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Vibrational Control of an Underactuated Mechanical Systems

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Abstract An open loop vibrational control of underactuated mechanical systems with amplitude and frequency modulations is investigated. The underactuated systems considered in the paper are assumed to have free joints with no brake. The active joints are positioned first by a linearizing control, and then periodic oscillatory inputs are applied to them to move the remaining free joints to their desired states. A systematic way of obtaining averaged systems for the underactuated systems with oscillatory vibrations is developed. A complete solution to the open loop control strategy in terms of determining amplitudes and frequencies for general system is still under investigation. However, a specific control design for 2R manipulator which is obtained the averaging system is demonstrated.

Keywords Averaging, Feedback Linearization, Open Loop Control, Underactuated Manipulator, Vibrational Control

1. INTRODUCTION

An underactuated mechanical system refers to the system with less number of actuators than the degrees of freedom that the system possesses. Therefore, a manipulator with free joints becomes naturally an underactuated system. Recently, control of underactuated systems draws great attention in view of reducing the number of actuators and/or sensors, and improving the reliability by a fault-tolerant design of fully-actuated manipulators working in hazardous areas or with dangerous materials. It is particularly important to control the failed joints in the event of the actuator failure in the case of space robots. Referring that an active joint is the one which is fully controlled via an actuator, and that a passive joint is the one which has no actuation but equipped with a brake, and that a free joint is the one even without a brake, the underactuated systems are defined as those with passive and/or free joints.

Vibrational control is a control technique which utilizes high frequency zero mean vibrations to modify the behavior of dynamical systems in a desired manner. The theory for nonlinear systems had been matured in the middle of 1980's by [1], [2]. The theory has also been extended to the parabolic partial differential equations [3], [6] and functional differential equations [4].

The underactuated system with passive joints has been investigated by several researchers. Recently, interesting papers which applied periodic oscillations to control manipulators with free joints have appeared [7], [8]. Note that the active joint variables appearing in the joints dynamics can be considered as varying system parameters. Therefore, the periodic movement of active joints provides a parametric vibrational control to the free joints dynamics. Suzuki et al. [7], [8] investigated an oscillatory control based on Poincare map analysis. De Luca et al. [5] also investigated a constructive open loop control which involves nilpotent approximation and iterative steps.

In this paper a prescribed end point steering problem for underactuated systems with free joints via partial feedback linearization and vibrational control is investigated. The control design consists of two stages. The first stage linearizes the system partially, and applies a feedback control to drive the active joints to their desired locations. At the end of first stage the positions of free joints will be arbitrary. Then periodic inputs to the active joints are applied to move the remaining free joints to their desired positions. Proper magnitudes and frequencies for the oscillatory inputs are determined through the averaging analysis.

The contributions of the paper are the following: Averaging analysis is extended to the system with the derivative of input, and a systematic way of obtaining averaged systems for underactuated systems is developed. The plant considered in the paper allows a drift term and assumes the free joints with no brake. The utilization of both magnitude and frequency of the vibrational inputs is proposed.

2. CONTROL PROBLEM

Consider an n degrees of freedom open loop mechanism with joint variables $q_1, \ldots, q_n$. It is assumed that each joint has a single degree of freedom and only $m < n$ joints are active. The joints which are capable of actuation are called an active joints. And the remaining $l = n - m$ joints with no actuation and no brake are called free joints. It is assumed that both joint variables, either active or free, can be measured.

Using the Lagrange method, one can derive the equations of motion of the system. We can rearrange the equations so that the coordinates for active joints are grouped in $q_1 \in R^m$ and the coordinates for passive/free joints are grouped in $q_2 \in R^l$. Hence the final form of equations of motion for an underactuated mechanical system is represented as

$$M_{11}\ddot{q}_1 + M_{12}\ddot{q}_2 + C_1(q,\dot{q}) + G_1(q) = f \quad (1)$$

$$M_{21}\ddot{q}_1 + M_{22}\ddot{q}_2 + C_2(q,\dot{q}) + G_2(q) = 0 \quad (2)$$

where the vector functions $C_1(q,\dot{q}) \in R^m$ and $C_2(q,\dot{q}) \in R^l$ contain Coriolis and centripetal terms, the vector functions $G_1(q) \in R^m$ and $G_2(q) \in R^l$ contain gravitational terms, $f \in R^m$ represents the input generalized force produced by the m actuators at the active joints. Hence like a fully actuated robot, the dynamic equation for an underactuated system can also be written as

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = Bf \quad (3)$$

where

$$q = [q_1^T, q_2^T]^T,\ M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},\ B = \begin{bmatrix} I_{m\times m} \\ 0_{l\times m} \end{bmatrix},$$

$$C = [C_1^T, C_2^T]^T,\ G = [G_1^T, G_2^T]^T$$

Note that M is a symmetric positive definite matrix. For notational simplicity we will henceforth not write the explicit dependence on q in M, C and G. It is emphasized again that the dynamics of underactuated systems is represented as standard dynamics of n link robots except that there is no control input to the l equations.

2.1 Partial Feedback Linearization

Now consider equation (2). The term M_{22} is an invertible $l\times l$ matrix as a consequence of the uniform positive definiteness of the inertia matrix M in (3). Therefore we may solve for $\ddot{q}_2$ as

$$\ddot{q}_2 = -M_{22}^{-1}(M_{21}\ddot{q}_1 + C_2 + G_2) \quad (4)$$

Substituting (4) into (1) yields

$$\overline{M_{11}}\ddot{q}_1 + \overline{C_1} + \overline{G_1} = f \quad (5)$$

where $\overline{M_{11}} = M_{11} - M_{12}M_{22}^{-1}M_{21}$, $\overline{C_1} = C_1 - M_{12}M_{22}^{-1}C_2$,

$\overline{G_1} = G_1 - M_{12}M_{22}^{-1}G_2$.

A partial feedback linearizing controller can therefore be defined for equation (5) according to

$$f = \overline{M_{11}}u + \overline{C_1} + \overline{G_1} \quad (6)$$

where $u \in R^m$ is an additional control input yet to be defined. Note that the $m\times m$ matrix $\overline{M_{11}}$ is itself symmetric and positive definite. The complete system up to this point may be written as

$$\ddot{q}_1 = u \quad (7)$$

$$M_{22}\ddot{q}_2 + C_2 + G_2 = -M_{21}u \quad (8)$$

Since the input-output relation from u to q_1 in equation (7) is linear, the active part of equation (1) has been completely linearized. However considering the full state vector q, only partial linearization has been obtained.

2.2 Averaging Analysis

The method of averaging is an asymptotic method which permits the analysis of dynamic behavior of a time-varying system via a time-invariant (averaged) system, which is obtained by averaging of the right hand side of the original time-varying system. In this paper once all the active joints reach their desired set points periodic, oscillatory inputs are applied to the active joints in view of moving the remaining free joints to their target positions. Since the input is periodic, each active joint returns back to its original position in each period. The design issue now becomes how to move the free joints to their set positions. Since the system is time varying the determination of input magnitudes and frequencies are calculable through the trajectory analysis of averaged system.

Consider the nonlinear system

$$\dot{x} = X(x, \lambda, \dot{\lambda}) \quad (9)$$

where $X: R^n \times R^m \times R^m \rightarrow R^n$; $x \in R^n$ is the state; $\lambda \in R^m$ are the system parameters in which vibrations are to be introduced; and $\dot{\lambda} = d\lambda/dt$.

Introduce an oscillatory input into (9) as follows

$$\lambda(t) \rightarrow \lambda_0 + \gamma(t),\quad \gamma(t) = \alpha f(\omega t) \quad (10)$$

where λ_0 and α are constants, and $\gamma(t)$ is a periodic vector function with the average value equal to zero. Then (9) becomes

$$\dot{x} = X(x, \lambda_0 + \gamma(t), \dot{\gamma}(t)) \quad (11)$$

It is assumed that (10) can be decomposed as

$$\dot{x} = X_0(x, \gamma(t)) + \omega X_1(x, v(t)) \quad (12)$$

where $X_0(x,\gamma(t)) = X_0(x, \lambda_0 + \gamma(t))$, and $v(t) = df(t)/dt$.

(12) is now transformed into the standard form of $\dot{x} = \varepsilon f(x,t)$, so that the time averaging of the right hand side can be executed. To make this transformation rigorously consider the following generating equation which is motivated from the second term in equation (12).

$$\dot{x} = X_1(x, v(t)) \quad (13)$$

Let $h(t,c): R\times R^n \rightarrow R^n$ be the general solution of (13) which is a T-periodic for some $v(\cdot)$ and $t \geq t_0$. Note that $c \in R^n$ can be uniquely defined once initial conditions $x(t_0) \in \Omega \subset R^n$ are provided.

Introduce a new variable $q(t)$ as in the following transformation (the Lyapunov substitution)

$$x(t) = h(t, q(t)). \quad (14)$$

Then

$$\dot{q}(t) = \left[\frac{\partial h(\omega t, q(t))}{\partial q}\right]^{-1} X_0(h(\omega t, q(t)), \alpha f(\omega t)) \quad (15)$$

is obtained. In slow time scale such that $\tau = \omega t$ with $z(\tau) = q(t)$ and $\varepsilon = 1/\omega$, the following standard form is obtained.

$$\dot{z}(\tau) = \varepsilon\left[\frac{\partial h(\tau, z(\tau))}{\partial z}\right]^{-1} X_0(h(\tau, z(\tau)), \alpha f(\tau)). \quad (16)$$

Finally, an averaged system is defined as

$$\dot{y} = \varepsilon\overline{Y}(y) \quad (17)$$

where

$$\overline{Y}(y) = \lim_{T\to 0}\frac{1}{T}\int_0^T\left[\frac{\partial h(\tau, y)}{\partial y}\right]^{-1} X_0(h(\tau, y), \alpha f(\tau))d\tau$$

By the theory of averaging it is known that there exists $\varepsilon_0 > 0$ such that for all $0 < \varepsilon \leq \varepsilon_0$, the hyperbolic stability properties of (16) and (17) are the same.

3. APPLICATIONS

In this section, one example of underactuated mechanical systems with free joint, a two-link manipulator is shown. Its averaged systems via partial feedback linearization is also demonstrated. It is noted that any linear control theory for the 2R manipulator is not applicable since its linearized system is not controllable.

3.1 A Planar 2R Manipulator

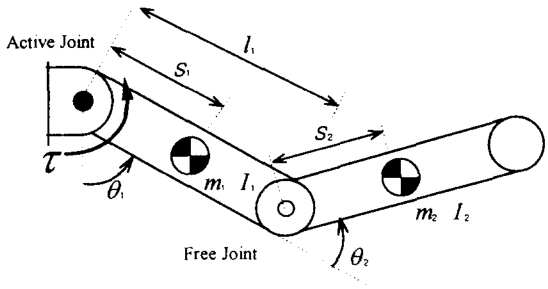

Fig. 1 A planar 2R manipulator

Fig. 1 shows a planar 2R manipulator on the horizontal plane [5], [8]. Using the Lagrange equation, the following equations of motion are obtained.

$$M_{11}(\theta_2)\ddot{\theta}_1 + M_{12}(\theta_2)\ddot{\theta}_2 + C_1(\theta_2, \dot{\theta}_1, \dot{\theta}_2) = \tau$$
$$M_{12}(\theta_2)\ddot{\theta}_1 + M_{22}(\theta_2)\ddot{\theta}_2 + C_2(\theta_2, \dot{\theta}_1, \dot{\theta}_2) = 0$$

where

$$M_{11}(\theta_2) = m_1 s_1^2 + m_2 l_1^2 + m_2 s_2^2 + 2m_2 s_2 l_1 \cos\theta_2 + I_1 + I_2$$
$$M_{12}(\theta_2) = m_2 s_2^2 + m_2 s_2 l_1 \cos\theta_2 + I_2, \quad M_{22} = m_2 s_2^2 + I_2$$
$$C_1(\theta_2, \dot{\theta}_1, \dot{\theta}_2) = -m_2 s_2 l_1 \sin\theta_2 (2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2)$$
$$C_2(\theta_2, \dot{\theta}_1, \dot{\theta}_2) = 2m_2 s_2 l_1 \dot{\theta}_1^2 \sin\theta_2$$

Note that the gravity term does not appear in the equations, and check that the linearized system is not controllable.

Following the procedure in Section Ⅱ, the following partially linearized system is obtained as

$$\ddot{\theta}_1 = u \tag{18a}$$
$$\ddot{\theta}_2 = -(1 + n\cos\theta_2)\ddot{\theta}_1 - n(\dot{\theta}_1)^2 \sin\theta_2 \tag{18b}$$

where $n = m_2 l_1 s_2 / (m_2 s_2^2 + I_2)$ is a constant. Now assume that the active joint θ_1 has been positioned at a desired location with an appropriate control algorithm. For instance,

$$u = \ddot{\theta}_{1d} + k_1(\dot{\theta}_{1d} - \dot{\theta}_1) + k_2(\theta_{1d} - \theta_1)$$

would suffice. Now restricting our control task to the second equation, $\ddot{\theta}_1$ and $\dot{\theta}_1^2$ become varying parameters in the θ_2 dynamics. Let $x_1 = \theta_2$, $x_2 = \dot{\theta}_2$. Then the state equation

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(1 + n\cos x_1)\ddot{\theta}_1 - n\sin x_1(\dot{\theta}_1)^2$$

is obtained. We apply oscillatory periodic inputs as $\dot{\theta}_1 = -\frac{\alpha}{\omega}\cos\omega t$. Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -n\sin x_1(\alpha\sin wt)^2 \end{bmatrix} + w\begin{bmatrix} 0 \\ -\alpha\cos wt(1 + n\cos x_1) \end{bmatrix} \tag{19}$$

is obtained. Note that equation (19) is in the form of equation (12). Therefore the generating equation of (13) takes the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\alpha\cos t(1 + n\cos x_1) \end{bmatrix} \tag{20}$$

The general solution of (20) is

$$h(t, c) = \begin{bmatrix} h_1(t, c) \\ h_2(t, c) \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 - \alpha(1 + n\cos c_1)\sin t \end{bmatrix}.$$

Therefore (15) becomes

$$\frac{dq}{dt} = \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{bmatrix} = \begin{bmatrix} q_2 - \alpha(1 + n\cos q_1)\sin wt \\ n\alpha\sin q_1 \sin wt(-q_2 + n\alpha\cos q_1 \sin wt) \end{bmatrix} \tag{21}$$

In slow time $\tau = wt$ with $q(t) = z(\tau)$, and $\varepsilon = 1/w$, (21) becomes

$$\dot{z}_1(\tau) = \varepsilon[z_2 - \alpha(1 + n\cos z_1)\sin\tau]$$
$$\dot{z}_2(\tau) = \varepsilon n\alpha\sin z_1 \sin\tau(-z_2 + n\alpha\cos z_1 \sin\tau) \tag{22}$$

Finally, by time-averaging the right hand side of (22) the following averaged system is obtained.

$$\dot{y}_1 = \varepsilon y_2$$
$$\dot{y}_2 = \varepsilon\frac{n^2\alpha^2}{4}\sin 2y_1 \tag{23}$$

3.2 Control Design for 2R Manipulator

Fig. 2 shows a vibrationally controlled motion of the second joint in the three dimensional space. Fig. 3 shows the trajectories of transformed system (22) starting at various initial conditions. As it can be compared from Fig.

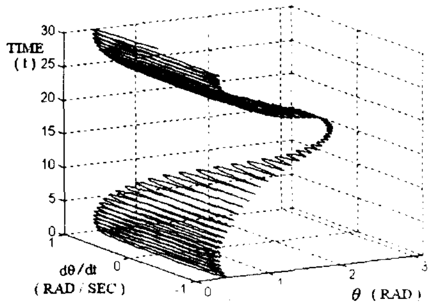

Fig. 2 Trajectories of Original System (19)
($\alpha = 0.5$, I.C.: $(\theta, \dot{\theta}) = (0.5, 0)$)

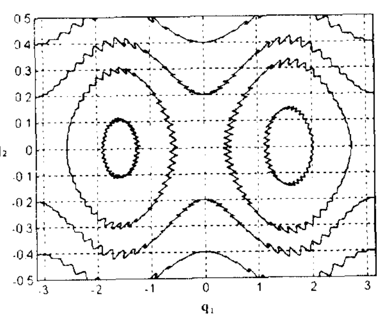

Fig. 3

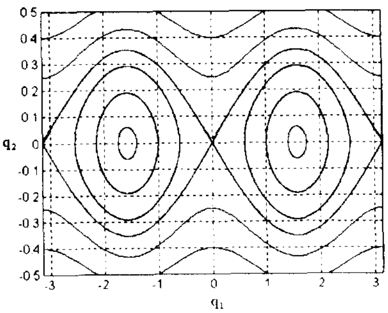

Fig. 4

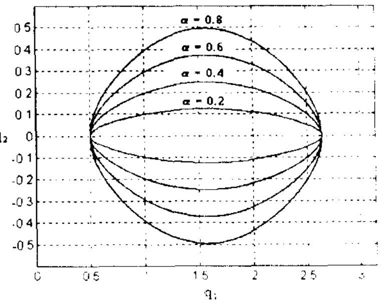

Fig. 5

Fig. 3 Trajectories of Transformed System (21) ($\alpha = 0.5$, $\omega = 4\pi$)

Fig. 4 Phase Portraits of Averaged System (23) ($\alpha = 0.5$, $\omega = 4\pi$)

Fig. 5 Phase Portraits of the Averaged System with Different α's ($\omega = 4\pi$, I.C.: $\theta_2(0) = 0.5$, $\dot{\theta}_2(0) = 0$)

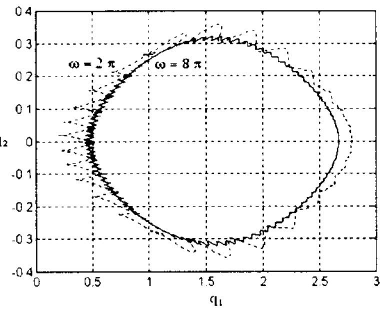

Fig. 6

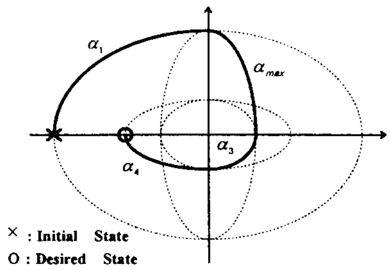

Fig. 7

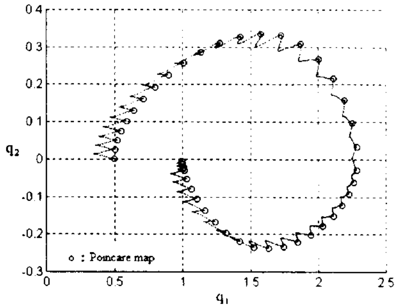

Fig. 8

Fig. 6 Trajectory of the Original System with different ω's. ($\alpha=0.5$)
Fig. 7 Control Strategy
Fig. 8 Simulations $(0.5, 0)\rightarrow(1.0, 0)$ $(\alpha_1=0.5,\ \alpha_2=1,\ \alpha_3=0.5)$

3 and Fig. 4, the behavior of transformed system (22) is well described by that of averaged system (23). Fig. 5 compares the trajectories of the averaged system with different input amplitudes α's. Fig. 6 shows the trajectories of the transformed system with two different input frequencies. With the same amplitude finer oscillations are observed in the case of high input frequency. Now consider a positioning problem in which the desired angle is larger than the intial angle as in Fig. 7. For instance, let the initial state be (0.5, 0), and the desired state be (1.0, 0). The following control strategies are summarized.

In the first departing quarter an arbitrary vibration with frequency ω and amplitude α_1 can be applied. When θ_2 reaches $\pi/2$, the input amplitude in the second quarter is switched from α_1 to α_{max} in order to reduce control time. If θ_2 begins to decrease, which corresponds to the point that the averaged trajectory crosses the horizontal axis, the amplitude is switched again to α_3 which is supposed to be smaller than α_1. Finally, when θ_2 becomes $\pi/2$, we now enter the last cruising quarter to the target position. The amplitude α_4 in the last quarter can be easily calculated from equation (23) as

$$\frac{n^2\alpha^2}{2}\cos^2 y_1 + y_2^2 = \frac{n^2\alpha^2}{2}\cos^2 y_{1d} + y_{2d}^2$$

where (y_{1d}, y_{2d}) are desired state. Note that the amplitude in the last quarter should be calculated in every step as follows.

$$\alpha = \sqrt{\frac{2(y_2^2 - y_{2d}^2)}{n^2(\cos^2 y_{2d} - \cos^2 y_2)}}$$

Finally, it is noted that the first link needs to be stopped at exact period of input in order to keep it at its desired position. It is also noted that once the second link crosses over its target position, there is no return and it has to go all the way around again. Therefore just before getting to the target position the input frequency needs to be increased. A fine step approach is shown in Fig. 8 at the last stage of control.

4. CONCLUSIONS

Open loop vibrational control of underactuated mechanical systems with free joints was investigated. The active joints were first moved to their desired position via partial feedback linearization, and then periodic vibrations were applied to them to move the remaining free joints. A systematic way of obtaining averaged systems via the generating equations was presented.

후기

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