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완전 선형화 가능한 미지구조를 가지는 비선형 시스템

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Completely Feedback Linearizable Families for Uncertain Nonlinear Sy

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Abstract - In this paper, we characterize the whole class of vector fields that can be linearized by a given nominal state transformation and a feedback linearizing controller. The necessary and sufficient condition for a given uncertain vector field to be so-called "completely linearizable by the nominal coordinate transformation" is given in terms of Lie Bracket of uncertain vector fields and some suitable vector fields of the nominal system.

1. Introduction

We consider the feedback linearizing co single input nonlinear systems that uncertainty. The principal limitation of linearization is that the system dynamics s known exactly for the nonlinearities successfully cancelled.

The main objective of this paper is to st necessary and sufficient conditions for nonlinear systems which can be transform the linear controllable system with u parameters. If a nonlinear system with uncertainty is transformed into a linear sy unknown parameters, then we apply robust techniques such as Kharitonov-based desi for linear systems to stabilize the uncertain system. Thus, using an only regular state-controller, the system can be stabilized in uncertainty.

2. Main Results

Consider a dynamical system:

$$\Sigma_p \ : \ \dot{x} = f_p(x,p) + g(x)u \ , \ x(0) = x_0 \, , \quad (1)$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}$, $p \in B_r^{p_0} \subset \mathbf{R}^m$, f_p, g are assu to be smooth vector fields on the smooth $\Omega \subset \mathbf{R}^n$.

Let the nominal nonlinear plant modelled b of the form:

$$\Sigma_{p_0} : \ \dot{x} = f_0(x,p_0) + g(x)u \quad (2)$$

where f_0 , g are assumed to be known s vector fields on the smooth manifold $\Omega \subset \mathbf{R}^n$ p_0 is a known nominal constant vector.

If the modelling error is defined as

$$\Delta f(x,p,p_0) = f_p(x,p) - f_0(x,p_0) \, , \quad (3)$$

the system (1) can be rewritten in terms nominal vector field f_0 and the modelling er is,

$$\dot{x} = f_0(x,p_0) + \Delta f(x,p,p_0) + g(x)u . \quad (4)$$

For notational simplicity, we use $f_0(x) \triangleq f(x,p_0)$ $f_p(x) \triangleq f(x,p)$.

Assumption 1

The nominal system (2) is input-state lin ([9]) in Ω. □

By Assumption 1, there exists a n coordinates transformation $\Phi^{p_0}: \ \Omega \to V \subset R^n$ form:

$$z = \Phi^{p_0}(x) = [\,T,\ L_{f_0}T \ldots,\ L_{f_0}^{n-1}T\,]^T, \quad (5)$$

where $T(x)$ is a smooth scalar function satisfies $\langle dT, ad_{f_0}^k g \rangle = 0, k = 0..n-2$ a $\langle dT, ad_{f_0}^{n-1} g \rangle \neq 0$, $\forall x \in \Omega$.

Since $\Phi^{p_0}: \ \Omega \to V \subset R^n$ is a diffeomorphism, define a tangent mapping $\Phi_*^{p_0}: T\Omega \to TV$ as foll

$$\Phi_*^{p_0}(x) = [\, dT(x) \ dL_{f_0}T(x) \ \cdot \ \cdot \ dL_{f_0}^{n-1}T(x)\,]^T . \quad (6)$$

A nominal feedback linearizing controller obtained as follows :

$$u = \frac{-L^n_{\bar f_n} T}{L_g L^{n-1}_{f_n} T} + \frac{1}{L_g L^{n-1}_{f_n} T} v \triangleq \alpha(x) + \beta(x) v. \quad (7)$$

with $\beta(x) \neq 0$, $\forall x \in \Omega$.

Hence, if we denote $\bar f_0(x) \triangleq f_0(x) + \alpha(x) g(x)$ $\bar g(x) \triangleq \beta(x) g(x)$, the feedback transformed s can be obtained as follows:

$$\bar\Sigma_{p_n}: \dot x = \bar f_0(x) + \bar g(x) v. \quad (8)$$

Using $z = \Phi^{p_n}(x)$, the closed-loop system (transformed into a linear controllable sy so-called Brunovsky controller form:

$$\dot z = Az + bu, \quad (9)$$

with

$$(\Phi^{p_n}_*(x)\ \bar f_0(x))(z) = Az,\ (\Phi^{p_n}_*(x) \bar g(x))(z) = b. \quad (10)$$

where (A, b) is Brunovsky normal pair.

Since for a diffeomorphism Φ^{p_n}: $\Omega \to V \subset R^n$ arbitrary smooth vector fields f_1, f_2 on Ω identity $\Phi^{p_n}_*[f_1, f_2](z) = [\Phi^{p_n}_* f_1, \Phi^{p_n}_* f_2](z)$ holds ([7]) have

$$(\Phi^{p_n}_* ad_{\bar f_0} \bar g)(z) = (\Phi^{p_n}_* [\bar f_0, \bar g])(z) = [Az, b] = -Ab \quad (11)$$

and similarly for $k \in \underline{n}$

$$(\Phi^{p_n}_* ad^k_{\bar f_0} \bar g)(z) = [\Phi^{p_n}_* \bar f_0, \Phi^{p_n}_* ad^{k-1}_{\bar f_0} \bar g](z)$$

$$= [Az, (-1)^{k-1} A^{k-1} b] = (-1)^k A^k b. \quad (12)$$

Now, we define a set Λ of the vector fiel as follows:

$$\Lambda \triangleq \left\{ X \mid X = \sum_{k=1}^{n} a_k ad^{k-1}_{\bar f_0} \bar g(x),\ a_k \in \mathrm{R} \right\},\ x \in \Omega. \quad (13)$$

If we choose an arbitrary vector field X i satisfies the following relation:

$$(\Phi^{p_n}_* X)(z) = (-1)^{n-1} a_n A^{n-1} b + \cdot + (-1) a_2 A b + a_1 b$$

$$= \sum_{k=1}^{n} (-1)^{k-1} a_{n-k+1} \frac{\partial}{\partial z_k},\ \forall x \in \Omega,\ a_i \in \mathrm{R}, i \in \underline{n}, \quad (14)$$

where $\partial/\partial z_1, \cdot\cdot, \partial/\partial z_n$ are the coordinates v fields associated with the z-coordinates.

From (14), the representation of a vec $ad^{k-1}_{\bar f_0} \bar g(x)$, $k \in \underline{n}$ in the coordinates of (5) ca easily obtained as follows:

$$(ad^{k-1}_{\bar f_0} \bar g)(z) = (-1)^{k-1} \frac{\partial}{\partial z_{n-k+1}}. \quad (15)$$

Definition 2

A smooth vector field on a manifold Ω is c completely linearizable vector field by a c transformation $z = \Phi^{p_n}(x)$, if there exists a matrix M in $\mathrm{R}^{n \times n}$ and constant vector Θ in such that

$$(\Phi^{p_n}_*(x)\ \hat f(x))(z) = Mz + \Theta. \quad (16)$$

Lemma 3

Let $\hat f(x)$ be a smooth vector field on Ω. Th vector field $\hat f$ is a completely linearizable vector field by a coordinates transformation $z = \Phi^{p_n}(x)$ if and only if

$$[\hat f, ad^{k-1}_{\bar f_0} \bar g] \in \Lambda,\ k \in \underline{n}. \quad (17)$$

Theorem 4

Suppose that the nonlinear system (1) Assumption 1. Then, by the nominal co transformation (5) and the nominal linearizing control (7), the uncertain syste be transformed into a linear system with parameters if and only if

$$[\Delta f, ad^{k-1}_{\bar f_0} \bar g] \in \Lambda,\ k \in \underline{n}. \quad (18)$$

□

Proof) Sufficiency:

Substituting (7) into (4) we obtain

$$\dot x = \bar f_0(x) + \Delta f(x, p, p_0) + \bar g(x) v, \quad (19)$$

Since, by lemma 3, the modeling error Δf completely linearizable vector field by the coordinates transformation (5), we have

$$\Phi^{p_n}_*(\bar f_0 + \Delta f)(z) = Az + M(p) z + \Theta(p),$$

$$\Phi^{p_n}_*(\bar g)(z) = b, \quad (20)$$

where $M(p) = \begin{pmatrix} m_{11}(p) & \cdot & m_{1n}(p) \\ \cdot & \cdot & \cdot \\ m_{n1}(p) & \cdot & m_{nn}(p) \end{pmatrix}$, $\Theta(p) = \begin{pmatrix} m_{01}(p) \\ \cdot \\ m_{0n}(p) \end{pmatrix}$,

$m_{ij}(\cdot)$: $\mathrm{R}^p \to \mathrm{R}$ ($i = 0, 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$).

Therefore, by the nominal coordinates trans (5) and the nominal feedback linearizing co the system (1) can be transformed into system:

$$\dot z = (A + M(p)) z + bv + \Theta(p) \quad (21)$$

Necessity:

Suppose the nominal coordinates transf $z = \Phi^{p_n}(x)$ transforms the nonlinear system ((34), if follows from Lemma 3 that the re is satisfied . □□□

In order to design control v which stabili linear system with uncertain parameters, th (32) should be stabilizable for all $p \in B^{p_n}_r$ following theorem gives a controllability con the linear uncertain system (34).

Theorem 5

Suppose that the nonlinear system (1) sat conditions of Theorem 4. Then the linear with uncertain parameters (35) is controlla

$p\in B_r^{p_0}$ *if and only if the distribution defined*

$$D_n^{f_p,g}=span\{g\ \ ad_fg\ \cdots ad_{f_p}^{n-1}g\},\qquad(22)$$

satisfies $\dim D_n^{f_p,g}=n$ *for all* $p\in B_r^{p_0}$ *and* $x\in\Omega$

□

To prove the above theorem, we need the f lemma.

Lemma 6

Suppose a distribution $D_k^{\bar f_p,\bar g}$ *where* $\bar f_p=f_0+\alpha g+\Delta f$ $\bar g=\beta g$ *is defined as*

$$D_k^{\bar f_p,\bar g}=span\{\bar g\ \ ad_{\bar f}\bar g\ \cdots ad_{\bar f_p}^{k-1}\bar g\},\ k\in\underline{n}.\qquad(23)$$

Then for each $p\in B_r^{p_0}$ *and* $x\in\Omega$*, if the distributions* $D_i^{\bar f_p,\bar g}$*(* $i=1,2,\ldots,k$ *) are all involutive, then*

$$D_k^{f_p,g}=D_k^{\bar f_p,\bar g},\ k\in\underline{n},\qquad(24)$$

at each $x\in\Omega$.

□

(Proof of Theorem 5)

Sufficiency: For the linear system with parameters (34), the distribution $D_1,\ldots,D_n$ a are the following flat distributions, defined

$$D_k^{A_m,b}=Im\{b|\ A_mb|\cdots|A_m^{k-1}b\}\subset R^n,\quad k\in\underline{n}\qquad(25)$$

where the superscripts A_m,b indicate dependence on the vector f $A_mz=(A+M(p))z$ and b.

Clearly, the distributions $D_k^{A_m,b}$, $k\in\underline{n}$, a involutive. Since it follows from (36) and th $(\Phi_*^{p_0}ad_{\bar f_p}^k\bar g)(z)=(-1)^kA_m^kb$ where $\bar f_p=\bar f_0+\Delta f$ that

$$D_k^{\bar f_p,\bar g}=(\Phi_*^{p_0})^{-1}D_k^{A_m,b}(\Phi^{p_0}(x))\qquad(26)$$

where $(\Phi_*^{p_0})^{-1}D_k^{A_m,b}(\Phi^{p_0}(x))=\{X(x)\mid\Phi_*^{p_0}X(x)\in D_k^{A_m,b}\}$, we conclude that all the distributions $D_k^{\bar f_p,\bar g}$ are also involutive. By Lemma 6, it follo involutiveness of distributions $D_k^{\bar f_p,\bar g}$, $k\in\underline{n}$ $D_n^{\bar f_p,\bar g}=D_n^{f_p,g}$. Therefore, we have

$$\dim D^{f_p,g}=\dim D_n^{A_m,B}=\dim Im\{b|\cdots|A_m^{n-1}b\}=n.\qquad(27)$$

Necessity: Assume that the linear system controllable. Then we have

$$\dim D_n^{A_m,b}=\dim Im\{b|\ A_mb|\cdots|A_m^{n-1}b\}=n,\ k\in\underline{n}\qquad(28)$$

However, the distributions $D_k^{A_m,b}$, $k\in\underline{n}$ equivalently involutive in Ω. Thus by Lemm have

$$\dim D_n^{A_m,B}=\dim D^{\bar f_p,\bar g}=\dim D^{f_p,g}=n.\qquad(29)$$

□□□

To stabilize the linearized system (34), we a new input v of the form

$$v=-k_1z_1-k_2z_2\cdots-k_nz_n\ k_i\in\mathrm{R},\ i\in\underline{n}.\qquad(30)$$

Thus, the complete closed-loop system of obtained as follows:

$$\begin{pmatrix}\dot z_1\\ \dot z_2\\ \vdots\\ \dot z_n\end{pmatrix}=\begin{pmatrix}m_{11} & \cdots & m_{1n}\\ m_{21} & \cdots & \cdot\\ \cdot & \cdots & \cdot\\ \cdot & \cdot & 1+m_{n-1n}\\ -k_1+m_{n1} & \cdot & -k_n+m_{nn}\end{pmatrix}\begin{pmatrix}z_1\\ z_2\\ \vdots\\ z_n\end{pmatrix}+\begin{pmatrix}m_{01}\\ m_{02}\\ \cdot\\ \cdot\\ m_{0n}\end{pmatrix}\qquad(31)$$

Now, we consider a design technique to s the system (44). We want to obtain cond all characteristic roots of the system to b in the left half plane despite the vari parameters $m_{ij},\ i=0,1..,n,\ j\in\underline{n}$. If we can ch the feedback gain $k_1,k_2,\cdots,k_n$ which forc family of characteristic polynomials of (4 Hurwitz, then the closed-loop system (44) i stabilized. The study of polynomial rob results has been active in recent years number of useful results have been obta Kharitonov's Theorem is a powerful resu enables the robust stability of an interval p to be determined by checking only four polynomials.

3. Conclusion

Necessary and sufficient conditions for t class of uncertain nonlinear systems that linearized by the given nominal state trans and the feedback linearizing controller h discussed. The proposed condition is differ previous matching conditions. If a u nonlinear system satisfies the proposed c the system can be transformed into a linea with uncertain parameters and thus be stabilized by a regular static feedback contr

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