Evaluation and Interpretation of the Fracture Toughness of Rocks

Hwanjo Baek

Dept. of Resources Engineering
Kangwon National University

1. Intoduction

Fracture toughness of rock materials, which generally violate the fundamental assumptions of LEFM, often depends on the specimen size and test method employed. Hence, a standardized procedure for testing and data interpretation for determining fracture toughness of rock materials is required. Special attention has been given by the International Society for Rock Mechanics (ISRM) to the difficulties in obtaining true fracture mechanics parameters for the wide variety of rock materials.

Rock samples are usually cored from drill holes, and core-based specimens are more cost-effective for determining rock fracture toughness. Accordingly, the single-edge-cracked round-bar-in-bending (SECRBB), semi-circular bending (SCB), notched Brazilian discs, and the chevron bend (CB) specimens are practical specimen types. Other core-based specimen types include the burst cylinder method, modified ring test, and the round compact tension. Among the various specimen geometries listed above, the SECRBB and CB specimens will be discussed here.

2. Specimen Geometries

2.1 Single-Edge-Cracked Round-Bar-in-Bending

The geometry of the SECRBB specimen is shown in Fig. 1. Assuming a linear elastic material behavior, the compliance, *C*, of the specimen is defined as:

$$C = \frac{\delta_F}{F} \tag{1}$$

where d_F is the load point displacement (LPD) and F is the applied load. Within the limits of the slender beam theory, the compliance of a SECRBB specimen is given by:

$$C_{secrbb} = \frac{S^3}{48E \cdot I_{secrbb}} = \frac{1}{ED} \cdot \frac{4(S/D)^3}{3\pi \cdot f(\alpha)} \tag{2}$$

where S is the support span and $f(a)$ is a dimensionless function derived from I_{secrbb}, the moment of inertia of the notched SECRBB cross-section. The mode I stress intensity factor for SECRBB specimen (S/D = 3.33) is then given by:

$$K_I = \sqrt{\frac{F^2}{4(a \cdot D - a^2)^{0.5} \cdot D^2} \cdot \frac{\partial(c \cdot E' \cdot D)}{\partial(a/D)}} = \frac{F}{D^{1.5}} \cdot Y'_{secrbb} \tag{3}$$

From a compliance calibration of the Ekeberg marble, Ouchterlony obtained Y'_{secrbb} as:

$$Y'_{secrbb} = 10.62 \cdot \alpha^{0.5} \cdot (1 + 19.65\alpha^{4.5})^{0.5} / (1-\alpha)^{0.25} \tag{4}$$

valid for $0 \leq a < 0.6$, and $S/D = 3.33$.

2.2 Chevron Bend Specimen

The chevron bend (CB) specimen is one of the two specimen geometries suggested by the ISRM. Chevron-notched specimens have several advantages over other specimen types, especially for materials which exhibit brittle fracturing: the ligament shape enables a stable crack propagation from a self-produced sharp crack. A stable crack growth up to certain distance from the initial chevron tip produces a naturally sharp crack, and the fatigue pre-cracking requirement of

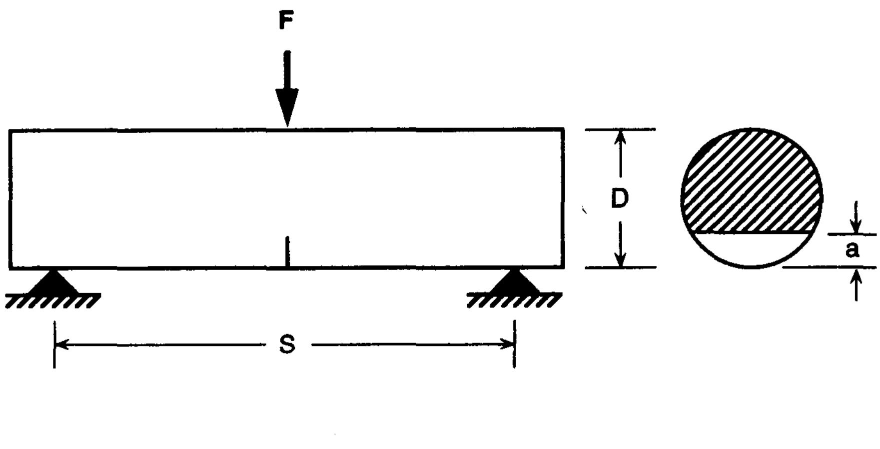

Fig. 1. Geometry of the SECRBB specimen

ASTM E 399 can be omitted. Furthermore, assuming a flat *R*-curve for generally brittle rock materials, fracture toughness is calculated from the maximum load and initial specimen dimensions.

The geometry of the CB specimen is shown in Fig. 2. In Level I testing, only the maximum test load, F_{max}, is recorded and the fracture toughness is calculated by (ISRM, 1988):

$$K_{Ic,cb} = A_{\min} \cdot F_{\max} / D^{1.5} \tag{5}$$

With the notations are given in Fig. 2, a dimensionless factor, A_{min}, is given as:

$$A_{\min} = \left[1.835 + 7.15(a_o / D) + 9.85(a_o / D)^2\right] \cdot (S / D) \tag{6}$$

3. Notch Sensitivity

Two different types of failure mechanisms act within a notched specimen under loading. The first is a strength failure governed by the maximum tensile stress at the crack tip and the tensile strength of the material. The second is a failure due to crack extension governed by the fracture toughness of material. Therefore, it is necessary to determine which failure mechanism is critical at the instant of failure.

Carpinteri (1982) employed the concept of notch sensitivity in analyzing fracture toughness test results of aggregative materials, assuming homogeneous, isotropic, and linear elastic material

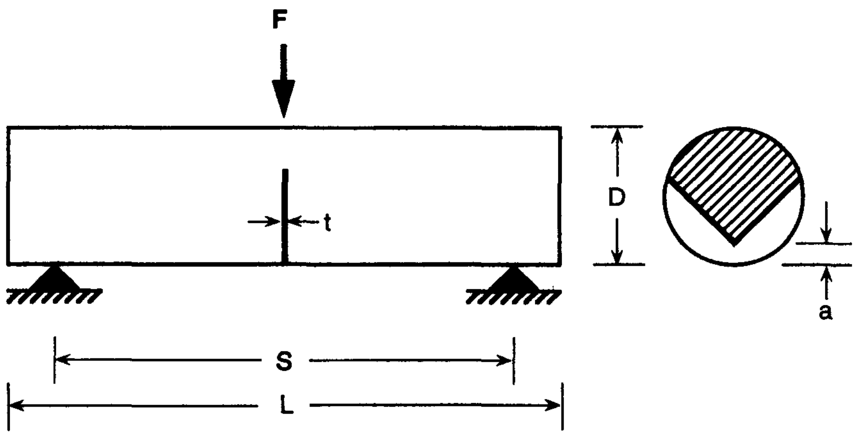

Fig. 2. Geometry of the Chevron Bend specimen.

behavior. He applied the dimensional analysis for physical similitude and scale modeling to define a *non-dimensional* brittleness number, s, given by:

$$s = \frac{K_{Ic}}{\sigma_t \cdot W^{1/2}} \tag{7}$$

where W is the specimen width.

For example, the stress intensity factor for the three-point bend specimen is:

$$K_I = \frac{F \cdot S}{B \cdot W^{3/2}} \cdot f(a/W) \tag{8}$$

where S is the support span and $f(a/W)$ is a dimensionless coefficient. At the critical load F_c, the generalized force for crack extension, $\hat{F}_{crack}$, is:

$$\hat{F}_{crack} = \frac{F_c \cdot S}{\sigma_t \cdot W^2 \cdot B} = \frac{s}{f(a/W)} \tag{9}$$

Assuming that the linear elastic slender beam theory is still applicable for the notched three-point bend specimen, the generalized force for ultimate strength failure, $\hat{F}_{strength}$, is:

$$\hat{F}_{strength} = \frac{F_c \cdot S}{\sigma_t \cdot W^2 \cdot B} = \frac{2}{3} \cdot \left(1 - \frac{a}{W}\right)^2 \tag{10}$$

It should be noted that the two generalized forces given in Eqs. (9) and (10) are both dimensionless and directly comparable. The notch sensitivity is then defined as a function of the brittleness number and crack length ratio, a/W: at a certain combination of these two parameters, if the generalized force for crack extension is smaller than that of ultimate strength failure, crack extension is more critical and the specimen is notch sensitive. A valid fracture toughness measurement for a given specimen geometry can be assured only if the specimen is notch sensitive. Carpinteri (1982) concluded that some recurring experimental inconsistencies, such as the variance of K_{Ic} with the crack length, specimen size, and test geometry, can be explained by the notch sensitivity of the specimen.

4. Notch Sensitivity of the CB and SECRBB Specimens

The coordinates of the centroid, $\bar{x}$ and $\bar{y}$, and the moment of inertia with respect to the y direction, I_y, of a plane area in Fig. 3 are calculated from the theory of elasticity as:

$$\bar{x} = \frac{Q_y}{A} = \frac{\int x \cdot dA}{\int dA}, \quad \bar{y} = \frac{Q_x}{A} = \frac{\int y \cdot dA}{\int dA} \tag{11}$$

$$I_y = \int x^2 \cdot dA \tag{12}$$

where Q_x and Q_y are the first moments of the area about the x and y axes, respectively, and A is the area of the plane. As shown in Fig. 4, the distance y' from the new centroid of the ligament plane of the notched specimens to the crack tip is calculated as:

$$y' = D \cdot h(\alpha) \tag{13}$$

where D is the specimen diameter and a is the crack length ratio. The dimensionless function $h(\alpha)$ depends on the specimen geometry as:

$$\begin{aligned} h(\alpha)_{cb} &= 0.56 - 0.51\alpha - 0.02\alpha^2 - 0.06\alpha^3 \\ h(\alpha)_{secrbb} &= 0.50 - 0.85\alpha + 1.11\alpha^2 - 1.75\alpha^3 \end{aligned} \tag{14}$$

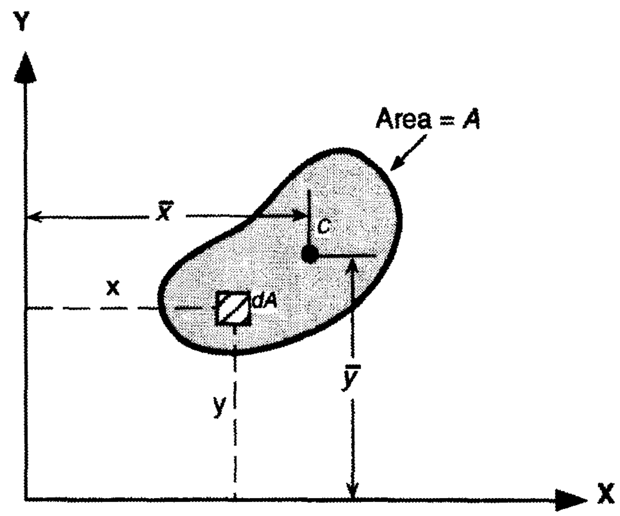

Fig. 3. Centroid of a plane area.

where the subscripts *cb* and *secrbb* represent the CB and SECRBB specimens, respectively. For the notched cross-section, the moment of inertia of the ligament plane is given by:

$$I = \pi \cdot D^4 \cdot i(\alpha) \tag{15}$$

where the dimensionless function *i(a)* also depends on the specimen geometry, such that:

$$\begin{aligned} i(\alpha)_{cb} &= 0.01 - 0.04\alpha + 0.05\alpha^2 - 0.02\alpha^3 \\ i(\alpha)_{secrbb} &= 0.02 - 0.02\alpha - 0.09\alpha^2 + 0.27\alpha^3 \end{aligned} \tag{16}$$

Assuming that the elastic slender beam theory is applicable for the notched beam, the tensile stress developed at the notch tip in the three-point bend specimen is :

$$\sigma_t = \frac{F \cdot S \cdot \bar{y}}{4I} \tag{17}$$

where S is the support span. From Eqs. (13), (15) and (17), the generalized force for the ultimate strength failure, $\hat{F}_{strength}$ of the CB and SECRBB specimens ($S/D = 3.33$) is given by:

$$\hat{F}_{strength} = \frac{F_{\max}}{\sigma_t \cdot D^2} = \frac{4\pi}{3.33} \cdot \frac{i(\alpha)}{h(\alpha)} \tag{18}$$

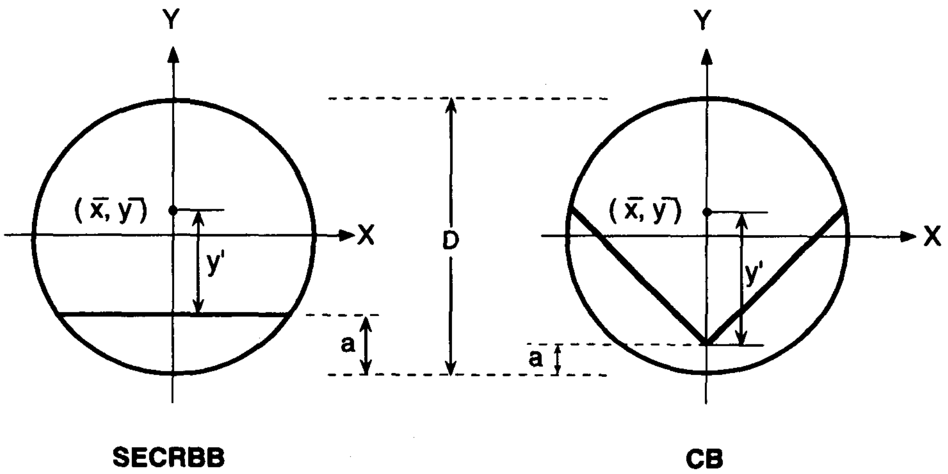

Fig. 4. Notched cross-section of the CB and SECRBB specimens.

From Eqs. (3) and (5), the fracture toughness formulas for the CB and SECRBB specimens (*S/D* = 3.33) can be written as:

$$K_{Ic} = \frac{F_{\max}}{D^{1.5}} \cdot f(\alpha) \tag{19}$$

The dimensionless stress intensity factor *f(a)* is given in Eq. (4) for the SECRBB specimen, and in Eq. (6) for the CB specimen. The generalized force for crack extension, $\hat{F}_{crack}$, is then given as:

$$\hat{F}_{crack} = \frac{F_{\max}}{\sigma_t \cdot D^2} = \frac{s}{f(\alpha)} \tag{20}$$

where the brittleness number, *s*, for the cylindrical specimens is:

$$s = \frac{K_{Ic}}{\sigma_t \cdot D^{0.5}} \tag{21}$$

5. Conclusion

Figure 5 shows the generalized crack extension force curves, for different brittleness numbers, and the generalized strength failure curve for the CB specimen. The range of the crack length ratio in the figure corresponds to the valid range of the dimensionless stress intensity factor for this specimen type. The region where the crack propagation failure curves are below the ultimate strength failure force curve defines the notch-sensitive range of the given specimen type. Within this region, the failure due to crack extension comes before the strength failure, and test data is valid.

The brittleness number is calculated from the tensile strength and diameter of the specimen, and the measured fracture toughness which depends on the crack length ratio. As shown in this figure, the brittleness number calculated from the measured fracture toughness of test specimens should be less than 0.45; otherwise, test result is invalid.

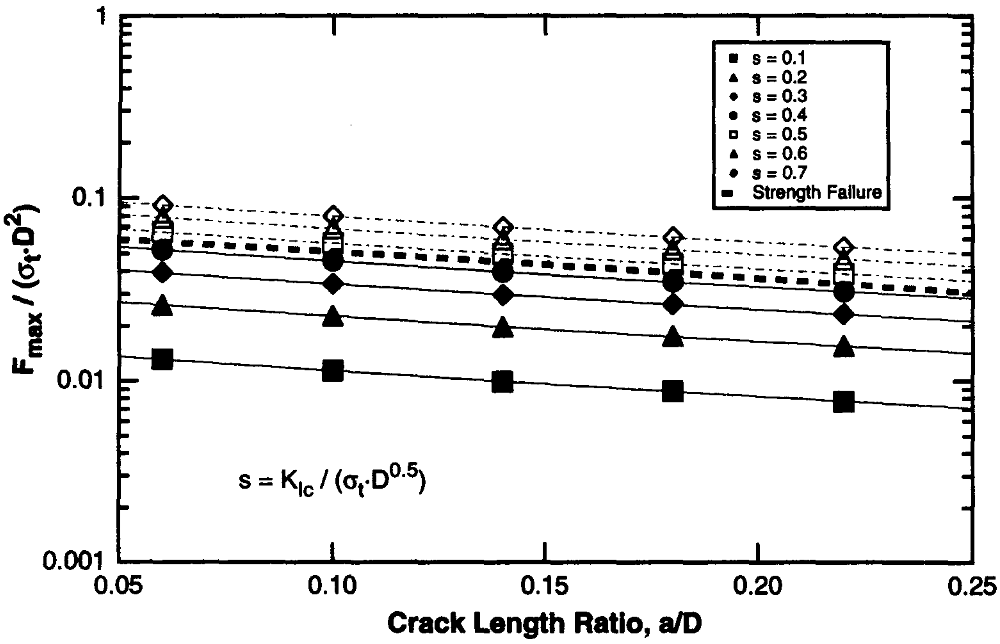

Fig. 5. Generalized force curves versus crack length ratio for the CB specimen, with varying brittleness number.

The same plot for the SECRBB specimen is shown in Fig. 6. Consider a SECRBB specimen for which the brittleness number is calculated as 0.5. If the crack length ratio is in the range of 0.2 to 0.3, fracture toughness testing on this specimen may be accepted, since the crack extension failure curve is tangent to the strength failure curve within this range. However, for other crack length ratios, for which the strength failure curve is located under the crack extension failure curve (s = 0.5), the ultimate strength failure is more critical and the test result is rejected. Thus, the notch sensitivity analysis with a fixed crack length ratio of the specimens can be applied to discriminate invalid fracture toughness values. Similarly, with an estimated fracture toughness for a rock material, the valid range of the initial crack length ratio can be determined prior to the testing.

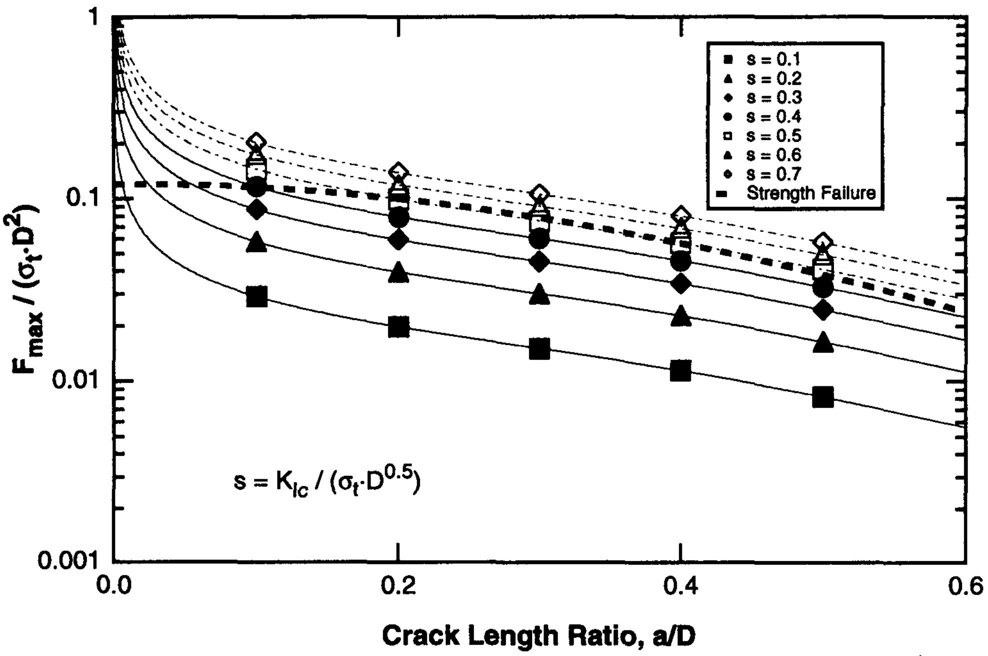

Fig. 5. Generalized force curves versus crack length ratio for the SECRBB specimen, with varying brittleness number.

6. References

ANSI/ASTM Designation: E399-78a, Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials.

Baek, H., 1994, Evaluation of Fracture Mechanics Properties and Microstructural Observations of Rock Fractures, Ph.D. Dissertation, Department of Civil Engineering, The University of Texas at Austin, 200 p.

Bush, A. J., 1976, "Experimentally Determined Stress-Intensity Factors for Single-Edge Round Bars Loaded in Bending," Experimental Mechanics, Vol. 16, No. 7, pp. 249-257.

Carpinteri, A., 1982, "Notch Sensitivity in Fracture Testing of Aggregative Materials," Engineering Fracture Mechanics, Vol. 16, No. 4, pp. 467-481.

ISRM Commission on Testing Methods, 1988, "Suggestive Methods for Determining the Fracture Toughness of Rock," Int. J. Rock Mech. Min. Sci. & Geomech. Abstr., Vol. 25, No. 2, pp. 71-96.

Ouchterlony, F., 1981, "Extension of the Compliance and Stress Intensity Formulars for the Single Edge Crack Round Bar in Bending," Fracture Mechanics Methods for Ceramics, Rocks, and Concrete, ASTM STP 745, ASTM, pp. 237-256.