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Simulation Procedure for Estimating the Reliability of a System with Repairable Units+

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Abstract

This paper propose a procedure to estimate the system lifetime distribution using simulation method in a parametric framework and also develop the criterion for terminating the simulation. We assume that a system is composed of many components whose lifetime and repair time distributions are general, and repair of each component is imperfect or not. General simulation algorithms can not be adopted for this case, due to the dependency of successive operating times and the discontinuity in base line intensity function of failure process. Then we propose algorithms for generating failure times subject to imperfect repair. We develop the event time tracking logic for identifying the system failure time, and also develop the criterion for terminating the simulation. Our procedure is composed of two phases. The first phase of the procedure is to generate the system failure times from the inputs. The second phase is to estimate the lifetime distribution of the system. The best model is selected by a fully automated procedure among well-known parametric families, and the required parameters are estimated. We give examples to show the accuracy of our procedure and the effect of repair effect of components to system MTTF(Mean Time To Failure).

1. Introduction

The objective of this paper is developing the simulation procedure for estimating the system lifetime distribution. A system is composed of many components which undergo failures and repairs as long as the system runs. When a component fails, repair begins immediately as long as the system runs. Repair of each component is either perfect or not. In this paper, repair is considered to improve the failed component. If repair has improved t years old component to new one, it is called perfect repair. Otherwise, it is called imperfect repair. A concept of imperfect maintenance was discussed by Nakagawa [9], Malik [8], and Lie & Chun [7]. Each time to failure depends on the previous one when the repair is imperfect.

In most cases, it is difficult to analyse the system reliability using exhaustive procedure. In particular, it is more difficult to derive the explicit form of the system reliability incase the system has a complex structure with many components, or when the state of components

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are operating or repairing alternatively. The simulation can be used without any additional degree of difficulty in analysing reliability for both Markovian and non-Markovian situations. This allows one to model extremely complex situations that are analytically intractable. Simulation is a powerful reliability evaluation method. But, when the repairs are imperfect, due to the dependency between operating times and the dynamic interaction among the states of components and the discontinuities in baseline intensity function of failure process, general simulation algorithms can not be adopted for this problem. A few articles have contributed to the simulation of nonhomogeneous Poisson processes in the presence of imperfect repairs. Several algorithms were proposed for the simulation of general nonhomogeneous Poisson process considering the imperfect repair. In these algorithms, it is needed to store all the occurrence probabilities of the previous events. If a system has many repairable components, a system failure can be a rare events. But, in this case, it is impossible to store all the occurrence probabilities for each component.

In this paper, we propose a procedure to estimate the system lifetime distribution using simulation method in a parametric framework and also develop the criterion for terminating the simulation. Proposed procedure can be used to identify the reliability characteristic of a system, and also it can be applied to validate analytical models.

The simulation method described in this paper is discrete event simulation method.

Our procedure is composed of two phases. The first phase of the procedure is to generate the system failure times from the input. The second phase is to estimate the lifetime distribution of the system. The best model is selected by a fully automated procedure among well-known parametric families, and the required parameters are estimated. The resulting distribution is presented in a handy format, so that it may be easily utilized to develop *effective maintenance programs for the system.*

Section 2 gives assumptions. And section 3 describes the simulation procedure for system failures, and the estimation procedure for the system lifetime distribution. The criterion for terminating the simulation is presented in section 4. In section 5, numerical examples are presented. Finally, our conclusions as well as future work are discussed in section 6.

2. Assumptions

1. Lifetime and repair time distributions of all components are given.
2. Each component can undergo imperfect repair. Improvement factors [7,8,9] of all components are given.
3. Each distribution is one of the parametric families in Table 1, and estimated lifetime distribution of the system is fitted to one of them.
4. Structure function of the system is given.
5. Whenever a component fails, repair begins immediately as long as the system is operating.

Notes:

1. The concept of improvement factor is described in Nakagawa [9], Malik [8], Lie [7]. A method of estimating the improvement factor is proposed in Shin [12].
2. In this paper, we adopt the optimal invariant selection(OIS) procedure as a procedure for distribution selection. OIS procedure was proposed by Quesenberry and Kent[10]. This procedure select the best fitting parametric class for a sample from exponential, gamma, Weibull, and lognormal distributions. For more detail, see [10].

Table 1. Event Time Generating Methods

Distribution	Generating Method		Result
expf(· ; β)	Inversion Method		$x = -\beta \cdot \ln u$
weif(· ; α, β)			$x = \beta \cdot [-\ln u]^{1/\alpha}$
gamf(· ; α, β)	$0<\alpha<1$	Generate Y using GS algorithm	$x = \beta \cdot Y$
	$1<\alpha$	Generate Y using GB algorithm	
lgnf(· ; α, β)	Generate Y using Polar method		$x = e^{Y}$

3. Simulation Procedure

Figure 1 shows the simulation procedure for estimating the system lifetime distribution. The simulation procedure consists of two major parts, simulator and estimator. Simulator generates system failure times. Simulator consists of event generator and event time tracking logic. Failure and repair times of each component are generated from given distribution by event generator. Failure and completion of repair are defined to be events. Event time tracking logic traces the events, and check the state of the system. System failure time is the elapsed time until the condition for the system failure is satisfied for the first time. Simulator runs until the criterion for terminating the simulation is satisfied. The lifetime distribution of the system is estimated from collect system failure times by estimator. Estimator selects the best one among parametric families, and estimates their parameters.

3.1 Event generator

Event times are generated by event generator. Event generator generates successive failure times x_{ik}'s and repair times y_{ik}'s (k=1,2,...) of component i until the system fails. The event times x_{ik}'s and y_{ik}'s are generated from given distribution $F_i(\cdot)$, $G_i(\cdot)$, respectively. In the imperfect repair model($\rho_i \neq 0$ case), failure rate function $\lambda(t)$ is adjusted between "good as new" and "bad as old" after completion of repair. In this paper, repair effect is assumed to reduce the age of a component. The amount of reduction is determined by the improvement factor ρ_i. Two algorithms are devised for the imperfect repair case. Algorithm 1 is used for generating Weibull random variates x_{ik}'s. Algorithm 2 is used for generating random variates whose distribution has no inverse function.

<u>Algorithm 1</u>

<u>initialize</u> k=1, Generate $u \sim U(0,1)$, $x_{ik} = \beta_i[-\ln(U)]^{\frac{1}{\alpha_i}}$, $t_{ik} = x_{ik}$

<u>step 1</u> $k \leftarrow k+1$, $\tau_{ik} \leftarrow (1-\rho_i)\cdot t_{i,k-1}$, $u_1 = F(\tau_{ik})$, Generate $u_2 \sim U(0,1)$

<u>step 2</u> $x_{ik} = -\beta_i[\ln\{(u_1-1)\cdot u_2 + u_1\}]/\alpha_i - \tau_{ik}$, $t_{ik} \leftarrow t_{i,k-1} + x_i$, go to <u>step 1</u>

<u>Algorithm 2</u>

<u>initialize</u> k = 1 Generate $x_{ik} \sim F_i(\cdot)$ using methods in Table 1, $t_{ik} = x_{ik}$

<u>step 1</u> $k \leftarrow k+1$, $\tau_{ik} \leftarrow (1-\rho_i)\cdot t_{i,k-1}$, Generate $s_{ik} \sim F_i(\cdot)$ using methods in Table 1

<u>step 2</u> If $s_{ik} > \tau_{ik}$, then $x_{ik} = s_{ik} - \tau_{ik}$, $t_{ik} \leftarrow t_{i,k-1} + x_{ik}$, go to <u>step 1</u>

else, repeat step 2

3.2 Event time tracking logic

The system failure can be occurred by failures generated by the event generator. It is difficult to identify the failure of a system with many components, because the state space of a system will be seriously large. For this reason, it is required to have a module for identifying the system failure in the simulator. And this module has to control the system clock in order to collect system failure times. It is event time tracking logic. The system clock time T denotes the time when the latest event happens. When the system fails, the system clock time is to be the system failure time. Time-advance mechanism in this simulation model is the next-event time advance mechanism. Event time tracking logic keeps up the clock time of each component T_i (i=1,2, .. , M). T_i is advanced as much as the next inter-event time of component i. A system failure can occur when the new event of a component, whose clock time T_i up to the new event is minimum among all T_i's, makes a transition from the operating state to the failed state. The algorithm used for generating system failure time is given below.

initialize For i = 1, 2, .. , M

$T_i = 0 \quad t_i = 0, \; r_i = 0$ (Set the state of each component to be operating)

Generate $x_i \sim F_i(\cdot)$ using methods in Table 1, $t_i = x_i \quad T_i = t_i$

Sort $\{ T_i, i=1, 2, .. , M \}$

step 1 $T = Min \{T_i, \; i=1, 2, .. , M\}$

If $r_j = 0$ for which $T_j = T$, then $r_j = 1$ and compute R value using $\phi(\bar{r})$

If R = 1, go to step 5 , else, go to step 2

else, go to step 3

step 2 Generate $y_j \sim G_j(\cdot)$ using methods in Table 1. $T_j = T_j + y_j$, go to step 4

step 3 $\tau_j = (1-\rho) \cdot t_j$, Generate x_j using algorithms in section 3.1.2

$t_j = t_j + x_j, \; T_j = T_j + x_j, r_j = 0$, go to step 4

step 4 Sort $\{ T_i, i=1, 2, .. , M \}$ by insertion of T_j, go to step 1

step 5 System failure time = T. Stop

3.3 Estimator

Estimator selects the best fitting parametric class for the simulated system failure time data from the exponential, gamma, Weibull, lognormal classes, and estimates its parameters. The selection procedure used in the estimator is optimal invariant selection(OIS) procedure.[10] Let F_i be a parametric class with scale parameter β, and let f_i be the density function corresponding to that member of F_i (i=1, 2, 3, 4). Since this procedure uses the scale invariant statistic S_i, scale parameters need not to be estimated, while shape parameters should be estimated. We use the maximum likelihood estimator(MLE). For each distribution, MLE's of the shape parameter and S_i's are given in [10].

4. Criterion for terminating the simulation

In general, it is required to determine the suitable run size of a simulation to guarantee the steady-state condition in simulation. In this section, we propose the criteria for terminating the simulation. In OIS procedure, shape parameters of gamma, Weibull, and lognormal distribution are estimated from system failure times. In the steady state, each process of estimated parameters would be a covariance stationary process. We propose a sequential procedure utilizing the stationarity of { $\hat{\alpha}_{ij}$, $i = 1, 2, 3, 4$, $j=1, 2, \ldots, N_0/m$ } as a criterion for terminating the simulation. In this procedure, the length of a simulation sequentially increases until an acceptable confidence interval can be constructed and uncorrelated MLE's for all candidated distributions can be obtained. Figure 2 shows the procedure for determining the simulation. We propose the autocorrelation of MLE $\hat{\alpha}_{ij}$'s and the relative precision as the criteria for terminating the simulation. The criteria are given below. The usual lag 1 estimators based on first N/m and last N/m batches with batch size m are noted by $\hat{\rho}_{i1}^{\,1}$ and $\hat{\rho}_{i1}^{\,2}$. $t(N/m-1,(1-c)/2)$ is t-value with significance level c and degree of freedom (N/m-1), and S is sample standard deviation of $\hat{\alpha}_{ij}$'s.

$$\bar{\rho}_{i1} = 2\cdot\hat{\rho}_{i1} - [\hat{\rho}_{i1}^{\,1} + \hat{\rho}_{i1}^{\,2}]/2, \quad \bar{\alpha}_i = \sum_{j=1}^{N/m} \hat{\alpha}_{ij}/(N/m)$$

$$\hat{\rho}_{i1} = \sum_{j=1}^{N/m-1} (\hat{\alpha}_{ij} - \bar{\alpha}_i)(\hat{\alpha}_{i,j+1} - \bar{\alpha}_i) / \sum_{j=1}^{N/m} (\hat{\alpha}_{ij} - \bar{\alpha}_i)^2$$

$$\delta_i = \left| [t(N/m-1,(1-c)/2)\cdot S/\sqrt{N/m}] \,/\, \bar{\alpha}_i \right|$$

5. Examples

In order to see whether the simulation procedure proposed in this paper is appropriate for estimating the system lifetime distribution, we present several examples.

[Example 1] The lifetime distributions are estimated for a system with the series structure of 2 parallel structures with 2 components. All $F_i(\cdot)$'s are expf($\cdot$;1) and all $G_i(\cdot)$'s are expf($\cdot$;1/10). Figure 3 plots the mixture of 10 simulated pdf's and exact pdf. A visual inspection shows that the estimated pdf is closely approximated to exact pdf.

[Example 2] The MTTF's for a system with parallel structure of 3 components are estimated 100 times for each case. All $F_i(\cdot)$'s are weif($\cdot$; 0.5, 0.5) with mean 1, and all $G_i(\cdot)$'s are expf($\cdot$;1/10). Figure 4 plots 95% confidence interval(CI) of system MTTF. Figure 4 shows that the system MTTF is geometrically increased as the ρ's of components increase.

[Example 3] For a system with the structure given in figure 5, the system lifetime distribution is estimated. Table 2 shows the estimation results. All $F_i(\cdot)$'s are given in table 2. For all i, $F_i(\cdot)$'s are identical and their mean is fixed to 1. All $G_i(\cdot)$'s are expf($\cdot$;1/10). The system has 8 components and 7 gates. Basic event denotes failure of a component. Component 6 is a redundant component.

Table 2. Estimated results in example 3.($N_0=2000$, $m=50$, $\gamma=0.1$, $c=0.05$)

Component	ρ_i	N	System pdf	MTTF
weif(.; 0.5, 0.50)	1	3450	weif(.; 0.5166, 0.0263)	0.0496
	0.5	3850	weif(.; 0.5199, 0.0261)	0.0487
	0	2050	weif(.; 0.5167, 0.0256)	0.0483
weif(.; 1.5, 2.03)	1	3800	weif(.; 1.5620, 0.8016)	0.7204
	0.5	7000	weif(.; 1.5424, 0.7805)	0.7025
	0	6650	weif(.; 1.5557, 0.7798)	0.7011
weif(.; 2.0, 3.55)	1	2050	weif(.; 2.0780, 1.8038)	1.5978
	0.5	2050	weif(.; 2.1152, 1.7988)	1.5931
	0	3500	weif(.; 2.1129, 1.7789)	1.5755
weif(.; 2.5, 5.55)	1	2100	weif(.; 2.7015, 3.2546)	2.8943
	0.5	2950	weif(.; 2.7052, 3.2069)	2.8520
	0	7000	weif(.; 2.6416, 3.1813)	2.8270
weif(.; 3.0, 8.04)	1	2100	weif(.; 3.3188, 5.1827)	4.6503
	0.5	3200	weif(.; 3.2587, 5.0662)	4.5416
	0	2050	weif(.; 3.3048, 5.0479)	4.5284
gamf(.; 0.5, 2.00)	1	3450	weif(.; 0.5604, 0.0661)	0.1094
	0.5	5550	weif(.; 0.5736, 0.0671)	0.1074
	0	7000	weif(.; 0.5622, 0.0650)	0.1071
gamf(.; 1.5, 0.67)	1	7000	weif(.; 1.3945, 0.3623)	0.3304
	0.5	2050	weif(.; 1.4169, 0.3639)	0.3310
	0	4200	weif(.; 1.4232, 0.3573)	0.3248
gamf(.; 2.0, 0.50)	1	2050	weif(.; 1.6779, 0.4422)	0.3949
	0.5	7000	weif(.; 1.7471, 0.4402)	0.3921
	0	7000	gamf(.; 2.5029, 0.1564)	0.3915
gamf(.; 2.5, 0.40)	1	3450	weif(.; 1.9376, 0.5059)	0.4487
	0.5	2100	weif(.; 1.9889, 0.5015)	0.4445
	0	4000	weif(.; 2.0156, 0.4970)	0.4404
gamf(.; 3.0, 0.33)	1	2050	gamf(.; 4.2393, 0.1113)	0.4718
	0.5	7000	gamf(.; 4.4769, 0.1053)	0.4714
	0	6700	gamf(.; 4.6043, 0.1016)	0.4678

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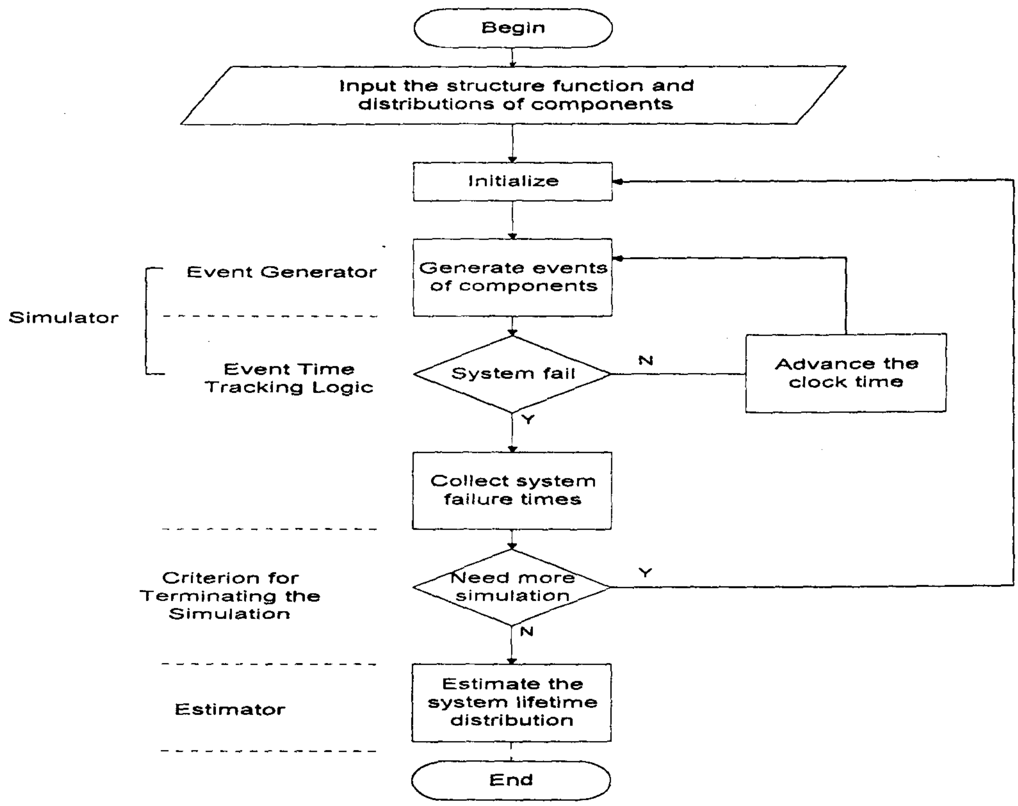

Figure 1. Simulation Procedure

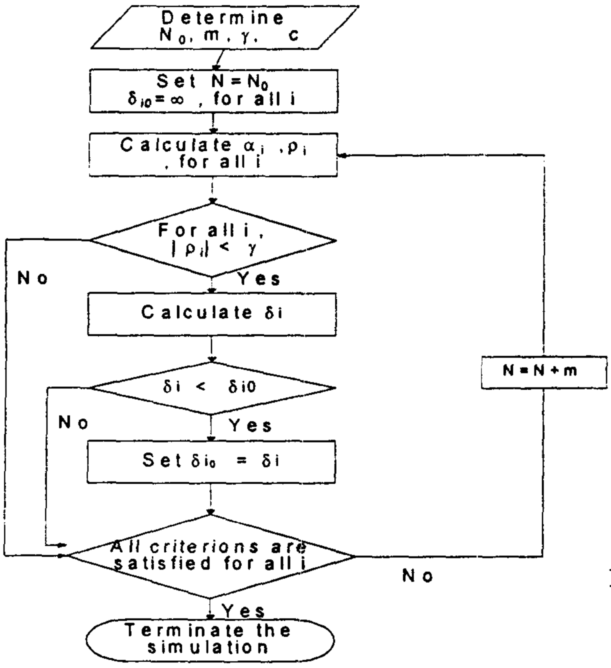

Figure 2. Sequential Procedure for terminating simulation

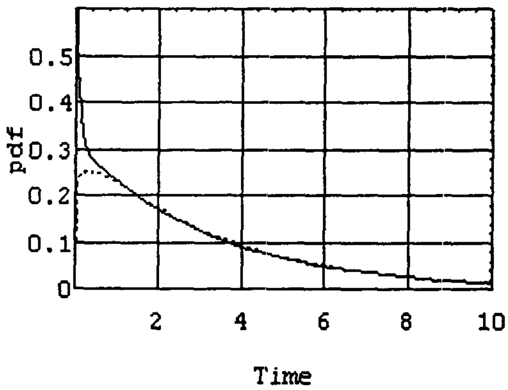

Figure 3. Comparison of estimated pdf with exact one in example 1

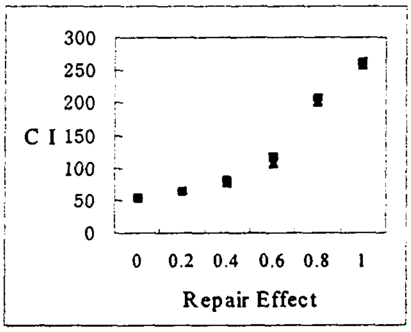

Figure 4. 95% Confidence Interval of System MTTF for different ρ in example 2

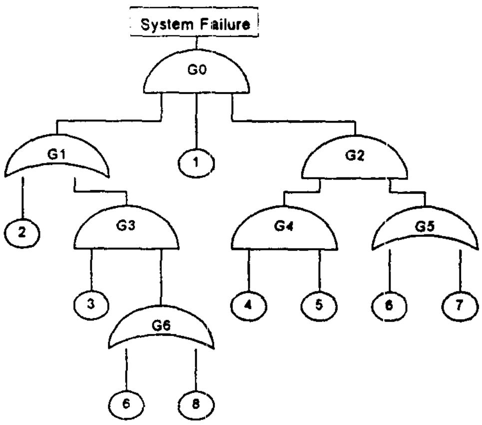

Figure 5. Fault tree for example 3