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파라미터 불확실성을 갖는 비최소위상 비선형 시스템의 강인 안정화 제어

°주진만[*], 최윤호[**], 주영훈[***], 박진배[*]
연세대학교 전기공학과[*], 경기대학교 전자공학과[**], 군산대학교 제어계측공학과[***]

Robust Stabilization of Nonminimum Phase Nonlinear Systems with Parametric Uncertainty

Jin Man Joo[*], Yoon Ho Choi[**], Young Hoon Joo[***], Jin Bae Park[*]
Yonsei University[*], Kyonggi University[**], Kunsan University[***]

Abstract - A control synthesis scheme is presented for nonlinear single-input-single-output (SISO) systems with parametric uncertainty which have completely unstable zero dynamics. The approach involves the derivation of an input-output linearizing control law which achieves internal stability for a nonlinear minimum phase approximation to the original system using Fliess normal form. A vector of unknown constant parameters is also considered. A Lyapunov-based additional control law is shown to stabilize the full system.

1. Introduction

In this paper, we consider the robust stabilization of nonminimum phase nonlinear systems with parametric uncertainty. Since the nonlinear systems must have stable zero dynamics for the input-output (IO) linearized system to be stable, we first derive a local nonlinear outer factor for the original nonlinear system by coordinate transformation and time reversal of unstable manifolds. And a dynamic IO linearizing control law renders the original nonlinear system into linear system with uncertain parameter vector. Next, we propose Lyapunov-based additional control law for the IO linearized system to guarantee asymptotic stability of the system.

2. Approximate IO linearization

2.1 Systems and definition

We consider the following SISO nonlinear dynamical systems which are affine in the control variable

$$\dot{x} = f(x,a) + g(x,a)\,u$$
$$y = h(x) \qquad (1)$$

where it is assumed that $f(x,a)$ and $g(x,a)$ are C^∞ vector fields defined on a dense submanifold $M \subset R^n$. It is also assumed that $h(x)$ is C^∞ on M. The vector a is the unknown parameter vector. It is assumed that f and g are smooth vector fields for every $a \in B_a \subset R^p$ where B_a is an admissible set of unknown parameter vectors. The nominal parameter vector a_n is assumed known and the perturbation about a_n are represented as

$$a = a_n + \delta a. \qquad (2)$$

Define a local nonlinear outer factor as follows [2]:

Definition 1 (Local Nonlinear Outer Factor): A local nonlinear outer factor (P_M) of a nonlinear system (P) exhibits the following properties at each equilibrium point along the steady state operating locus:

1) The poles of the Jacobian linearization of P_M coincides exactly with the poles of the Jacobian linearization of P.

2) The Jacobian linearization of P_M has zeros at the location of the left-half plane zeros of the Jacobian linearization of P.

3) The Jacobian linearization of P_M has stable zeros corresponding to a reflection across the imaginary axis of the right-half plane zeros of the Jacobian linearization of P.

4) The static (nonlinear) gain of P_M is equal to the static (nonlinear) gain of P.

5) The unforced dynamics of P_M are exactly equal to the unforced dynamics of P.

2.2 Basic assumptions

We need the following assumptions.

Assumption 1: For a given system (1), the nominal system

$$\dot{x} = f(x,a_n) + g(x,a_n)\,u$$
$$y = h(x) \qquad (3)$$

has the well-defined relative degree $r < n$.

Assumption 2: The unforced zero dynamics of (3) is completely unstable, i.e., Jacobian linearization of the zero dynamics has all its roots in the right-half plane.

Assumption 3: The parameter vector α appears linearly in (1).

2.3 Local nonlinear outer factor

Assumption 1 implies that for the system (3) there exists a local diffeomorphism T such that the system can be transformed into the Byrnes-Isidori normal form [5]. However, the Byrnes-Isidori normal form is not useful in the derivation of a local nonlinear outer factor because we cannot retain the pole dynamics in the system but merely alter the zero dynamics.

A more useful structure for the derivation of a local nonlinear outer factor is the Fliess controller canonical form [1] or observability canonical form [3]. This normal form has a property that the pole dynamics and the zero dynamics manifest themselves separately in the Jacobian approximation of the nonlinear system. As with the Byrnes-Isidori normal form, successive differentiations of the output are considered; however, in this case, the differentiations continue past the rth term (relative degree) until the full nth derivative. The resulting coordinate transformations are a function of both the original state and the control input u and its derivatives, i.e.,

$$z = T(x, u, \dot{u}, \ddot{u}, \ldots, u^{(n-r)}). \tag{4}$$

Define $\hat{u} = [\, u, \dot{u}, \ddot{u}, \ldots, u^{(n-r)}\,]^T$ and consider the dependence of the nominal parameter vector on the coordinate transformation function, then (4) can be rewritten as

$$z = T(x, \alpha_n, \hat{u}). \tag{5}$$

Via the coordinate transformation function (5), the nominal system (3) can be written

$$\dot{z} = \frac{\partial T}{\partial x}\{f(x,\alpha_n) + g(x,\alpha_n)u\} + \frac{\partial T}{\partial \hat{u}}\dot{\hat{u}}$$

and apply $x = T^{-1}(z, \alpha_n, \hat{u})$. Or

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= F(z, \alpha_n, \hat{u}) \\ y &= z_1. \end{aligned} \tag{6}$$

Since the system (3) has completely unstable zero dynamics, the transfer function of the Jacobian linearization of (6) has $(n-r)$ unstable zeros.

In the linear case, the unstable zeros of the original system are related to the corresponding zeros of the outer factor by a reflection across the imaginary axis in the frequency domain. For nonlinear systems, this concept is extended in a straightforward manner through a reflection in the time domain. From a linear perspective, one can observe that the spans of the set of stable and unstable eigenvectors of the Jacobian linearization can be reflected by time reversal. Thus the reflection of the unstable zeros of P across imaginary axis to yield the zeros of P_M can be viewed as a symmetry in time.

A local nonlinear outer factor, P_M, of a nonminimum phase system, P, can be constructed from the original system by time reversal of the unforced zero dynamics. This is accomplished for a system in the form of (6) by substituting the value $(-1)^i u^{(i)}$ for each occurrence of the derivative of u as follows

$$\begin{aligned} \dot{z}_1 &= z_2 \\ &\vdots \\ \dot{z}_{n-1} &= z_n \\ \dot{z}_n &= F(z, \alpha_n, u, -\dot{u}, \ldots, (-1)^{(n-r)}u^{(n-r)}) \\ y &= z_1. \end{aligned} \tag{7}$$

The above local nonlinear outer factor satisfies the five criteria of the Definition 1 and we let S be the set of initial conditions such that for all $x_0 \in S$, the steady state behavior of a local nonlinear outer factor satisfies the five criteria.

2.4 Dynamic input-output linearizing control law

For the local nonlinear outer factor, one can calculate the input-output linearizing controller of the original nonlinear system by solving the following ordinary differential equation along trajectories of the closed-loop system [1], [2]

$$F(z, \alpha_n, u, -\dot{u}, \ldots, (-1)^{(n-r)}u^{(n-r)}) = v \tag{8}$$

where v is a control variable to be determined.
Then the nonlinear system (3) can be expressed as the following linear system

$$\begin{aligned} \dot{z} &= Az + Bv \\ y &= Cz \end{aligned} \tag{9}$$

where (A, B, C) are in the Brunovsky canonical form. The input v will not only be utilized to stabilize the nominal linear system, but it will be used to asymptotically stabilize the system once the parametric uncertainty is introduced.

3. Robust stabilization

Since the original system (1) has uncertain parameter vector, a robust control strategy is considered. We first begin by expanding $\dot{z}$ as a Taylor series in α about its nominal value, i.e., $\alpha = \alpha_n$. For convenience, rewrite (6) as

$$\dot{z} = Az + N(z, \alpha_n, \hat{u}) \tag{10}$$

where $N(z, \alpha_n, \hat{u}) = BF(z, \alpha_n, \hat{u})$. Taylor series of (10) in α in the new coordinates yields

$$\dot{z} = Az + N + \left[\frac{\partial N}{\partial \alpha}\right]_{\alpha=\alpha_n} \delta\alpha + O(\|\delta\alpha\|^2). \tag{11}$$

If we ignore the higher order terms and apply the dynamic IO linearizing control law (8), (11) results in the perturbed system

$$\dot{z} = Az + Bv + \eta(z, \hat{u}) \tag{12}$$

where

$$\eta(z,\hat{u}) = \left[\frac{\partial N}{\partial \alpha}\right]_{\alpha=\alpha_n} \delta\alpha. \tag{13}$$

The first step in the stabilization of the nonlinear system is to place the poles of the nominal linear system (9) into the left-half plane. This can be achieved via linear quadratic regulator theory or simple pole-placement methods (e.g., Ackermann's formula). We assume the existence of a feedback gain matrix.

Assumption 4: For the given matrix pair (A, B), there exists a constant $1\times n$ matrix K such that

$$A_c = A + BK$$

is stable.

Thus, if we let $v = Kz + p$, (12) can be written

$$\dot{z} = A_c z + Bp + \eta(z,\hat{u}) \tag{14}$$

where p is an additional control to stabilize the uncertain vector. Next, we assume the bounds on the uncertainty $\eta(z,\hat{u})$.

Assumption 5: There exists a nonnegative scalar function $\rho(z,\hat{u},t)$ such that

$$\|\eta\| \le \rho(z,\hat{u},t). \tag{15}$$

The function ρ can be explicitly written from (13) and (15) as

$$\rho(z,\hat{u},t) = \left\|\frac{\partial N}{\partial \alpha}\right\|_{\alpha=\alpha_n} \sup\|\delta\alpha\|. \tag{16}$$

In (16) all the values are known *a priori* except $\sup\|\delta\alpha\|$. However, the unknown value is determined once one decides on a relative parameter deviation to consider. For example, if one wishes to determine the effects of 20% relative uncertainty for α, then $\sup\|\delta\alpha\| = 0.2\alpha_n$.

We are now ready to design the additional control p. The following theorem gives us the control law and the stability result for the system.

Theorem 1: Under Assumptions 1-5, the equilibrium point $z = 0$ is asymptotically stable with respect to initial conditions lying in S if p satisfies

$$p = \begin{cases} \dfrac{-\rho(z,\hat{u},t)}{\|B\|}, & z^T PB \ge 0 \\ \dfrac{\rho(z,\hat{u},t)}{\|B\|}, & z^T PB < 0 \end{cases} \tag{17}$$

where P is the unique symmetric positive definite solution to the Lyapunov equation

$$PA_c + A_c^T P + Q = 0 \tag{18}$$

with Q a given symmetric positive definite matrix.

Proof: Consider the Lyapunov function for the nominal linearized system

$$V(z) = z^T P z. \tag{19}$$

Differentiating (19) along solutions of (14) and utilizing (18), we obtain

$$\begin{aligned}\dot{V} &= 2z^T PA_c z + 2z^T PBp + 2z^T P\eta(z,\hat{u},t) \\ &= -z^T Qz + 2z^T PBp + 2z^T P\eta(z,\hat{u},t) \\ &= -z^T Qz + 2z^T P(Bp + \eta(z,\hat{u},t))\end{aligned} \tag{20}$$

and from (10)-(13) we observe the following equality holds

$$\eta(z,\hat{u},t) = B\tilde{\eta}(z,\hat{u},t) \tag{21}$$

where $\tilde{\eta}$ is a scalar function. A bound on $\tilde{\eta}$ can be obtained as

$$\|\eta\| = \|B\tilde{\eta}\| = |\tilde{\eta}|\cdot\|B\| \le \rho(z,\hat{u},t). \tag{22}$$

From (22) we get

$$-\frac{\rho(z,\hat{u},t)}{\|B\|} \le \tilde{\eta} \le \frac{\rho(z,\hat{u},t)}{\|B\|} \tag{23}$$

which leads to

$$\dot{V} \le \begin{cases} -z^T Qz + 2z^T PB\left(p + \dfrac{\rho}{\|B\|}\right), & z^T PB \ge 0 \\ -z^T Qz + 2|z^T PB|\left(p - \dfrac{\rho}{\|B\|}\right), & z^T PB < 0 \end{cases} \tag{24}$$

With the choice of control law in (17), we have [6]

$$\dot{V} \le -z^T Qz < 0. \tag{25}$$

This completes the proof. ■

4. Example

To demonstrate the approach, we consider the following nonlinear dynamic system

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - 0.2x_1^2 - 4x_2 - 9x_3 - x_4 + \alpha u \\ \dot{x}_3 &= 2x_3 + 2x_3 - x_4 \\ \dot{x}_4 &= x_3 + 0.3x_1x_2 \\ y &= x_1\end{aligned} \tag{26}$$

where α is a scalar uncertain parameter. The system is already in the Byrnes-Isidori normal form, and the system has relative degree 2. The unforced zero dynamics is completely unstable. This can be verified that the characteristic roots of the Jacobian of the zero dynamics are located both at $+1$. As outlined in section 2.3, we define the new state variable

$$\begin{aligned}z_1 &= x_1 \\ z_2 &= x_2 \\ z_3 &= -x_1 - 0.2x_1^2 - 4x_2 - 9x_3 - x_4 + \alpha_n u \\ z_4 &= 4x_1 - 3x_2 + 17x_3 - 4\alpha_n u + \alpha_n \dot{u} \\ &\quad + 0.8x_1^2 - 0.7x_1x_2.\end{aligned} \tag{27}$$

With these variables, the nominal nonlinear system can be transformed into the canonical form of (6)

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= \alpha_n u + 2\alpha_n \dot{u} + \alpha_n \ddot{u} - z_1 - 0.2z_1^2 - 4z_2 \\ &\quad + 4.1z_1z_2 - 0.7z_2^2 - 12z_3 - 0.7z_1z_3 - 2z_4 \\ y &= z_1.\end{aligned} \tag{28}$$

The outer factor is obtained by switching the sign on $\dot{u}$ in (28). Then using (8) we can design the dynamic IO linearizing control law. Next, we need to design robust control law as explained in section 3. We let $\alpha_n = 1$,

$$\eta(z, \hat{u}) = [\,0, 0, 0, u + 2\dot{u} + \ddot{u}\,]^T \delta a. \quad (29)$$

And the bound function on η can be obtained as

$$\rho(z, \hat{u}, t) = \| \eta(z, \hat{u}) \| \sup | \delta a |. \quad (30)$$

Moreover, the feedback gain matrix is selected such that the closed loop poles are all located at -1. Figure 1 shows the outputs when we consider the relative parameter deviation 30% and 80% respectively. And figure 2 shows corresponding nonlinear control input u. In figure 1, we observe that the asymptotic stabilization is achieved when the parameter has relatively large uncertainty.

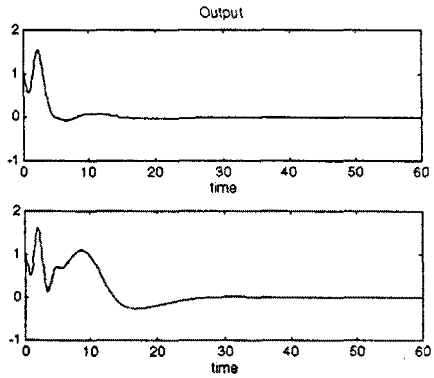

Fig. 1 System outputs: Above: 30% uncertainty, Bottom: 80% uncertainty

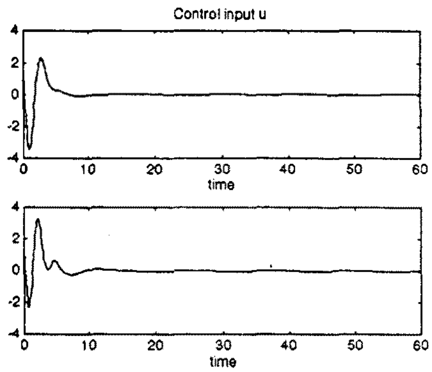

Fig. 2 Nonlinear control inputs

5. Conclusion

In this paper, we present a factorization approach to constructing a local nonlinear outer factor for nonlinear systems with completely unstable zero dynamics and the dynamic IO linearizing control law. In addition, we also outline the Lyapunov-based robust control law to stabilize the nonlinear uncertain term. The results are local in nature, as the required coordinate transformations are only defined locally.

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