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Some Studies on Stress Field in Dissimilar Materials

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ABSTRACT

Stress singularities appear at the interface edge in dissimilar materials also under thermal loading. First, these singularities when an interface meets a free side surface with an arbitrary angle are studied for a two-dimensional problem. The singular properties under thermal loading are made clear and the concrete singular field are obtained. Secondly, the dependence of stress field on elastic constants in axisymmetric dissimilar materials are discussed. That is, it is shown that three elastic constants mutually independent are necessary, in general, to characterize the stress field of axisymmetric dissimilar materials, although Dunders' parameters defined for two-dimensional dissimilar materials have been often applied correspondingly also to axisymmetric problems.

Key Words : stress field, dissimilar materials, interface, two-dimensional problem, thermo elasticity, singularity, Dunders' parameter, axisymmetric problem

1. Introduction

Recently, various kinds of bonded dissimilar materials are used widely in many engineering fields, so the stress evaluations of them have become important to let them have sufficient strengths. However, there still exist some difficult problems to be solved in their stress analyses. For example, stress singularity appears not only under mechanical loading but also under thermal loading at an interface edge and interface crack tip, and the actual states in the neighbourhood of an interface are not suitable to two-dimensional analyses in many cases. Three-dimensional analyses are necessary and, moreover, plastic deformation also should be taken into consideration.

In this paper, two problems are dealt with. The stress singularities at the free edge of the interface of a bonded joint under thermal loading are studied first. It is important to know them when the process of the production of dissimilar materials is taken into account. Thermal stress when an interface meets a free side surface with an arbitrary angle is solved by replacing it with the stress from traction forces on the free boundary. The properties of the singularity are discussed and the general expression of singular field about the problem are given.

On the other hand, the stress field in two-dimensional dissimilar materials are characterized by two elastic constants called Dunders' parameters[1] and these Dunders' parameters have been often applied correspondingly also to axisymmetric dissimilar materials because of two-dimensional- like properties of axisymmetric problems. It is discussed secondly how many parameters mutually independent are necessary for an axisymmetric problem and it is shown theoretically and numerically that three parameters mutually independent are generally necessary in the problem as for the general three-dimensional problem.

2. Thermal Stress Singularities at an Interface Edge

2.1 *Thermal Stress Model* Consider the case where an interface meets a side surface with an arbitrary angle as is shown in Fig.1. In this configuration, it can be easily shown that the thermal stress problem caused due to a

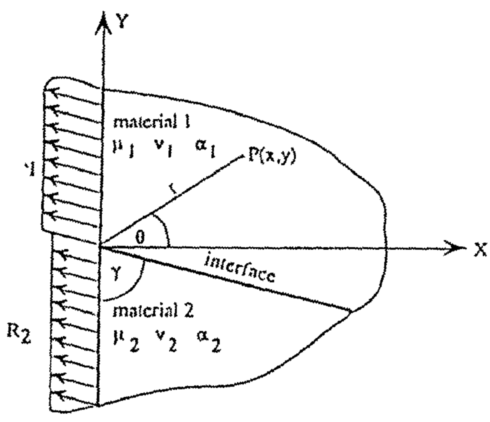

Fig.1 Thermal stress model

uniform temperature change ΔT can be replaced by the problem under the tractions R_1 and R_2 on the free surface given by

$$R_i = E_i^* \alpha_i^* \Delta T \sin\gamma \tag{1}$$

$$E_i^* = \frac{E_i}{1-(\nu_i^*)^2},\ \alpha_i^* = (1+\nu_i^*)\alpha_i^*,\ \nu^* = \frac{\nu_i}{1+\nu_i} \quad \text{: plane strain}$$

$$E_i^* = E_i, \alpha_i^* = \alpha_i, \nu^* = \nu_i \quad \text{: plane srtess}$$

as shown in the figure, where E_i, α_i, ν_i and $\mu_i(=E_i/2(1+\nu_i))$ denote Young's modulus, thermal expansion coefficient, Poisson's ratio and shear modulus, respectively. Subscript i takes 1 and 2 and it means that the quantity is for material i.

2.2 *Singularity Analysis* The Goursat formulas for thermal stress problem are given, in a polar coordinate system, by

$$\sigma_{i\theta} + \sigma_{ir} = 4\,\mathrm{Re}[\phi_i'(z)]$$

$$\sigma_{i\theta} - \sigma_{ir} + 2i\tau_{ir\theta} = 2e^{2i\theta}\left[\bar{z}\phi_i''(z) + \varphi_i'(z)\right] \tag{2}$$

$$2\mu_i\left[u_{ir} + iv_{i\theta} - i\varepsilon_{i0}e^{i\theta}\right] = e^{-i\theta}\left[\kappa_i\phi_i(z) - z\phi_i'(z) - \varphi_i(z)\right]$$

$$\varepsilon_{i0} = (1+\nu_i)\alpha_i\Delta T,\quad \kappa_i = 3-4\nu_i \quad \text{: plane strain}$$

$$\varepsilon_{i0} = \alpha_i\Delta T,\quad \kappa_i = (3-\nu_i)/(1+\nu_i) \quad \text{: plane stress}$$

where σ_{ir}, $\sigma_{i\theta}$ $\tau_{ir\theta}$ and u_{ir}, $v_{i\theta}$ are stress components and displacement components in a polar coordinate system, respectively. The conditions of continuity and boundary conditions in the problem here are given by

$$(u_{1r} + iv_{1\theta})_{\theta=\gamma-\pi/2} = (u_{2r} + iv_{2\theta})_{\theta=\gamma-\pi/2}$$

$$(\sigma_{1\theta} + i\tau_{1r\theta})_{\theta=\gamma-\pi/2} = (\sigma_{2\theta} + i\tau_{2r\theta})_{\theta=\gamma-\pi/2}$$

$$(\sigma_{1\theta} - i\tau_{1r\theta})_{\theta=\pi/2} = R_1(\sin\gamma - i\cos\gamma)$$

$$(\sigma_{2\theta} - i\tau_{2r\theta})_{\theta=\pi/2} = R_2(\sin\gamma - i\cos\gamma) \tag{3}$$

Here, let the complex stress functions be expressed by

$$\varphi_i(z) = \sum_{m=0}^{n}\left[\left(b_{1i,m}z^{\lambda} + b_{2i,m}z^{\bar{\lambda}}\right)(\log z)^m\right]$$

$$\phi_i(z) = \sum_{m=0}^{n}\left[\left(a_{1i,m}z^{\lambda} + a_{2i,m}z^{\bar{\lambda}}\right)(\log z)^m\right] \tag{4}$$

By substituting Eq.(4) into Eq.(2) and applying Eq.(3) to the results, a system of equations can be obtained for the coefficients of the complex stress functions in the following matrix form.

$$[P]\{d_n\} = \{0\}$$
$$[P]\{d_{n-1}\} = \{q_{n-1}\}$$
$$\vdots$$
$$[P]\{d_0\} = \{q_0\} \tag{5}$$

where $\{d_k\}$ $(k=0,1,\cdots,n)$is the column matrix of coefficients in stress functions, the matrix $[P]$ is given by

$$[P] = \begin{bmatrix}
\sin\gamma & \lambda & 1/e^{2i\gamma} & 0 & -1 & -\lambda & -1/e^{2i\gamma} & \sin 2\gamma \\
\lambda & 1/\sin\gamma & 0 & e^{2i\gamma} & -\lambda & -1 & \sin 2\gamma & -e^{2i\gamma} \\
(1+\alpha)\sin\gamma & 0 & 0 & 0 & -(1-\beta) & -(\alpha-\beta)\lambda e^{-2i\gamma} & (\alpha-\beta)e^{-2i\gamma} & \cos\gamma \\
0 & (1+\alpha)/\sin\gamma & 0 & 0 & -(\alpha-\beta)\lambda e^{2i\gamma} & -(1-\beta) & \cos\gamma & (\alpha-\beta)e^{2i\gamma} \\
1 & \lambda & 1 & 0 & 0 & 0 & 0 & 0 \\
1/\lambda & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \lambda & 1 & 0 \\
0 & 0 & 0 & 0 & \lambda & 1 & 0 & 1
\end{bmatrix} \tag{6}$$

and $\{q_t\}$ $(t=0,1,\cdots,n\text{-}1)$ is defined by

$$\{q_{n-1}\} = -{}_nC_1\frac{d}{d\lambda}[P]\{d_n\}$$
$$\vdots$$
$$\{q_{n-r}\} = -{}_nC_r\frac{d^r}{d\lambda^r}[P]\{d_n\} - {}_{(n-1)}C_{r-1}\frac{d^{r-1}}{d\lambda^{r-1}}[P]\{d_{n-1}\} - \ldots - (n-r+1)\frac{d}{d\lambda}[P]\{d_{n-r+1}\}$$
$$\vdots$$
$$\{q_0\} = \{R\}\frac{d^n}{d\lambda^n}[P]\{d_n\} - \frac{d^{n-1}}{d\lambda^{n-1}}[P]\{d_{n-1}\} - \ldots - \frac{d}{d\lambda}[P]\{d_1\}$$

Here, α and β are the Dunders' parameters and the defined by

$$\alpha = \frac{\mu_1(\kappa_2+1) - \mu_2(\kappa_1+1)}{\mu_1(\kappa_2+1) + \mu_2(\kappa_1+1)}$$

$$\beta = \frac{\mu_1(\kappa_2 - 1) - \mu_2(\kappa_1 - 1)}{\mu_1(\kappa_2 + 1) + \mu_2(\kappa_1 + 1)} \tag{8}$$

The matrix $[P]$ is the characteristic matrix for the problem here, so the solution of $\{d_k\}$ varies with $[P]$. The rank of $[P]$ depends on the values of Dunders' parameters.

2.3 *Result* By observing the stress functions closely, the logarithmic singularities can be stated when λ=1. Thus, let us deal with the case where $\alpha\{\alpha-\beta(1+\mathrm{cosec}\gamma)\} \leqq 0$ and λ=1, first. In this case, the rank of $[P]$ becomes 6 and the final solution for stress components can be obtained, when n=1, as

$$\begin{aligned} \sigma_r &= 2(1 - \alpha \sin\gamma) F_2 (1 - \cos 2\theta) \log r \\ \sigma_\theta &= 2(1 - \alpha \sin\gamma) F_2 (1 + \cos 2\theta) \log r \\ \tau_{r\theta} &= 2(\alpha \sin\gamma - 1) F_2 \sin 2\theta \log r \end{aligned} \tag{9}$$

When $\alpha\{\alpha-\beta(1+\mathrm{cosec}\gamma)\} \geqq 0$ and λ=1, the rank of $[P]$ becomes 8 and there exist no solutions for the homogeneous equation in Eq.(5). When $\alpha\{\alpha-\beta(1+\mathrm{cosec}\gamma)\} > 0$ and $\lambda \neq 1$ the rank of $[P]$ becomes 7 and there exist solutions for $\{d_k\}$. By considering both homogeneous and non-homogeneous equations in Eq.(5), it is seen that the solutions of $\{d_k\}$ can be obtained for n=1 and n=0. However, according to the non-homogeneous equations, the value of λ becomes 0 when n=0, so only the case of n=1 can be considered for the evaluation of stress field and its singularities. The value of λ can be determined as the solution of following equation.

$$A_1 \lambda^2 + B_1 \lambda + C_1 = 0 \tag{10}$$

where

$$\begin{aligned} A_1 =& \alpha^2\beta^2 \left[\alpha^4(1 - \sin 2\gamma + \cos\gamma) + \beta^2 \sin 3\gamma\right] \\ & \left[\alpha\beta(\sin\gamma + \cos\gamma) + \alpha^2(\sin 3\gamma - \cos\gamma)\right] \\ B_1 =& \alpha^3\beta^2 \left[3\beta^3(1 - \sin\gamma\cos\gamma) - 11\alpha\beta^2(\sin 2\gamma + \cos\gamma) \right. \\ & \left. - 6\alpha^2(\sin\gamma - \cos 2\gamma)\right] \left[\beta^2(\sin\gamma + \cos\gamma) + 2\alpha\beta^2 \sin\gamma \right. \\ & \left. + 3(\sin\gamma - \cos 2\gamma)\right] \\ C_1 =& 6\alpha^2\beta^3 \left[(\sin\gamma - \cos 2\gamma)(4\alpha - \beta^2)\right] \left[\beta \sin\gamma \right. \\ & + \alpha\beta(\sin 2\gamma + \cos\gamma) + \alpha^2(\sin 2\gamma + \cos 2\gamma) \\ & \left. + 5(\sin\gamma + \cos\gamma)\right] + f(\alpha) \end{aligned}$$

and the singular field is obtained as

$$\begin{aligned} \tau_r =& 2\left[1 - \alpha(\sin\gamma + \cos\gamma)\right] F_2 (1 - \cos 2\theta) \log r \\ & + \left[1 + \alpha^2(\sin^2\gamma - 2\sin 2\gamma\cos\gamma)\right] F_3 (1 + \sin 2\theta) r^\lambda \\ \tau_\theta =& 2\left[1 - \alpha(\sin\gamma + \cos\gamma)\right] F_2 (1 + \cos 2\theta) \log r \\ & + \left[1 - \alpha^2(\sin^2\gamma - 2\sin 2\gamma\cos\gamma)\right] F_3 (1 + \cos 2\theta) r^\lambda \\ \tau_{r\theta} =& 2\left[\alpha(\sin\gamma + \cos\gamma) - 1\right] F_2 \sin 2\theta) \log r \\ & + \left[1 - \alpha^2(\sin^2\gamma - 2\cos 2\gamma\cos\gamma)\right] F_3 (1 + \cos 2\theta) r^\lambda \end{aligned} \tag{11}$$

Here, F_2 and F_3 in Eqs.(9) and (11) depend on Dunders' parameters, thermal expansion coefficient, temperature change, specimen shape and boundary conditions.

3. Dependence of Stress Field in Axisymmetric Dissimilar Materials on Elastic Constants

3.1 *Theoretical Consideration* Consider the axisymmetric dissimilar materials as shown in Fig.2 under an axisymmetric loading. In an axisymmetric problem, the stress components σ_{ir}, $\sigma_{i\theta}$, σ_{iz} and τ_{irz}, that are not zero, in material i are generally given, by using Michell's stress function ϕ_i, as

$$\begin{aligned} \sigma_{ir} &= \frac{\partial}{\partial z}\left(\nu_i \nabla^2\phi_i - \frac{\partial^2\phi_i}{\partial r^2}\right) \\ \sigma_{i\theta} &= \frac{\partial}{\partial z}\left(\nu_i \nabla^2\phi_i - \frac{1}{r}\frac{\partial\phi_i}{\partial r}\right) \\ \sigma_{iz} &= \frac{\partial}{\partial z}\left((2-\nu_i) \nabla^2\phi_i - \frac{\partial^2\phi_i}{\partial z^2}\right) \\ \tau_{irz} &= \frac{\partial}{\partial r}\left((1-\nu_i) \nabla^2\phi_i - \frac{\partial^2\phi_i}{\partial z^2}\right) \\ \nabla^2 &= \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \end{aligned} \tag{12}$$

and the displacement components u_i and w_i in r and z directions are given by

$$\begin{aligned} u_i &= -\frac{1+\nu_i}{E_i}\frac{\partial^2\phi_i}{\partial r \partial z} \\ w_i &= \frac{1+\nu_i}{E_i}\left\{(1-2\nu_i)\nabla^2\phi_i + \frac{\partial^2\phi_i}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_i}{\partial r}\right\} \end{aligned} \tag{13}$$

On the other hand, these quantities should be satisfied with the conditions of continuity on the interface expressed as

$$\begin{aligned} \sigma_{1z} &= \sigma_{2z}, & \tau_{1rz} &= \tau_{2rz} \\ u_1 &= u_2, & w_1 &= w_2 \end{aligned} \tag{14}$$

Therefore, four equations between ϕ_1 and ϕ_2 are obtained and, from these equations, the quantities related to the stress in material 1 can be expressed, by the corresponding quantities related to the stress in material 2, as

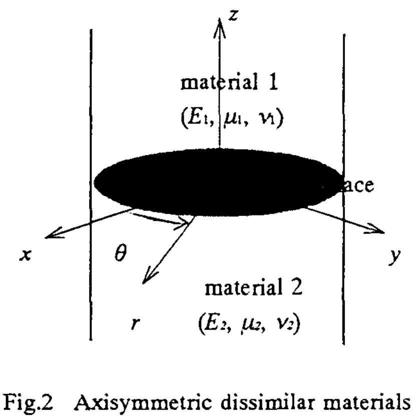

Fig.2 Axisymmetric dissimilar materials

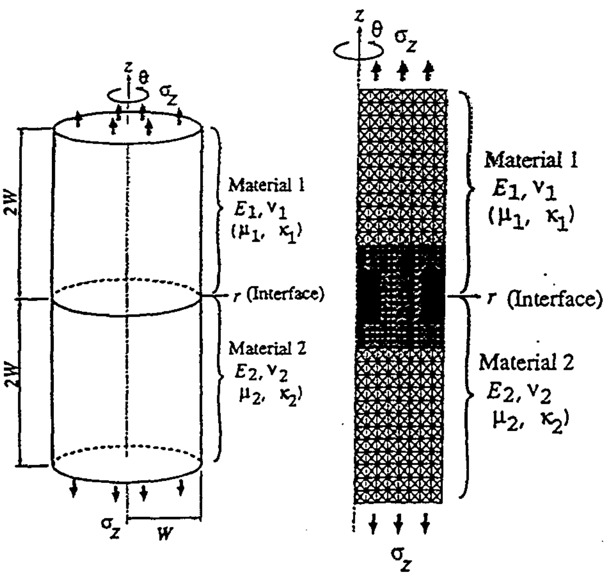

Fig.3 Axisymmetric dissimilar materials for analysis

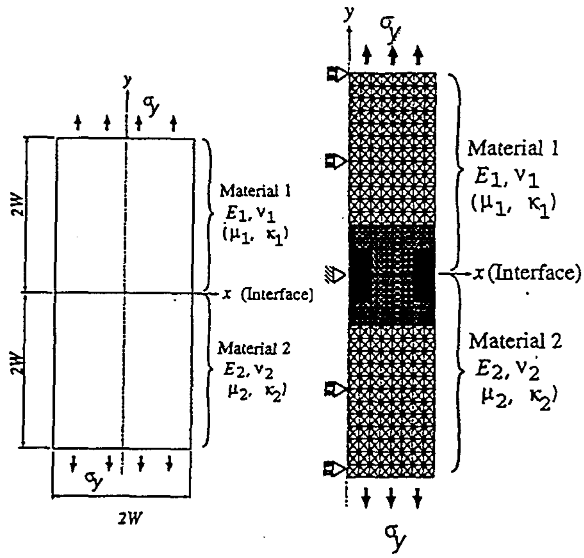

Fig.4 Two-dimensional dissimilar materials for analysis

$$\frac{\partial^3\phi_1}{\partial z^2\partial r}=\frac{2(1-\nu_2)(1-\Gamma)}{\Gamma}\frac{\partial^3\phi_2}{\partial r^3}+\frac{2(1-\nu_2)(1-\Gamma)}{\Gamma}\frac{1}{r}\frac{\partial^2\phi_2}{\partial r^2}+\frac{(1-2\nu_2+2\Gamma\nu_2)}{\Gamma}\frac{\partial^3\phi_2}{\partial z^2\partial r}$$

$$\Gamma\frac{\partial^3\phi_1}{\partial r^2\partial z}=\frac{\partial^3\phi_2}{\partial r^2\partial z}$$

$$\frac{\partial^3\phi_1}{\partial z^3}=\left\{\frac{2-\nu_2}{1-\nu_1}-\frac{2-\nu_1}{(1-\nu_1)\Gamma}\right\}\frac{\partial^3\phi_2}{\partial r^2\partial z}+\left\{\frac{2-\nu_2}{1-\nu_1}-\frac{2-\nu_1}{(1-\nu_1)\Gamma}\right\}\frac{1}{r}\frac{\partial^2\phi_2}{\partial r\partial z}+\frac{1-\nu_2}{1-\nu_1}\frac{\partial^3\phi_2}{\partial z^3}$$

$$\frac{\partial^3\phi_1}{\partial r^3}+\frac{1}{r}\frac{\partial^2\phi_1}{\partial r^2}=\left\{\frac{(2\nu_1-2\nu_1\Gamma+\Gamma)(1-\nu_2)}{\Gamma(1-\nu_1)}\right\}\frac{\partial^3\phi_2}{\partial r^3}+\left\{\frac{(2\nu_1-2\nu_1\Gamma+\Gamma)(1-\nu_2)}{\Gamma(1-\nu_1)}\right\}\frac{1}{r}\frac{\partial^2\phi_2}{\partial r^2}+\left\{\frac{\nu_1-2\nu_1\nu_2-\nu_2\Gamma+2\nu_1\nu_2\Gamma}{\Gamma(1-\nu_1)}\right\}\frac{\partial^3\phi_2}{\partial z^2\partial r} \tag{15}$$

where $\Gamma=\mu_2/\mu_1$. In the equation above, the quantities related to the stress in material 1 are connected with the quantities related to the stress in material 2 through three elastic constants mutually independent and this means that three parameters are necessary to characterize the stress field in axisymmetric dissimilar materials.

Table 1 Combination of Materials

	Γ	ν_1	ν_2
Standard (Si_3N_4/S45C)	0.6620	0.2700	0.3000
Combination A	0.6900	0.3580	0.3583
Combination B	0.6800	0.3275	0.3376
Combination C	0.6393	0.1938	0.2534

3.2 *Discussion based on Numerical Analyses* Finite element analyses were carried out in order to confirm the result above numerically. The model of axisymmetric dissimilar materials shown in Fig.3 was analyzed under uniform tension. Two-dimensional model in Fig.4 was also analyzed for comparison. As a standard, the combination of Si_3N_4 as the material 1 and S45C as the material 2 was adopted. Their Young's moduli and Poisson's ratios are 304.0GPa, 206.0GPa, 0.27 and 0.30, respectively, and α and β are 0.1832 and 0.0403, respectively, in this case. Moreover, the material combinations as shown in Table 1 were supposed. In these combinations, the values of α and β are the same as those of standard combination.

The distributions of the stress normalized by the stress for the standard combination are shown in Figs.5 and 6 with the distance from the interface edge. All the results for two-dimensional model in Fig.5 coincide well each other and this verifies the Dunders' theory[1] that the stress field in two-dimensional problem can be characterized by Dunders' parameters. However, the results for axisymmetric model in Fig.6 show that the stress distribution depends on the material combination, and this is considered to correspond to the theoretical consideration in the previous section.

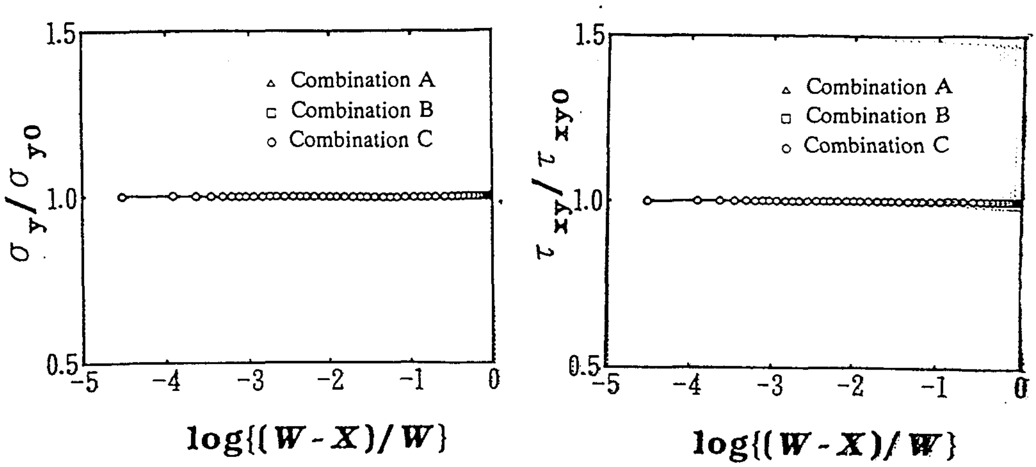

Fig.5 Stress distributions on the interface in two-dimensional model

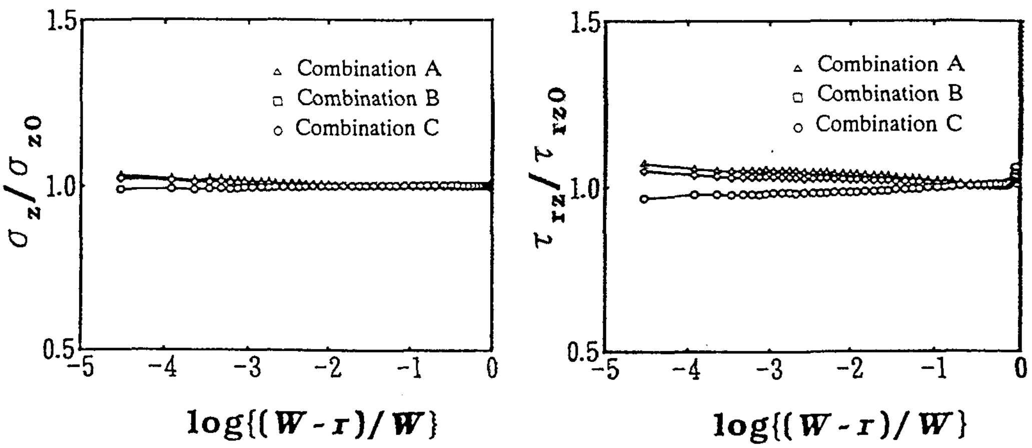

Fig.6 Stress distributions on the interface in axisymmetric model

4. Conclusion

Two-dimensional thermal stress singularities at the interface were studied and the concrete expression of stress field was derived. Moreover, the effect of elastic constants on stress field in axisymmetric dissimilar materials was discussed and it was shown that three parameters are necessary to characterize the stress in the problem different from in two-dimensional problem.

Reference

1) Dunders, J., J. Appl. Mech., 36(1969), 650.