# 가변속 전동기구동을 위한 새로운 반포화 PI 제어기

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## New Anti-windup PI Control for Variable-Speed Motor Drives

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#### Abstract

The windup phenomenon appears and results in the performance degradation when the PI controller output is saturated. A new anti-windup PI controller is proposed to improve the control performance of the variable-speed motor drives and it is experimentally applied to the speed control of a vector-controlled induction motor driven by a pulse width modulated(PWM) voltage source inverter (VSI). The integral state is separately controlled corresponding to whether the PI controller output is saturated or not. The experimental results show that the speed response has the much improved performances such as small overshoot and fast settling time over the conventional anti-windup technique. Although the operating speed command is changed, the similar control performance can be obtained by using the PI gains selected in the linear region.

#### Nomenclature

B: Friction coefficient.

J: Moment of inertia of total system.

 $k_p$ : Proportional gain of PI speed controller.

 $k_T$ : Torque constant.

q : Integral state of PI speed controller.

 $T_L$ : External load torque.

 $\tau_I$ : Integral time constant of PI speed controller.

 $\tau_m$ : Mechanical time constant(= J/B).

*u*: Output of PI speed controller.

 $U_m$ : Limitation of plant input.

v : Plant input, i.e. torque-producing current command.

 $\omega_r$ : Motor speed.

 $\omega_{\star}^{\star}$ : Motor speed command.

#### I. INTRODUCTION

Proportional-integral(PI) control scheme has been widely used for the speed control of the variable-speed motor drives. When a current control scheme is employed in an inner feedback loop for the purpose of fast dynamics and current limitation, the outer speed controller generates a current command for the current controller. This current command is limited to a prescribed maximum value due to the converter protection, the magnetic saturation, and the motor overheating[1]. Therefore, there exists a saturation-type nonlinearity in the speed control loop.

Since the PI speed controller is usually designed in a linear region ignoring the saturation-type nonlinearity, the closed-loop performance will be significantly deteriorated with respect to the expected linear performance. This performance deterioration is referred to as windup phenomenon[2], which causes large overshoot, slow settling time, and sometimes even instability in the speed response[3][4].

To overcome the windup phenomenon, a number of the anti-windup techniques have been proposed in the literature. In the Krikelis intelligent integrator[4], the integral action is limited with the dead-zone nonlinearity whose two parameters are the designer's choices. However, such freedom vanishes when the intelligent integrator is applied to the variable-speed motor drives, so that the undesirable overshoot occurs in the speed response[5]. An anti-windup controller based on the

conditioning technique is proposed to restore the consistency of the controller states in the presence of the nonlinearities by Hanus et al.[6] and its usefulness is compared with other anti-windup controllers through a computer simulation[2]. When the conditioning technique is applied to the variable-speed motor drives, the control performance cannot meet the specifications determined by the PI gains selected in the linear region. This problem may occur because the integral state accumulates the speed error even during the plant input saturation, and it will be experimentally shown in a later section. Furthermore, because the conditioning technique can undergo performance degradation in the presence of both upper and lower restrictive saturation levels, Walgama and et al. have modified this technique by introducing a designer-chosen parameter[7].

Recently, Kothare et al. have presented a general framework for anti-windup design[8]. The design criteria are as follows: 1) the nonlinear closed loop system must be stable; 2) when there is no saturation, the closed-loop performance should meet the specifications for linear design; and 3) when the saturation occurs, the closed-loop performance should degrade gracefully from the linear performance. For an ideal anti-windup PI control, it is desirable that the control performance satisfies the specifications determined by the PI gains in the linear region.

In this paper, a new anti-windup PI speed controller is proposed by feeding back the PI controller output ant the stability conditions are presented. The integral state is separately controlled corresponding to whether the PI controller output is saturated or not. The proposed control scheme is applied to the speed control of a vector-controlled induction motor driven by a PWM-VSI and its usefulness is experimentally verified and compared with the conventional anti-windup technique.

### II. ANTI-WINDUP PI SPEED CONTROL

The current controller is usually designed to have a much faster dynamics than the speed controller. If a fast current control scheme is employed, the current dynamics can be neglected and the variable-speed motor drives can be considered as a first-order system given by

$$\dot{\omega}_r = -\frac{1}{\tau_m} \omega_r + k_t v - T_l \tag{1}$$

where  $k_t = k_T / J$ ,  $T_l = T_L / J$  and v denotes the plant input, namely the torque-producing current command. It is assumed that the plant input v is limited by a saturation-type nonlineraity as

$$v = \begin{cases} u & \text{if } |u| \le U_m \\ U_m \cdot sgn(u) & \text{if } |u| > U_m \end{cases}$$
 (2)

where  $sgn(\cdot)$  denotes a sign function.

The output of PI speed controller u can be written as  $u = k_D e + q$  (3)

where  $e=\omega_r^*-\omega_r$  and q denotes the integral state. The PI controller output u may be saturated if the speed command is given a large step change or a large external torque is loaded. When it happens, the integral state is not consistent with the plant input, which may give rise to the windup phenomenon. Therefore, in order to overcome the windup phenomenon, the integral state is separately controlled corresponding to whether the PI controller output is saturated or not. If the PI controller output is saturated, the integral state is reset to zero with a rate of the integral time constant by negatively feeding back the controller output. Otherwise, the integral state accumulates the speed error and the PI action is activated.

Fig.1 shows the proposed anti-windup PI speed controller and the plant dynamics. The integral state q is given as

$$\dot{q} = \begin{cases} \frac{k_p}{\tau_I} e & \text{if } u = v \\ \frac{k_p}{\tau_I} e - \frac{1}{\tau_I} u & \text{if } u \neq v. \end{cases}$$
 (4)

In the followings, it will be called as a linear region and a saturation region when u=v and  $u\neq v$ , respectively, and it is assumed that the integral time  $\tau_I$  is much faster than the mechanical time constant  $\tau_m$ .

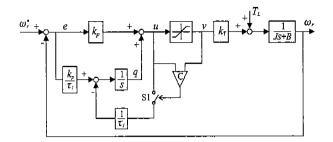


Fig. 1 Block diagram of proposed anti-windup PI speed controller (S1 is open if u = v).

#### III. STABILITY CONDITIONS

The anti-windup PI speed controller in (3) and (4) operates in the saturation or linear region. When the speed command or the external load torque is given a large step change, the speed controller may operate in the saturation region. In this region, the plant input is clamped at a prescribed maximum value and the integral state rapidly converges to zero. When the speed error lies inside of some error bound, the speed controller operates in the linear region and the linear PI action is activated. Therefore, in order to show the stability of the proposed anti-windup PI speed controller, it is sufficient to find the conditions for ensuring both attractiveness to the linear region from the saturation region and asymptotic stability in the linear region.

#### A. Attractiveness condition

For a step command  $\omega_r^*$ , the speed error equation can be written as

$$\dot{e} = -\frac{1}{\tau_m} e - k_t v + \frac{1}{\tau_m} \omega_r^* + T_l.$$
 (5)

In the saturation region, the integral state q converges to zero, from (3) and (4), with a dynamics given by

$$\dot{q} = -\frac{1}{\tau_I} q \ . \tag{6}$$

Since  $\tau_I \ll \tau_m$ , the speed error dynamics is much slower than that of the integral state. Hence, the integral state q can be neglected and the PI controller output u can be written from (3) as

$$u = k_p e. (7)$$

Therefore, there exists a speed error bound  $E_b$  which determines the operating regions of the PI controller and  $E_b$  can be defined as

$$E_b = \frac{U_m}{k_p}. (8)$$

If  $|e| > E_b$ , the PI controller operates in the saturation region. Otherwise, the PI controller operates in the linear region.

In order to obtain the attractiveness condition to the linear region from the saturation region, consider the Lyapunov function given by

$$V(e) = \frac{1}{2}e^2. (9)$$

Then, the time derivative of the Lyapunov function can be written as

$$\dot{V}(e) = e\dot{e}$$

$$= -\frac{1}{\tau_m}e^2 + e\left\{-k_t v + \frac{1}{\tau_m}\omega_r^* + T_l\right\}$$
(10)

If  $k_p$  is a positive gain, substituting (2) and (7) into (10) yields that

$$\dot{V}(e) = -\frac{1}{\tau_m} |e|^2 - k_t U_m sgn(k_p e) e + e \left\{ \frac{1}{\tau_m} \omega_r^* + T_l \right\}$$

$$= -\frac{1}{\tau_m} |e|^2 - k_t U_m |e| + e \left\{ \frac{1}{\tau_m} \omega_r^* + T_l \right\}$$

Hence,

$$|\dot{V}(e)| \le -\frac{1}{\tau_m} |e|^2 + |e|^2 - k_t U_m + \frac{1}{\tau_m} |\omega_r^*| + |T_l|^2.$$
 (11)

From  $\dot{V}(e) \leq 0$ , the maximum error can be written as

$$|e|_{max} = |\omega_r^{\star}| + \tau_m |T_l| - k_t \tau_m U_m. \tag{12}$$

In order that the PI controller may transfer from the saturation region to the linear region, the maximum error should be less than the error bound  $E_b$  in (8). Therefore, the attractiveness condition can be expressed as

$$|\omega_r^{\star}| + \tau_m |T_l| < (\frac{1}{k_p} + k_t \tau_m) U_m \tag{13}$$

and it can be rewritten as

$$B|\omega_r^*| + |T_L| < (k_T + \frac{B}{k_p})U_m.$$
 (14)

If the attractiveness condition in (14) is satisfied, the speed error converges to the inside of the error bound  $E_b$  and the PI controller will operate in the linear region. Fig. 2 shows the range of the speed command and the load torque satisfying the attractiveness condition.

## B. Asymptotic stability condition

In the linear region, the PI action is activated and the integral state accumulates the speed error. From (4) and (5), the error equation in the linear region can be expressed as

$$\dot{e} = -\left\{\frac{1}{\tau_m} + k_p k_t\right\} e - k_t q + \frac{1}{\tau_m} \omega_r^* + T_l.$$
 (15)

To obtain the asymptotic stability condition, consider the Lyapunov function given by

$$V(e,q) = \frac{1}{2} \frac{1}{k_t} e^2 + \frac{1}{2} \frac{\tau_I}{k_p} (q - q_{ss})^2$$
 (16)

where  $k_p$  is a positive gain and  $q_{ss}$  denotes a steady state value of the integral state q. Then, the time derivative can be written, from (2) and (15), as

$$\dot{V}(e,q) = -\frac{1}{k_t} \left( \frac{1}{\tau_m} + k_t k_p \right) e^2 + e \left\{ \frac{1}{k_t} \left( \frac{1}{\tau_m} \omega_r^* + T_l \right) - q_{ss} \right\}$$
(17)

Since the integral state will have a suitable value  $q_{ss}$  given by

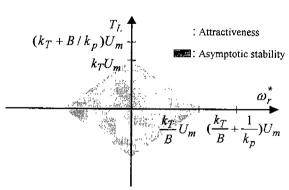


Fig.2 Operating ranges for satisfying attractiveness condition and asymptotic stability.

$$q_{ss} = \frac{1}{k_t} \left( \frac{1}{\tau_m} \omega_r^* + T_l \right), \tag{18}$$

the stability condition such that  $\dot{V}(e,q) \leq 0$  will be satisfied for the unlimited plant input[8]. However,  $q_{ss}$  should be less than  $U_m$  for the limited plant input. The asymptotic stability condition can , therefore, be obtained as

$$\frac{1}{\tau_m} |\omega_r^*| + |T_l| \le k_t U_m \tag{19}$$

and can be rewritten as

$$B|\omega_r^*| + |T_L| \le k_T U_m. \tag{20}$$

If the operating conditions satisfy the inequality in (20), the error dynamics becomes asymptotically stable in the linear region though the PI controller output is saturated. The asymptotic stability range for the speed command and the load torque is shown in Fig. 2.

#### IV. DESIGN GUIDELINE

#### A. PI Gains for linear region

Although the speed command and the load torque satisfy the stability condition in (20), the PI controller may transfer from the linear region to the saturation region unless the PI gains are properly selected. Therefore, a guideline for choosing the PI gains is needed.

For a small step speed command r such that  $|r| \le E_b$ , the closed-loop transfer function can be calculated, in the linear region, as

$$\frac{U(s)}{R(s)} = k_p G(s) \tag{21}$$

where

$$G(s) = \frac{s^2 + (\frac{1}{\tau_m} + \frac{1}{\tau_I})s + \frac{1}{\tau_m \tau_I}}{s^2 + (\frac{1}{\tau_m} + k_t k_p)s + \frac{k_t k_p}{\tau_I}}.$$
 (22)

In (22), R(s) and U(s) denote the Laplace transforms of r and u, respectively. The PI controller output u should be less than the limitation of the plant input  $U_m$  in order that the speed error may remain in the linear

region. Therefore, since  $k_p|r| \le k_p E_b$ , the transfer function G(s) should satisfy from (8) that

$$|G(j\omega)| \le 1, \quad \forall j\omega$$
 (23)

and the ranges of the PI gains can be easily derived as (see Appendix)

$$k_p \ge \frac{1}{\tau_m k_t} \tag{24}$$

$$\frac{1}{\tau_I} \le k_t k_p \left\{ \sqrt{2 \left( 1 + \frac{1}{\tau_m} \frac{1}{k_t k_p} \right)} - 1 \right\}. \tag{25}$$

The inequality in (25) can be approximately written from (24) as

$$\frac{1}{\tau_I} \le \left(\sqrt{2} - 1\right) k_t k_p \,. \tag{26}$$

Therefore, the PI gains for remaining within the linear range can be rewritten as

$$k_p \ge \frac{B}{k_T} \tag{27}$$

$$\tau_{J} \ge \left(\sqrt{2} + 1\right) \frac{J}{k_{T} k_{p}}.$$
 (28)

## B. Design summary

After the PI gains are obtained by using a poleplacement technique or an optimal control theory, etc., the selected PI gains are checked according to the following steps.

#### Step 1

Considering the maximum load torque and the maximum operating speed range in Fig. 2, determine the plant input limitation  $U_m$ .

Step 2

Table 1. Parameters of induction motor.

1[hp], 220[V], 4[pole], 60[Hz], 1730[rpm] 
$$R_s = 1.985[\Omega], \quad R_r = 1.730[\Omega], \quad X_m = 38.43[\Omega]$$

$$X_s = 40.38[\Omega], \quad X_r = 41.35[\Omega], \quad V_{dc} = 306[V]$$

$$J = 2.1 \times 10^{-3} [Kgm^2], \quad B = 0.96 \times 10^{-3} [Kgm^2 / s]$$

Determine  $k_p$  satisfying  $k_p \ge \frac{B}{k_T}$ .

Step 3

Determine 
$$\tau_I$$
 satisfying  $\left(\sqrt{2}+1\right)\frac{J}{k_T k_p} \leq \tau_I << \tau_m$ .

It is noted that the PI control with saturation-type nonlinearity becomes a bang-bang control if a very large  $k_p$  is selected. As  $k_p$  increases, the settling time becomes faster but the linear region becomes narrower, which may cause a chattering phenomenon in the plant input.

#### V. EXPERIMENTAL RESULTS

The proposed anti-windup PI speed control scheme is applied to the speed control of an induction motor driven by a PWM-VSI and it is experimentally verified and compared with the conventional conditioning technique in [2] and [6]. The parameters of an 1[hp] induction motor are listed in Table 1. Fig. 3 shows the block diagram of an experimental system. In the vector control method[9][10], the induction motor is controlled like a separately excited dc motor. The control algorithm is fully implemented in a software with 80196MC CPU which includes an A/D converter, a 3-phase waveform generator and an event processor array for the shaft encoder signal processing. The 3-phase currents are controlled to be settled within 2[msec] by using a synchronous PI regulator[11]. The PWM frequency is 10 [KHz] and the sampling time of speed control loop is 0.5[msec]. The shaft encoder has 300 pulses per revolution.

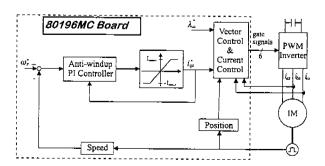


Fig.3 Block diagram of experimental system.

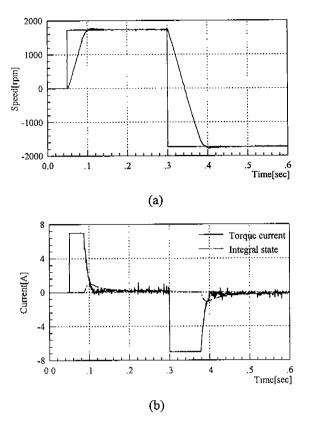


Fig. 4 Experimental responses of proposed anti-windup PI controller when  $k_p = 14.18$ ,  $\tau_I = 31.7[ms]$ .

Fig. 4 and Fig. 5 show the experimental results of the proposed and the conventional anti-windup PI controllers, respectively, when  $\omega_r^* = 1730[\text{rpm}]$  at t = 0.05[sec]and  $\omega_r^* = -1730[\text{rpm}]$  at t = 0.3[sec]. The torqueproducing current command is limited to  $I_{smax} = 7[A]$ and the rotor flux is controlled to be settled within 0.05[sec]. In the conventional scheme, the integral state becomes large at the start of the linear region because it accumulates the speed error even in the saturation region. This superfluous integral state results in a large overshoot and slow settling time in the speed response as shown in Fig. 5. In the proposed scheme, the integral state is reset to zero with a rate of the integral time constant during the saturation and the linear PI action is activated only in the linear region. Therefore, the speed control performance is much improved by the proposed control scheme as shown in Fig. 4.

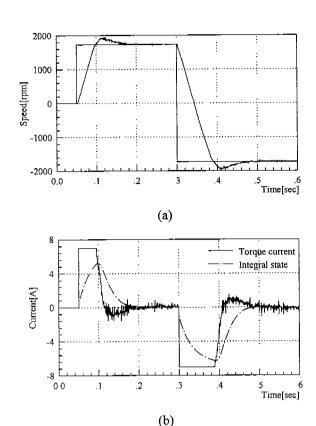


Fig. 5 Experimental responses of conventional antiwindup PI controller when  $k_p = 14.18$ ,

 $\tau_I = 31.7 [ms]$ .

Fig.6 shows the experimental comparisons of the speed responses corresponding to various operating speed commands when  $k_p = 16.6$  and  $\tau_I = 22 [\text{ms}]$ . In the conventional scheme, the speed control performances such as percent overshoot and settling time are largely changed due to the varying speed commands since the integral action starts with a different initial state in the linear region. On the other hand, the proposed control scheme shows the similar speed responses for the different speed commands because the integral action is activated only in the linear region. Fig. 7 shows the speed responses corresponding to several PI gains when the step speed command is 1730[rpm]. As the PI gains increase, the speed response becomes faster and has smaller overshoot.

As a result, the proposed anti-windup PI speed control scheme shows the much improved performances such as small overshoot and fast settling time over the conventional anti-windup conditioning technique.

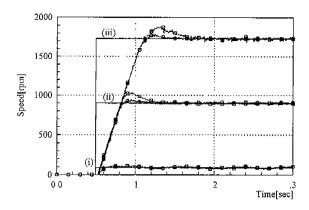


Fig. 6 Comparative experimental results corresponding to various operating speeds; (i)  $\omega_r^* = 90$  [rpm],

(ii)  $\omega_r^* = 900 [\text{rpm}]$ , (iii)  $\omega_r^* = 1730 [\text{rpm}]$ .

Although the plant input is saturated and the speed command is changed, the similar speed responses can also be obtained by using the PI gains selected in the linear region.

#### VI. CONCLUSIONS

A new anti-windup PI control scheme for the variable-speed motor drives has been proposed in order to overcome the windup phenomenon. The stability conditions and the design guideline for choosing the PI gains have been also presented. The integral state is separately controlled corresponding to whether the PI controller output is saturated or not. The proposed control scheme has been applied to the speed control of a vector-controlled induction motor driven by a PWM-VSI and its usefulness has been experimentally verified.

The experimental results show that the proposed antiwindup PI control has the much improved performances such as small overshoot and fast settling time over the conventional anti-windup technique. Although the operating speed command is changed, the similar control performances can also be obtained by using the PI gains which are properly selected in the linear region ignoring the plant input saturation.

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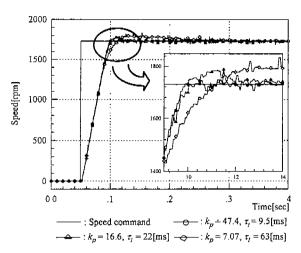


Fig. 7 Step speed responses of proposed control scheme.

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## **Appendix**

Consider the second-order transfer function given as

$$\frac{U(s)}{R(s)} = G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$
 (A1)

where  $a_i$  and  $b_i$  are real coefficients. Then, for  $|U(j\omega)| \le |R(j\omega)|$ ,  $\forall j\omega$ , the transfer function should satisfy that  $|G(j\omega)| \le 1$ ,  $\forall j\omega$ . Therefore,

$$|G(j\omega)|^2 = \frac{(b_0 - b_2\omega^2)^2 + b_1\omega^2}{(a_0 - a_2\omega^2)^2 + a_1\omega^2} \le 1, \quad \forall j\omega.$$
 (A2)

This inequality yields that

$$|a_2| \ge |b_2|, |a_0| \ge |b_0|, a_1^2 - b_1^2 - 2(a_0a_2 - b_0b_2) \ge 0$$
. (A3)