

폐-루우프 피드백을 가진 전-차수 관측기에 기준한 SM-MF 제어기를 이용한  
 다기 안정기 설계 : Part 5  
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**Design of Multimachine Stabilizer using Full-Order Observer-based  
 SM-MF Controller including CLF : Part 5**

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**[Abstract]**

In this paper, the power system stabilizer(PSS) using the sliding mode observer-model following(SMO-MF) including closed-loop feedback(CLF) for single machine system is extended to multimachine system.

**Keywords :** Sliding Mode Observer-Model Following, Closed Loop Feedback, Multimachine Power System Stabilizer

**1. Introduction**

In many situations, the entire state vector cannot be measured and the control law must be based on an estimate of the state, rather than the actual state. To solve these problems of the full state feedback[1-4], the sliding mode observer-model following(SMO-MF) for unmeasurable plant state variables is developed in this paper. The values of the output vector can be obtained by measuring angular velocity. In this paper, the power system stabilizer(PSS) using the sliding mode observer-model following(SMO-MF) including CLF for single machine system is extended to multimachine system.

**2. Multimachine model**

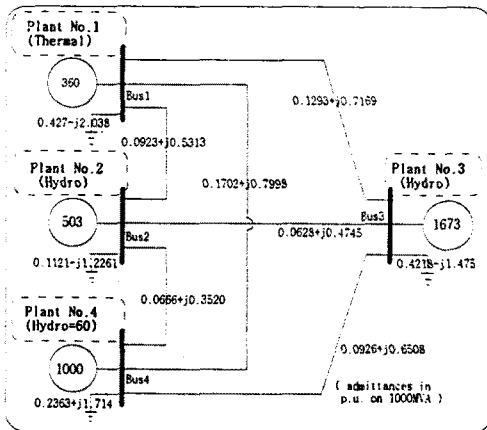


Fig. 1 Three-machine/infinite busbar system.

The 12-th order state equation for a reference model can be expressed as

$$x_m = \begin{bmatrix} \Delta\delta_{m1}, \Delta\omega_{m1}, \Delta e'_{m1}, \Delta e_{FDm1}, \\ \Delta\delta_{m2}, \Delta\omega_{m2}, \Delta e'_{m2}, \Delta e_{FDm2}, \\ \Delta\delta_{m3}, \Delta\omega_{m3}, \Delta e'_{m3}, \Delta e_{FDm3} \end{bmatrix}^T \quad (1)$$

The 12-th order state equation for the controlled plant can be expressed as

$$x_p = \begin{bmatrix} \Delta\delta_{p1}, \Delta\omega_{p1}, \Delta e'_{p1}, \Delta e_{FDp1}, \\ \Delta\delta_{p2}, \Delta\omega_{p2}, \Delta e'_{p2}, \Delta e_{FDp2}, \\ \Delta\delta_{p3}, \Delta\omega_{p3}, \Delta e'_{p3}, \Delta e_{FDp3} \end{bmatrix}^T \quad (2)$$

**3. Multimachine SMO-MF controller including CLF**

The state equation for a reference model can be expressed as

$$\dot{x}_m(t) = A_m \cdot x_m(t) + B_m \cdot u_m(t) \quad (3)$$

where  $x_m \in R^n$  is a state vector for model and  $u_m \in R^m$  is a control input for model. The control input of a reference model with  $r_m$  can be expressed as

$$u_m(t) = -K_m \cdot x_m(t) + r_m(t) \quad (4)$$

where  $K_m$  is a  $m \times n$  feedback gain for model and can be obtained by pole placement. And  $r_m \in R^m$  is a reference input vector for model.

The closed loop feedback system for a reference model is

$$\dot{x}_m(t) = (A_m - B_m \cdot K_m) \cdot x_m(t) + B_m \cdot r_m(t) \quad (5)$$

$$\text{Let } A_m = A_m - B_m \cdot K_m \quad (6)$$

The state equation for the reference model including CLF can be reformed as

$$\dot{x}_m(t) = A_m \cdot x_m(t) + B_m \cdot r_m(t) \quad (7)$$

where  $A_m$  is a  $n \times n$  system matrix including CLF for model.

The state equation for the controlled plant with the parameter variations and the output equation can be formed as

$$\begin{aligned} \dot{x}_p(t) &= (A_p + \Delta A_p) \cdot x_p(t) + (B_p + \Delta B_p) \cdot u_p(t) \\ &= \tilde{A}_p \cdot x_p(t) + \tilde{B}_p \cdot u_p(t) \end{aligned} \quad (8)$$

$$y_p(t) = C_p \cdot x_p(t) \quad (9)$$

where  $\tilde{A}_p = A_p + \Delta A_p$  is a  $n \times n$  system matrix with the parameter variations for plant and  $\tilde{B}_p = B_p + \Delta B_p$  a  $n \times m$  control matrix with the parameter variations for plant. And  $C_p$  is the  $m \times n$  output matrix for plant.

The following linear full-order observer equation of the controlled plant for unmeasurable state variables can be expressed as

$$\begin{aligned} \dot{\hat{x}}_p(t) &= \tilde{A}_p \cdot \hat{x}_p(t) + \tilde{B}_p \cdot u_p(t) + L_p \cdot (y_p(t) - C_p \cdot \hat{x}_p(t)) \\ &= (\tilde{A}_p - L_p \cdot C_p) \cdot \hat{x}_p(t) + \tilde{B}_p \cdot u_p(t) + L_p \cdot y_p(t) \end{aligned} \quad (10)$$

where  $\hat{x}_p \in R^n$  is the estimated state for plant.

$$L_p = P_p \cdot C_p^T \cdot R_p^{-1} \quad (11)$$

is the  $n \times m$  output injection matrix for plant.  $P_p$  is the symmetric positive definite solution of

$$\tilde{A}_p^T \cdot P_p + P_p \cdot \tilde{A}_p - P_p \cdot C_p^T \cdot R_p^{-1} \cdot C_p \cdot P_p + Q_p = 0 \quad (12)$$

$Q_p$  and  $R_p$  are positive definite matrices chosen by the designer.

The input control vector with a feedback gain for unmeasurable state is expressed

$$\begin{aligned} u_p(t) &= u_{clp}(t) + u_{smo}(t) \\ &= -K_p \cdot \hat{x}_p(t) + u_{smo}(t) \end{aligned} \quad (13)$$

Substituting eq.(13) into eq.(10), for unmeasurable state, the following full-order observer equation of the controlled plant including CLF can be expressed as

$$\begin{aligned} \dot{\hat{x}}_p(t) &= (\tilde{A}_p - L_p \cdot C_p) \cdot \hat{x}_p(t) + \tilde{B}_p \cdot u_p(t) + L_p \cdot y_p(t) \\ &= (\tilde{A}_p - L_p \cdot C_p) \cdot \hat{x}_p(t) + \tilde{B}_p \cdot u_{smo}(t) + L_p \cdot y_p(t) \end{aligned} \quad (14)$$

where  $\tilde{A}_p = (\tilde{A}_p - \tilde{B}_p \cdot K_p)$  is a  $n \times n$  system matrix with the parameter variations including CLF for plant.

The error vector and the differential error vector can be expressed as

$$e(t) = x_p(t) - \hat{x}_p(t) \quad (15)$$

$$\dot{e}(t) = \dot{x}_p(t) - \dot{\hat{x}}_p(t) \quad (16)$$

By substituting eq.(7) and eq.(14) into eq.(16), we have

$$\begin{aligned} \dot{e}(t) &= \dot{x}_p(t) - \dot{\hat{x}}_p(t) \\ &= [A_m \cdot x_p(t) + B_m \cdot r_p(t)] - [(\tilde{A}_p - L_p \cdot C_p) \cdot \hat{x}_p(t) \end{aligned}$$

$$+ \tilde{B}_p \cdot u_{smo}(t) + L_p \cdot y_p(t)] \quad (17)$$

$$x_p(t) = e(t) + \hat{x}_p(t) \quad (18)$$

By substituting eq.(18) into eq.(17), we have

$$\begin{aligned} \dot{e}(t) &= A_m \cdot x_p(t) + B_m \cdot r_p(t) - (\tilde{A}_p - L_p \cdot C_p) \cdot \hat{x}_p(t) \\ &\quad - \tilde{B}_p \cdot u_{smo}(t) - L_p \cdot y_p(t) \\ &= A_m \cdot e(t) - (\tilde{A}_p - L_p \cdot C_p - A_m) \cdot \hat{x}_p(t) + B_m \cdot r_p(t) \\ &\quad - \tilde{B}_p \cdot u_{smo}(t) - L_p \cdot y_p(t) \end{aligned} \quad (19)$$

Suppose the sliding mode exists on all hyperplanes. The sliding surface vector and the differential sliding surface vector can be expressed as

$$s(e(t)) = G^T \cdot e(t) \quad (20)$$

$$\dot{s}(e(t)) = G^T \cdot \dot{e}(t) \quad (21)$$

where  $G^T$  is the sliding surface gain.

To determine a control law that keeps the system on  $s(e(t)) \Rightarrow 0$ , we introduce the Lyapunov's function

$$V(e(t)) = s^2(e(t)) / 2 \quad (22)$$

The time derivative of  $V(e(t))$  is given by

$$\dot{V}(e(t)) = \dot{s}(e(t)) \cdot s(e(t)) \quad (23)$$

$$= G^T \cdot e(t) \cdot G^T \cdot \dot{e}(t) \quad (24)$$

$$\begin{aligned} &= G^T \cdot e(t) \cdot [G^T \cdot A_m \cdot e(t) \\ &\quad - G^T \cdot (\tilde{A}_p - L_p \cdot C_p - A_m) \cdot \hat{x}_p(t) + G^T \cdot B_m \cdot r_p(t) \\ &\quad - G^T \cdot \tilde{B}_p \cdot u_{smo}(t) - G^T \cdot L_p \cdot y_p(t)] \leq 0 \end{aligned} \quad (25)$$

From eq.(25), the control input vector with switching for the controlled plant can be represented by

$$\begin{aligned} u_{smo}(t) \geq (G^T \cdot \tilde{B}_p)^{-1} \cdot [G^T \cdot A_m \cdot e(t) - G^T \cdot (\tilde{A}_p - L_p \cdot C_p - A_m) \cdot \hat{x}_p(t) \\ + G^T \cdot B_m \cdot r_p(t) - G^T \cdot L_p \cdot y_p(t)] \text{ for } G^T \cdot e(t) > 0 \end{aligned} \quad (26)$$

$$\begin{aligned} u_{smo}(t) \leq (G^T \cdot \tilde{B}_p)^{-1} \cdot [G^T \cdot A_m \cdot e(t) - G^T \cdot (\tilde{A}_p - L_p \cdot C_p - A_m) \cdot \hat{x}_p(t) \\ + G^T \cdot B_m \cdot r_p(t) - G^T \cdot L_p \cdot y_p(t)] \text{ for } G^T \cdot e(t) < 0 \end{aligned} \quad (27)$$

From eq.(26) and eq.(27), the control input vector with sign function for the controlled plant can be reformed

$$\begin{aligned} u_{smo}^{**}(t) &= [SE_{sm} \cdot e(t) + SP_{sm} \cdot \hat{x}_p(t) + SU_{sm} \cdot r_p(t) + SO_{sm} \cdot y_p(t)] \\ &\quad \cdot \mu \cdot \text{sign}(s(e(t))) \end{aligned} \quad (28)$$

where  $\mu$  is the bias gain.

$$SE_{sm} = (G^T \cdot \tilde{B}_p)^{-1} \cdot G^T \cdot A_m \quad (29)$$

is a sliding equal error feedback gain.

$$SP_{pm} := -(G' \cdot \bar{B}_s)^{-1} \cdot G' \cdot (\bar{A}_m - A_m - L \cdot C_s) \quad (30)$$

is a sliding equal estimated plant feedback gain.

$$SU_{pm} := (G' \cdot \bar{B}_s)^{-1} \cdot G' \cdot B_m \quad (31)$$

is a sliding equal input gain.

$$SO_{pm} := -(G' \cdot \bar{B}_s)^{-1} \cdot G' \cdot L_s \quad (32)$$

is a sliding equal measured output gain.

#### 4. Multimachine data analysis

In this paper, the values of the  $12 \times 12$  system matrix  $A_m$  are decomposed into the 4-block form

$$A_m = \begin{bmatrix} A_{m11} & A_{m12} \\ A_{m21} & A_{m22} \end{bmatrix}$$

where

$$A_{m11} = \begin{bmatrix} 0 & 377 & 0 & 0 & 0 & 0 \\ -0.147 & -0.039 & -0.013 & 0 & 0.022 & 0.004 \\ -0.266 & -0.393 & -0.922 & 1 & -0.087 & 0.754 \\ -30.30 & -309.14 & -60.943 & -20 & 24.599 & -91.99 \\ 0 & 0 & 0 & 0 & 0 & 377 \\ 0.004 & -0.034 & -0.087 & 0 & -0.149 & 0.032 \end{bmatrix} \quad A_{m12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.046 & 0.02 & 0.003 & 0 \\ 0.024 & -0.029 & 1.191 & 0.872 & 0 & 0 \\ -3.901 & 0.2051 & -1.675 & -10.194 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.008 & 0 & 0.079 & -0.028 & 0 & 0 \end{bmatrix}$$

$$A_{m21} = \begin{bmatrix} 0.121 & 1.131 & 0.021 & 0 & -1.6 & -1.885 \\ -18.48 & -64.47 & -12.55 & 0 & 106.09 & -516.11 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.001 & -0.017 & -0.003 & 0.017 & -0.01 & 0 \\ 0.083 & 0 & -0.002 & 0.22 & 0 & 0 \\ -10.1 & -33.93 & -6.78 & 0 & 17 & -46.37 \end{bmatrix} \quad A_{m22} = \begin{bmatrix} -0.21 & 1 & 0.46 & 0.754 & 0.06 & 0 \\ -2.67 & -20 & 16.59 & -17.91 & -1.41 & 0 \\ 0 & 0 & 0 & 377 & 0 & 0 \\ 0 & 0 & -0.056 & -0.017 & -0.009 & 0 \\ 0.011 & 0 & -12 & -1.151 & -0.19 & 1 \\ -2.1 & 0 & 70.1 & -893.49 & -544 & -20 \end{bmatrix}$$

The  $12 \times 3$  control matrix  $B_m$  is given

$$B_m = \begin{bmatrix} 0 & 0 & 0 & 800 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 900 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1000 & 0 \end{bmatrix}$$

The values of  $K_m$  are

$$K_m = \begin{bmatrix} 42.80 & 2310 & 5.57 & .12 & 9.7 & -1265 & .53 & .008 & -1.01 & -316.5 & 499 & .01 \\ -74.31 & 8064 & -4.65 & .04 & 42.8 & 811 & 2.04 & .075 & -68.8 & -9434 & 4.11 & .04 \\ 5.80 & 1408 & 1.89 & .02 & 8.8 & -76.3 & .08 & .0009 & 22.5 & -586.2 & 2.76 & .08 \end{bmatrix}$$

Then for simulations, the controlled plant system matrix and the plant input vector are given by adding the plant parameter uncertainties with reference model.

$$\bar{A}_m = A_m + \Delta A_m = A_m + 10\% \text{ of } A_m$$

$$\bar{B}_m = B_m + \Delta B_m = B_m + 10\% \text{ of } B_m$$

The values of the output  $C_s$  are obtained by measuring angular velocity

$$C_s = [0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$$

The output injection gain  $L$  is

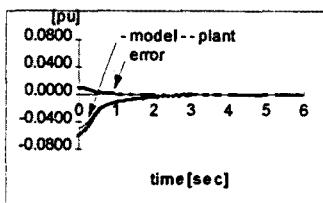
$$L = [10^{-3} \ -0.03 \ -0.07 \ 0.00 \ 0.19 \ -3.01 \ -0.03 \ 0.01 \ 0.09 \ -2.27 \ -0.01 \ 0.00 \ 0.05 \ 0.17]$$

The  $12 \times 3$  sliding surface matrix including CLF is obtained as

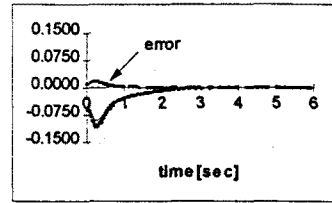
$$G = \begin{bmatrix} .01 & -2590 & 144 & 1 & -41 & -5770 & 343 & 0 & -52 & -5680 & 608 & 0 \\ 9.19 & 110 & 343 & 0 & -3.54 & -529 & 141 & 1 & -3.95 & -444 & .1 & 0 \\ 1.08 & 3430 & 603 & 0 & 3.67 & -993 & .0973 & 1 & -21.5 & -12100 & 72.2 & 1 \end{bmatrix}$$

#### 5. Time domain simulation

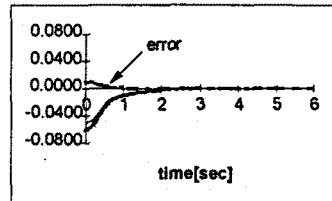
For the torque angle of machine #1, #2 and #3, the time domain simulations are carried out for 6 sec.



(a) torque angle of machine #1.



(b) torque angle of machine #2.



(c) torque angle of machine #3.

Fig. 2 Torque angle waveforms.

Fig. 2 shows that the proposed multimachine SMO-MF PSS for unmeasurable state variables is able to achieve asymptotic tracking error between the reference model state and the estimated plant state at different initial conditions.

#### 6. Conclusion

The sliding mode observer-model following (SMO-MF) power system stabilizer (PSS) including closed-loop feedback (CLF) for single-machine power system has been extended to multimachine systems. The multimachine SMO-MF PSS for unmeasurable state variables has been designed not only to damp out the low frequency oscillations of the multimachine power system by including CLF, but also to achieve asymptotic tracking error between the reference model state and the estimated plant state at different initial conditions.

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