written as in the next equation.

$$\hat{y}(t) = y(t) - a_2(\alpha) \cdot z^2(t) - a_3(\alpha) \cdot z^3(t) \quad (13)$$

Fig.2 is the block diagram showing the linearization method by use of polynomial compensation function. Let us assume the nonlinear system consists of a linear part followed by a nonlinear element(Wiener type nonlinear system), and the nonlinear system can be expanded up to 3rd order Volterra kernels. Input $u(t)$ is fed to polynomial compensation function f_2 and f_3, and produces a_2, a_3. Nonlinear output is subtracted by the effect of 2nd and 3rd compensation ,and the linearized output $\hat{y}$ is obtained.

5. Simulation

We have applied this method of obtaining polynomial compensation function of nonlinear system and it's linearization to nonlinear saturation system which is shown in Fig.3. This system has saturation nonlinearity put in cascade with a linear element $g(t)$. The saturation element is those element in which for $|z| \geq 0.5$, $|y| = 0.5$. Fig.4, Fig.5 show the crosscorrelation function of this system, when $\alpha = 0.4$, $\alpha = 0.8$, respectively.

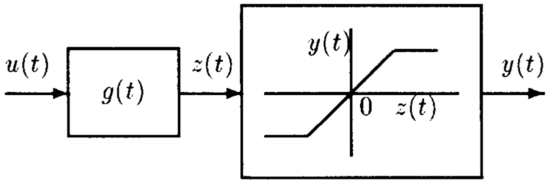

Figure 3: Nonlinear system having saturation element

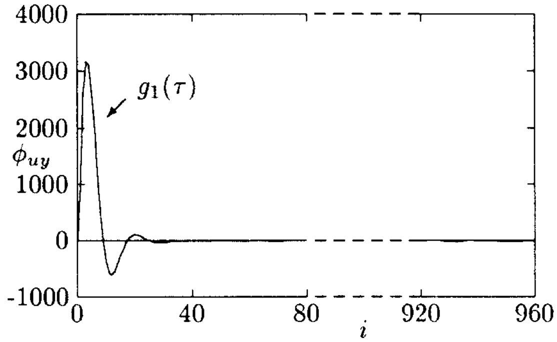

Figure 4: Crosscorrelation function of saturation system when $\alpha = 0.4$

We found the 3rd order Volterra kernel at $i \simeq 935$ in Fig.5. This nonlinearity is odd function,so 2nd,4th,... order Volterra kernels did not appear in these figures. $a_3(\alpha)$ is calculated by use of Eq.(12) from these crosscorrelation functions shown in Fig.6. $a_3(\alpha)$ is approximated by the next equation, and we take this equation as polynamial compensation function.

$$a_3(\alpha) = \begin{cases} -\frac{0.54}{0.8}u - 0.27 & (u < -0.4) \\ 0 & (-0.4 \leq u \leq 0.4) \\ -\frac{0.54}{0.8}u + 0.27 & (0.4 < u) \end{cases}$$

We carried out simulation using this compensation function. Fig.7 shows the result of simulation. Here, solid line is the linear output $z(t)$,and $\circ$ is the linearized output $\hat{y}(t)$. Fig.8 shows the case where we just use constant a_3 for $\alpha = 0.8$. From this figure, we have a better agreement in case of compensation function than constant a_3. We calculate square sum of the error between $\hat{y}(t)$ and $z(t)$ for evaluating the effect of linearization.

$$\begin{aligned} e &= \sum_{i=1}^{N}(\hat{y}(i\Delta t) - z(i\Delta t))^2 \qquad (14) \\ N &= 2000 \end{aligned}$$

In the case of constant a_3, $e = 2.690$, and in case where we use compensation function, $e = 1.080$, showing good result in case of compensation function.

5.Conclusion

A nonlinear system can be expressed in Volterra series expansion as in Eq.(1). In this case, if those Volterra kernels can be obtained beforehand, and a polynomial compensation function is known, we can linearize the nonlinear system simply by subtracting the effects of high order Volterra kernels from the system output. The authors applied the method of obtaining Volterra kernels by use of M-sequence and the linearization method to saturation-type nonlinear system. The simulation results show that our method of linearization technique of nonlinear system is quite useful in practical cases.

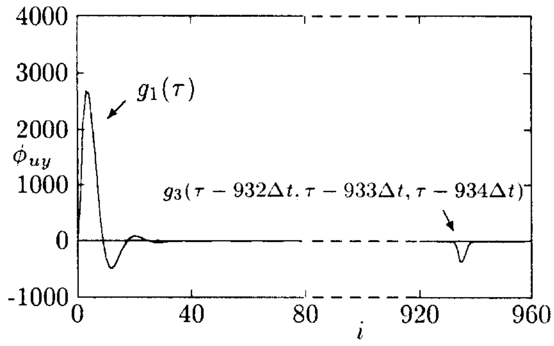

Figure 5: Crosscorrelation function of saturation system when $\alpha = 0.8$

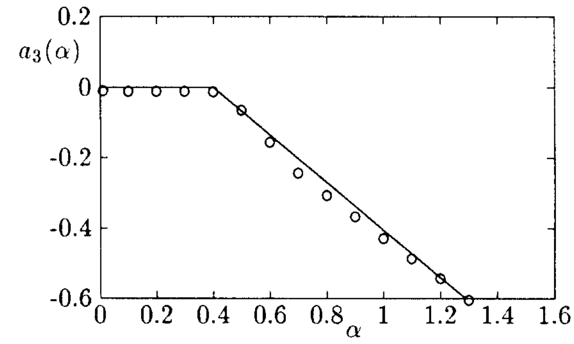

Figure 6: Coefficient $a_3(\alpha)$

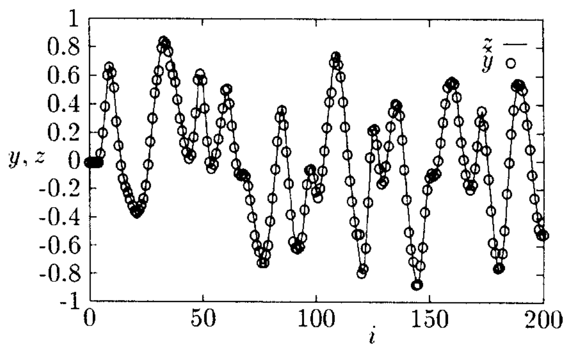

Figure 7: Simulation result of linearizing a saturation system when a_3 is variable

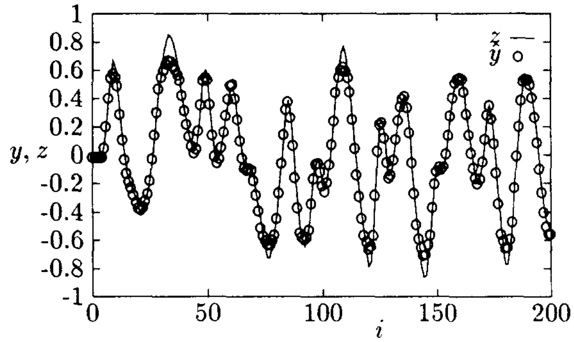

Figure 8: Simulation result of linearizing saturation system when a_3 is constant

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